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Filozofická Fakulta Universita Karlova v Praze

Review
of the Ph.D. Thesis
“Logical Foundations of Fuzzy Mathematics”
submitted by Mgr. Libor Běhouněk

Almost the last two decades now have seen a tremendous development in infinite-valued logics over the real unit interval as truth degree structure, and quite often with a commutative semigroup operation – particularly a (left) continuous t-norm – together with their residuation, understood as non-idempotent conjunction and implication operations. These logics are commonly referred to as (t-norm based) mathematical fuzzy logics because they arose out of investigations toward the foundations and applications of fuzzy set theory, and they proceed in the style of traditional mathematical logic.

So it is quite natural to ask to which extent these logics now can suitably be used to develop in a systematic manner mathematical theories based upon these logics. And it is just this question the author tries to solve, or better: to attack – because a complete solution of this question is impossible by the very nature of this question.

With his approach he revitalizes older approaches toward some kinds of systematic developments of fuzzified mathematical theories – approaches which also this reviewer years ago had been working on. The actual advantage over these older approaches is the existence of well developed mathematical fuzzy logics. And this allows for a unification and also a suitable generalization of these older approaches, done here by the author. However, and that should be said immediately together with these remarks, the author also extends these older approaches - e.g. toward fuzzy classes, fuzzy relations, or fuzzy topology - with interesting new general results and notions. And he offers also new approaches in further fields, not discussed previously; e.g. for erotetic logic.

Of course, one can generally proceed in such a way: but I would prefer to avoid to call the result of such an approach "fuzzy mathematics". The reason is that such an approach gives nothing but a not really well motivated generalization of notions and approaches from classical mathematics. Fuzzy mathematics, on the other hand, if

this term should have any reasonable meaning, should arise out of the needs of applications of fuzzy sets techniques, and of the mathematical reflections about such

applications. In the very early stages of such an approach it is completely open whether a later systematization of such a kind of fuzzy mathematics just yields "fuzzifications" (in the authors sense) of classical mathematical theories, or whether one should do foundations in other ways - even if it is natural to assume that for the case of a specification of the underlying logic to the case of classical logic such fuzzy mathematical theories should reduce to known areas of classical mathematics.

Besides this question of naming, the author is well aware of these problems of his approach: and he reflects upon them nicely and critically in his introductory remarks, with focus on the core points. This I welcome explicitly, it shows that the author is not only able to do logic and mathematics, he is also able to reflect critically what he is doing. That not everybody will share his points of view, this is quite normal in this more methodological realm.

The initial parenthetical remarks, contained in his "Thesis description" part, put the following collection of papers, which essentially constitutes this thesis, into a more general perspective. They show nicely that the author is aware of the limitations of his approach, they also prove that the author well knows the actual state of the art in fuzzy logic and in the mathematical approaches toward application related uses of fuzzy sets, and that he also knows well the main lines of development which constitute the past of the topic. They also show, in the parts which discuss future work, that he is well aware that important aspects even for the expressive powers of mathematical fuzzy logics are still unclear at present.

It is not necessary to sketch the contents of the papers which the author has collected into this thesis. They all have been peer reviewed and accepted for well established international journals and conferences.

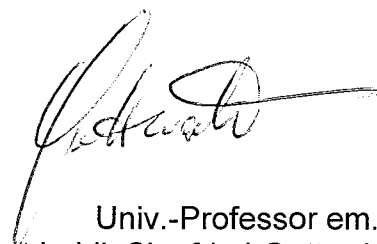
In any case, the formal results which the author presents here are correct and interesting in themselves. And they provide a satisfactory framework of the kind of unified presentation of fuzzified mathematical theories he is interested in, i.e. for doing "fuzzy mathematics" inside the frame set by mathematical fuzzy logics.

Summing up, there is a wealth of interesting and, I guess, important ideas and results which the author presents in this thesis.

The present thesis shows not only that the author has well mastered the field the work intends to contribute to, it also shows that the author is able to solve open problems within this field and to extend it in interesting directions with new results and approaches.

I propose to the Filozofická Fakulta of the Charles University of Prague to accept this work as a doctoral thesis, and I evaluate it with the predicate *magna cum laude*.

Leipzig, June 16, 2009



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