Evaluation Report of the Doctoral Thesis by Barbora Bezdeková

In this thesis, the propagation of light rays in the limit of geometric optics in curved spacetimes is studied for a variety of dispersive media, including non-magnetic cold plasma, which forms the basis of much of the discussion. The studies are carried out for a wide range of stationary and axially symmetric metrics and include the study of shadows in plasma environments, allowed and forbidden regions for light rays, and their dynamics in moving dispersive media. The thesis is original and I consider that it presents relevant results that significantly contribute to the understanding of electromagnetic radiation propagation in dispersive environments. The results presented in the thesis have led to 4 articles that have been published in a prestigious international journal (Physical Review D) and another that has been submitted to the Journal of Mathematical Physics. The thesis is divided into six chapters. The first one provides a very detailed review of the propagation of light rays in optical media following Synge's formalism. This is not commonly found in the literature, and the presentation is very pedagogical and detailed. Chapter 2 is, from my point of view, one of the main chapters of this thesis, as it presents the conditions for a spacetime containing a dispersive medium to allow the separability of the Hamilton-Jacobi equation, which enables finding generalized Carter-like constants. The next chapter shows various examples of the application of these techniques, including a hairy Kerr metric and the Thorne-Hartle metric. Additionally, a comparison is made between the results in the different spacetimes. Chapter 4 shifts the focus to analyze moving dispersive media (and non-dispersive as a particular case), which generally influences the dynamics of light rays in a non-trivial way. As an example, more general dispersive media than those describing non-magnetic cold plasma are discussed, as well as a general rotating dispersive medium in the equatorial plane of a spherically symmetric spacetime. This type of analysis is original and shows the differences with the propagation of light rays in non-magnetic cold plasma, where the fluid movement does not influence the propagation of light rays. Finally, in the last two chapters, the propagation of light rays in cold plasma in the Kerr spacetime for equatorial plane orbits is discussed.

The thesis is well-structured and coherently written. The bibliography is adequate and complete. The results presented are valuable and contribute to the understanding of the propagation of light rays in curved spacetimes when a dispersive medium is present.

I consider that the thesis is in a condition to be defended and that the candidate Barbora Bezdeková deserves the PhD title.

I only have a few questions that can be answered on the day of the defense.

1) Regarding the discussion in Chapter 2 about Carter-type constants, it is mentioned how this is related to the existence of a Killing tensor, which is associated with hidden symmetries of the metric. This is related to the geometric part of the constant. The other contributions to the Carter-like constant come from the presence of certain non-magnetic cold plasma profiles and do not seem to come from a geometric object. My question is whether it is possible to construct some geometric object (a Killing tensor or a Killing-Yano tensor) associated with an optical metric related to the spacetime and plasma that gives rise to the Carter-like constant. I believe the answer is affirmative at least for non-dispersive media,

where one can construct a Gordon optical metric, allowing light rays to follow null geodesic orbits. In the case of spherical symmetry, it is even easy to demonstrate the existence of a Killing-Yano tensor associated with such an optical metric, and surely something similar can be done for axially symmetric spacetimes for particular non-dispersive media. But for dispersive media, the answer does not seem so clear to me. Would a similar approach be possible using for example Finsler-Randers type metrics?

2) Regarding the discussion in Chapter 4 about moving dispersive media and their influence on light propagation, is there any known medium with these characteristics that could be used to characterize astrophysical systems?

3) Regarding the discussion in Chapter 5 about forbidden regions in Kerr spacetime, do you consider it possible to conduct a similar analytical discussion at least for nearly equatorial orbits (i.e., where the angular coordinate θ varies but with values close to pi/2)? How sensitive would the stability of these orbits be to different plasma profiles (especially due to their theta dependence)? Could anything be said about which forms of general profiles would lead to light ray orbits diverging from the equatorial plane?

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