Peer review of bachelor thesis
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Hausdorff and Capacitary Dimensions

The thesis under review defines the Hausdorff and the capacitary dimension, and proves that in the setting of the Euclidean space they coincide. The material is compiled from several sources and is sufficiently rich for the author to demonstrate his mathematical writing skills.

The thesis consists of three sections, first focuses on the definition and properties of the Hausdorff measure, second defines the Hausdorff dimension and third introduces the capacitary dimesion and a proof that it is in the Euclidian space equivalent to the Hausdorff dimension. It is a question if the first section is necessary, since the Hausdorff measure is a well known object and is covered in several graduate level courses. On the other hand, the third section might be expanded by definining terms such as the dyadic cubes or explaining in detail on which spaces is the weak star convergence realized.

Overall, the author showed that he can work with multiple sources and write a good mathematical manuscript. Unfortunately, the thesis has multiple problems. Besides few minor typos and problematic formulations, there are several more serious errors, which we list at the end of this review. However, despite these shorcomings I believe that the thesis exceeds the standards of the bachelor thesis and I recommend it to be accepted.

List of problems:

1) In the introduction, the author presents image of Sierpinski triangle, but never defines it or returns to the concept. It was possible to use this or similar fractal set to demonstrate the properties of the capacitary dimension.

2) Theorem 1.12 is wrong, the collection $\mathcal{F}$ is not a Vitali covering, first it does not cover the set A and second it does not have the Vitali property, therefore it cannot be Vitali covering of any set. Should be a disjoint system $\mathcal{F}$.

3) Last paragraph on page 12 and the footnote 2. The author claims that it is possible to cover a general open set by disjoint open dyadic cubes. This is not true, the correct formulation is that it may be covered by closed dyadic cubes with disjoint interiors. The dyadic cubes are object of fundamental importance comparable with for example the Vitali lemma and should have been defined separately and their properties explained.

4) In the section 1.2.1, the author first states, that the symmetrization will be defined for a compact set, however in the definition 1.13, he attempts to do it for a general set. It is not a typo, as in lemma 1.14 he works first with general set and then with a measurable set. It is not possible to define the symmetrization for a general set, as the intersections with line segments might not have defined measure. Even if A was a measurable set, some segmets still might not be measurable or have infinite measure and the definition would need to address this.

5) In proof of the Lemma 1.11, first formula, the author uses notation $S_j(b)$. However, S_j is defined two lines above as a set function, while b is a point. Moreover, I do not understant what would be the correct replacement.

6) In Lemma 1.15 the author claims that $\mathcal{F}'$ is a covering, the system $\mathcal{F}'$ does not cover the set A. Moreover, in the proof, on the first line under the second formula uses $\mathcal{F}$ instead of $\mathcal{F}'$.

7) On page 18, in the last formula, the second and third sum will be often sums over uncountable set and therefore infinity. This needs to be reformulated.

8) In Lemma 3.6, the author makes a claim about weak star convergence without first clarifying on what space he works and what is its predual. This confusion gets worse, as he presents a variant of Riesz representation theorem for positive functionals, while the weak star convergence works with continuous functionals. This discrepancy should have been explained in detail.

9)In the Banach-Alaoglu theorem 3.7, X should be normed *linear* separable space. It is not clear, however, if this if a good version of the theorem. The predual space is never fully specified, but from the context it seems to be C_c, which is not normed. Therefore it would be better to introduce version of Banach-Alaoglu for topological linear spaces.

10) In the proof of theorem 3.13, in the proof of the converse implication the author assumes that A is compact, but never explains how to pass to a general Borel set.

11) In the proof of theorem 3.13, after formula (6), should be weak star convergence, with the space and its predual clearly identified.

12) In the bibliography [2] should be P.R. Halmos, while [6] should have the name of the author.

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