Review by the opponent of the bachelor thesis	
Thesis title:	Two-dimensional integer trigonometry
Thesis author:	Anna Růžičková

SUMMARY OF THE THESIS CONTENT

The thesis deals with integer trigonometry in two dimensions, which is a lattice analogue of Euclidean geometry on planes. The thesis contains basic results concerning lattices, and an introduction of notions in integer trigonometry, for example the integer trigonometric functions $\ell\sin$, $\ell\cos$, $\ell\tan$, $\ell\arctan$, as well as the the lattice length sine (LLS) sequence. Main results established in the thesis includes an integral analogue of the sine rule (Lemma 2.35), a criterion for the integer congruence of rational angles in terms of the LLS sequence (Theorem 2.38), as well as a criterion for the existence of an integer triangle with given integer angles (Theorem 3.7). There are also ample examples throughout the thesis which illustrate the introduced concepts.

OVERALL EVALUATION OF THE THESIS

Thesis topic. The difficulty of the thesis is appropriate for a bachelor's thesis. While the topic builds on little prerequisites (mostly just basic group theory), it requires considerable dexterity to manipulate the various notions in integer trigonometry, which are often defined in a convoluted way.

Author's contribution. The author's contribution includes formal definitions of the basic notions and tools concerning lattices and integer geometry, providing proofs that such notions are well-defined where necessary. The author also vastly expanded and rewritten the proofs of the main results as well as the preliminary results; most of the proofs in the source [2] consist of just brief sketches. Furthermore, the author illustrated the notions and theorems with clearly presented figures. The author's contribution is clearly declared in the introduction of the thesis.

Mathematical level. The mathematical content is correct, and the definitions and the proofs are presented in a rigorous way.

Work with sources. The sources are correctly cited, and the thesis does not contain verbatim copied passages.

Formal editing. The presentation is largely clear, and is easily understandable. There are almost no typos, and in any case they do not affect the presentation.

COMMENTS AND QUESTIONS

1. Proof of Lemma 2.27, (a)⇒(b): How does $\triangle ABC$ being empty imply the parallelogram spanned by AB and AC is empty?

CONCLUSION

I consider the thesis to be excellent and I recommend that it be accepted as a bachelor's thesis. I recommend the grade 1.

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