Abstract

In this thesis we study the Aronszajn and special Aronszajn trees, their existence and nonexistence. We introduce the most common definition of special Aronszajn tree and some of its generalizations and we examine the relations between them. Next we study the notions of the tree property and the weak tree property at a given regular cardinal κ. The tree property means that there are no Aronszajn trees at κ and the weak tree property means that there are no special Aronszajn trees at κ. We define and compare two forcings, the Mitchell forcing and the Grigorieff forcing, and we use them to obtain a model in which the (weak) tree property holds at a given cardinal. At the end, we show how to use the Mitchell forcing to construct a model in which the (weak) tree property holds at more than one cardinal.