



*Review of the Habilitation Dissertation of Dr. Souček*

In accordance with the Czech Act on Higher Education Institutions, this review has been conducted with the standards expected of the rank "Associate Professor".

The thesis concerns the mathematical modelling and simulation of crystalline ice, restricted to the conditions of relevance for near-surface ice sheets at planetary scales. This context is of broad interest, both from the perspective of understanding the large glacial structures on our own planet (in particular the Antarctica and Greenland ice sheets), but also of relevance for several other planetary objects in our solar system. In particular, the author applies his method to understand problems of relevance on Mars, Saturn's moon Enceladus and Jupiter's moon Europa.

The introduction to the thesis is nicely written, and provides an excellent introduction to mathematical modelling of glacial dynamics.

The thesis studies three main topics in detail.

- 1) The thesis addresses the problem of the shallow ice approximation for gravity-driven glacial flows. In particular, the thesis applies a hybridization between low- and high-order shallow ice approximations, wherein successive approximations (iterations) within the shallow ice approximation framework are used to approximate the full 3D solution. This approach, which is part of the author's PhD thesis work, will have a computational cost equal to the shallow ice approximation whenever this approximation is sufficiently accurate, and will require a modest computational overhead when somewhat increased accuracy is needed. Such iterative approaches on lower-dimensional solution spaces are very valuable in many applications and the author justifies the applicability also in the setting of gravity-driven glacial flows. In this habilitation, the method is applied in two different directions: Firstly to shed light on the quality of results obtained with various methods on the SMIP-HEINO benchmark, and secondly to gain understanding of the possibility of glacial history within the Isidis Planitia on Mars.
- 2) The second part of the thesis leaves the shallow ice approximations, and considers the full 3D deformation of ice sheets, in the context of gravitational tidal waves. This deformation is associated with substantial friction, leading to the production of heat. A major contribution in this direction is the development of a finite element simulation


code (utilizing the FEniCS library) allowing for large-scale computations to be conducted. It is understood that the author's primary role in this collaboration was related to this code development and the subsequent numerical simulations. Unfortunately, as often happens in such interdisciplinary collaborations, the details of the numerical implementation is only summarized in the supporting information of papers P3 and P5, and it would have been good to see this topic further elaborated in one of the papers.

- 3) Finally, the thesis considers ice melt, which as the author points out, shares mathematical similarities with mantle melt on earth. This is a complex topic, combining important phase change effects with the poromechanical deformation of the solid/fluid structure. Again, the direction of the thesis is to exploit the possibilities of numerical modelling and simulation to gain insights into the processes that may be important on distant planetary moons.

As a personal interest, I would have liked to see that the thesis would explore in more depth the limitations of the chosen approach. In particular, I have concerns whether the great uncertainties that exist related to model and domain simplifications, model parameterizations and numerical simulation are fully acknowledged. Certainly, the experience from earth is that forecasting of complex geophysical processes is challenging, even in the presence of significantly more detailed parameters, and calibration data, than in the applications considered in this thesis.

Nevertheless, the thesis is comprehensive, and presents a broad contribution to various aspects of numerical simulation in the context of glacial dynamics in our solar system. The included papers display great creativity in finding intriguing observations within its stated scope, and exploring the relevance of glacial dynamics on the observed phenomena. In my assessment, there is no doubt that, the quantity, quality and originality of the presented work is consistent with the level of associate professor. As such, my assessment is that the habilitation thesis is of high scientific quality and should be accepted.

The thesis has been checked by a plagiarism checking tool, which reveals nothing unusual. In my own assessment, there is no reason to doubt the originality of the material contained in the thesis.

 Jan M. Nordbotten