The aim of the thesis is to give complete and thorough proofs of some well-known results from the measure theory. Oftentimes, arguments from functional analysis will be used to prove these results. For example, we will use an enhanced version of Schur's theorem to prove the Nikodym theorem and the Vitali-Hahn-Saks theorem. Then we will focus on the weak compactness in L_1 and we will present a proof of the Biting Lemma and its corollary Rosenthal's subsequence splitting lemma.