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Review of the thesis "Variational strategies in material sciences: Analysis & Numerics" by Mgr. Antonín Češík

For centuries, continuum mechanics has posed complex mathematical challenges, with many fundamental problems remaining unresolved despite ongoing efforts from the scientific community. Evolutionary problems, in particular, represent one of the most difficult areas within this field. The difficulties stem from various physical and mechanical constraints imposed on mathematical models in large-strain settings, including the nonconvexity of the set of admissible deformations, the requirement for injectivity of deformations, and principles like frame-indifference. Nonetheless, variational methods, grounded in energy-based approaches, have allowed significant progress. A. Češík has notably extended and further developed a two-time-scale method initially designed by B. Benešová, M. Kampschulte, and S. Schwarzacher for fully dynamic problems in viscoelasticity and fluid-structure interactions.

This thesis is based on one published paper and three submitted manuscripts (available as preprints), co-authored by Antonín, along with an introductory section. The first two works focus on modeling and analyzing dynamic contact (collision) of viscoelastic bodies. The first paper, published in Calculus of Variations and Partial Differential Equations, assumes that the reference domain has a $C^{1,\alpha}$ boundary. This is then extended, in a nontrivial way, to domains with Lipschitz boundaries. The contact zone is not known a priori, and the contact force arising from the non-interpenetration condition is represented as a vector-valued measure over the Cartesian product of the process time interval and the boundary of the reference configuration.

The third manuscript addresses stepwise time approximations for nonlinear hyperbolic initial value problems. The primary applications are in nonlinear elastodynamics. The evolution follows a variational structure that is exploited through stepwise minimization. For a wide class of elastic energies, the manuscript shows that the scheme is stable, even accommodating high-order nonlinearities. When the highest-order terms are assumed to be linear, it is proven that the limit solutions are regular and that the minimizing movements scheme converges with an optimal linear rate. This provides a foundation for the further development of fully discrete numerical schemes based on the two-time-scale method for hyperbolic problems with applications to FEM.

The final manuscript addresses a fluid-structure interaction problem, where slip boundary conditions are allowed on the interface between the fluid and the structure. Specifically, only the non-interpenetration condition is imposed, requiring that the normal component of the velocity be zero. The authors prove the existence of weak solutions and show that sufficiently regular weak solutions are strong ones.

The thesis is well-written, with only a few minor typographical errors in the introductory section. It is standard to refer to **materials science** when discussing the physical subject matter.

Finally, I would like to ask two questions: *Could you elaborate on Remark 2.3 in the first paper, where you discuss the transition between the Dirichlet and Neumann parts of the boundary? Are there situations in which one might demonstrate better regularity of the contact force?*

All four works contain significant results deeply extending our current knowledge and have a strong potential to initiate many follow-up results.

In my assessment, A. Češík's work unquestionably meets all the requirements for a doctoral thesis and fully merits the awarding of a Ph.D. degree.

Martin Kružík