

REVIEW BY THE OPPONENT OF THE MASTER THESIS

**Thesis title:** Products of Boolean Clones up to Minion Homomorphisms  
**Thesis author:** Radek Olšák

SUMMARY OF THE THESIS CONTENT

The author consider a generalization of clones (called multi-sorted clones) where instead of operations we consider tuples of operations of the same arity and composition is applied coordinatewise. He studies boolean multi-sorted clones that are defined by specifying a usual boolean clone for every coordinate. The goal of the thesis was to understand the lattice of such multi-sorted clones modulo minion homomorphism. Recall that the lattice of all single-sorted boolean clones modulo minion homomorphism is known and this result was the groundwork for the thesis.

The author consider two preorders  $A \leq B$  and  $A \preceq B$ : the first one comes from the existence of a minion homomorphism, and the second one is a weaker version where we only require for every coordinate of  $B$  the existence of a minion homomorphism from some coordinate of  $A$ . Surprisingly, the authors prove that this weaker version is equivalent to the original in this concrete case. To prove this, first the author describes the lattice modulo the preorder  $\preceq$  and then by showing concrete identities proves that  $A \preceq B$  implies  $A \leq B$ .

Thus, the main result of the thesis is a complete characterization of such boolean multi-sorted clones modulo minion homomorphism. The lattice turned out to be rather simple and the author draws it in the thesis.

On the negative side, the thesis contains quite a few typos and inaccuracies:  $X$  and  $Y$  in Lemma 1.32, the beginning of Observation 1.39,  $n$  and  $i$  in Definition 2.1,  $j$  and  $i$  are switched in the last line of page 11, binary negation of  $x$  in Observation 2.5, and many others. Also, sometimes articles are missing, for instance before minimum and maximum.

Probably the main disadvantage of the paper is that we still do not know whether the fact that two preorders coincide is a local fact that holds only for boolean clones or it is true in general. For instance, it would be great to have a counter example for larger domains. Also, it would be great to have a universal method to build distinguishing identities as it is hard to see regularity in the identities presented by the author.

OVERALL EVALUATION OF THE THESIS

**Thesis topic.** The question considered in the thesis is a very natural next step in studying the lattice of clones modulo minion homomorphism.

**Author's contribution.** The main contribution of the author is a surprising discovery that these two preorders coincide and finding concrete identities proving that  $A \preceq B$  for all the required cases.

**Mathematical level.** Mathematical level of the paper is high, the proofs seem to be correct, and the definitions are precise. Nevertheless, a few things were confusing to me:

- What is the  $\min(X_1^1, X_2^1, \dots)$ ? It is written that this is the infimum of the chain but I don't see why it exists unless some other properties of  $\leq$  for single-sorted clones are used.
- $R$  sometimes means a relation, sometimes a set of relation.

- Definition 1.29. It was strange to me that one identity may contain many equalities but not independent equalities.
- Definition 1.11. I would be more careful introducing the notation  $A^n \rightarrow B$  for  $A = (A_1, A_2, \dots, A_k)$ , as it is not a set.

**Work with sources.** The citations seem to be correct.

#### CONCLUSION

I consider the thesis to be very good and I recommend it to be accepted as a master's thesis.

*The classification will be communicated by the opponent to the chair of the examination committee.*

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