Review Report on the Thesis "Maximal Non-Compactness of Operators and Embeddings" by Anna Kneselová

Introduction

The thesis "Maximal Non-Compactness of Operators and Embeddings" by Anna Kneselová delves into the advanced topic of the ball measure of non-compactness $\alpha(T)$ for various embedding operators in sequence spaces, with a particular focus on Lorentz sequence spaces. This work is rooted in functional analysis and aims to elucidate the conditions under which identity operators can be considered maximally non-compact. The thesis is structured into three main sections: the first section introduces the necessary notation and definitions, the second section investigates the maximal non-compactness of embedding operators in sequence spaces, and the final section, which is the main part of the thesis, focuses on the detailed study of Lorentz sequence spaces.

Structure and Content

The thesis is well-organized and logically structured. The initial section lays the groundwork by defining essential concepts such as norms, quasinorms, α-norms, compact operators, and the ball measure of non-compactness. This section ensures that the reader is well-equipped with the foundational knowledge required to understand the subsequent sections.

The second section of the thesis examines the maximal non-compactness of identity operators in classical sequence spaces. Here, Kneselová attempts to provide necessary and sufficient conditions for these operators to be maximally non-compact. This section is characterized by good exposition and clear writing, except for the aforementioned incorrect statements.

The third and most substantial section focuses on Lorentz sequence spaces $\ell_{p,q}$. It investigates the inclusions $\ell_{p_1,q_1} \hookrightarrow \ell_{p_2,q_2}$ depending on the parameters p and q. The norms of identity operators between these spaces are described with proofs, and the conditions for maximal non-compactness are obtained (Section 3.2). Examples of cases where the embeddings are not maximally non-compact are also mentioned (Section 3.3).

Strengths

1. Clarity: - The thesis is notable for its clarity in defining complex concepts and its rigorous approach to proving theorems in Section 3.

2. Mathematical Depth: - The thesis exhibits a good understanding of functional spaces and analysis. The rigorous proofs and detailed exploration of maximal non-compactness reflect Kneselová's grasp of the subject matter.

Areas for possible improvement/further continuation

I think that this work could be a good foundation for possible master thesis.

1. Proposition 2.6 During reading I've spotted that in the recent form Proposition 2.6 is not true. See this contra-example:

Let l be the space equipped with the norm:

$\|y^i\|_l := \left(\sum_{i=1}^{\infty} |y^i|^p\right)^{1/p}$ *(i.e., $l = l^p$), and let w be the space equipped with the norm* $\|(y^i)\|_w := \left(|y^1|^p + \frac{1}{2}\sum_{i=2}^{\infty} |y^i|^p\right)^{1/p}$ *(i.e., a weighted sequence space). Clearly, we can see that for the embedding $I : l \to w$, we have $\|I\| = 1$, and that conditions 1-3 from Proposition 2.6 are satisfied. It is straightforward to see that in this case $\alpha(I) = 1/2$, i.e., I is not maximally non-compact.*

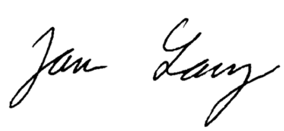
It seems that, perhaps, some conditions gets missing during typing.

2. Further continuation:

I would like to recommend that the author consider continuing the study of non-compactness. Investigating non-compactness in spaces beyond locally convex spaces, as well as related problems involving strictly singular maps, could serve as an intriguing and valuable topic for a master's thesis. Delving into these areas could provide deeper insights and contribute to the broader understanding of functional analysis. Additionally, exploring the implications of non-compactness in various applications and connecting these theoretical concepts with practical problems could enhance the significance of the research. Pursuing this line of study has the potential not only to yield interesting results but also to lay a strong foundation for future academic and professional endeavors in the field of mathematics.

Conclusion

Anna Kneselová's thesis is well-typed, and it is evident that she has studied and mastered the concepts of functional analysis related to non-compact operators and embeddings in sequence spaces. The work is characterized by its clarity and good structure, and the exposition on Lorentz sequence spaces (Section 3) is particularly well done. Simply the work goes over and beyond requirements expected from BA thesis.



POSUDEK VEDOUCÍHO/OPONENTA BAKALÁŘSKÉ PRÁCE

Název: Maximal non-compactness of operators and embeddings
Autor: Anna Kneselova

Shrnutí obsahu práce

Viz příloha.

Celkové hodnocení práce

Téma práce. Viz příloha.
-Náročnost je přiměřená a odpovídající. Zpracování velmi dobré. či až na problémy zmíněné
-Téma bylo zpracováno přiměřeně zadání.

Vlastní příspěvek. Práce obsahuje zobecnění některých existujících tvrzení a obsahuje dobrý souhrnem výsledků které jsou jinak rozptýlené a často nekompletně zmiňované v literatuře.

Matematická úroveň. Matematická úroveň práce je velmi dobrá.

Práce se zdroji. Zdroje jsou dobře citované. Nezdá se, že by práce byla kopii existující prace.

Formální úprava. Velmi dobrá

Připomínky a otázky

1. Najít chybu v důkaze Prop. 2.6 a modifikovat Prop. 2.6.

Závěr

Bohužel neznám standarty na MFF UK pro BA thesis proto se zdržím hodnocení ale dle mého názoru práce jde jasně nad standarty které by mohly být očekávané pro BA práce. Pokud autorka dokáže adresovat mojí připomínku pak by tato práce měla být dobrý kandidát na “The Highest Grade”.

Návrh klasifikace vedoucí/oponent sdělí předsedovi zkušební (sub)komise.

Oponent: Jan Lang (The Ohio State University), June 14, 2024

Jan Lang

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Date: 2024.06.15 14:55:38 -04'00'