Report on the master's thesis of Bc. Šárka Stejskalová

Title: The tree property at more cardinals

Supervisor: Dr. Radek Honzík

The tree property arises from the study of the existence of "tall but narrow" trees of height ω and ω_1 by König and Aronszajn at the beginning of the 20th century, respectively. Later this concept has been abstracted and extended to all regular cardinals. Nowadays they are called κ-Aronszajn trees, and κ is said to have the tree property if there are no κ-Aronszajn trees.

In her thesis Stejskalová presents the main known results on the tree property at successor cardinals, from classical by now results of Baumgartner, Mitchell, and Silver from 70's and 80's till the most recent ones of Friedman, Honzik, Unger, etc. This is a very interesting area of set theory with really heavy methods: most of the main results presented in the thesis involve the combination of the large cardinal combinatorics, forcing, elementary embeddings, and lifting thereof. This material goes beyond any logic courses I can imagine, which means that Stejskalová has a great ability to work through a difficult staff on her own. The presentation of the results is friendly towards a reader: she adds many small but important details to the proofs (like, e.g., in Theorem 5.19), at the same time some premises are weakened in order to highlight the main ideas of the proofs, see, e.g., Theorem 5.35. This shows her deep understanding of the material.

The initial 4 sections form an introduction for the fifth one, which is the core of the thesis. They contain several useful facts extending classical results at ω to all regular cardinals. Among them are Lemmata 2.21, 2.23, 2.24, 3.27, and some others. Special mentioning deserves paragraph 4.3, where the Grigorieff forcing is applied to the study of the tree property at κ^{++} for measurable κ. As far as I am concerned, Theorems 4.44 and 4.45 are *new*. Even though their proofs are patterned after those of corresponding results of Kanamori achieved with help of the Sacks forcing, I find them to be very *interesting*. This is because the application of the Grigorieff forcing might help to obtain in addition some new properties like inequalities between cardinal characteristics, projective wellorders, etc.

The only tiny criticism I have is that there are misprints and inaccuracies in a couple of the formulations (e.g., λ_i should be Mahlo and not just inaccessible in Theorem 5.22). But these are inessential and do not prevent from understanding what was really meant.

Based on the above, I conclude that Bc. Stejskalová deserves the Master's degree in Logic and suggest the best possible grade for her thesis.

Sincerely,
Lyubomyr Zdomskyy