

**Abstract:**

This doctoral thesis focuses on synthetic geometry in various dimensions. We start with plane geometry to show how synthetic geometry can be used in proofs. We demonstrate the advantages of synthetic geometry on two different geometric proofs of the Pappus–Pascal theorem, the construction of the osculating circles of an ellipse at any point of the ellipse and the graphical solution of a quadratic equation. Moreover, the thesis describes visualisation of the  $n$ -dimensional space using the "behind" view method and perpendicular layering. Furthermore, the thesis focuses on visualisation of 4-dimensional space. It describes two possible methods: a generalisation of Monge's projection (orthogonal projection onto two mutually perpendicular subspaces) and a generalisation of linear perspective. Finally, the thesis contains applications of the visualisation of 4-dimensional space. For example, the usage of the generalised Monge's projection and 4-dimensional perspective for representation of the complex number plane. The application for visualisation of shadows in 4-dimensions synthetically and algebraically, and for representation of 4-dimensional objects using 3D printing.