REPORT OF OPPONENT OF MASTER'S THESIS

Title: Domain decomposition methods for the solution of partial differential equations using discontinuous Galerkin method
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SUMMARY OF THE WORK

This thesis studies an application of the domain decomposition method (DDM) to the discontinuous Galerkin (DG) finite element method for a linear partial differential equation. For the DDM method the thesis uses a two-level additive Schwarz method with a coarse and fine solver steps. The thesis focuses on the effect of the mesh (both fine and coarse) size, polynomial degree, and PDE coefficients on the condition number of the matrices resulting from the methods. First a summary of existing analysis on the condition numbers, along with certain modifications necessary for a slightly different formulation considered in this thesis, is presented. Then, numerical experiments are performed to try to validate these results.

This is an interesting analytical and numerical problem. The mathematical background work is good and well structured. The numerical results are detailed and useful, despite the fact they do not perfectly validate the analysis.

OVERALL EVALUATION OF THE WORK

Thesis topic. The topic is interesting and suitable for a thesis. It has been completed to a very good level.

Author's contribution. The author has summarised existing analysis, and made modifications to a few lemmas and theorems to allow for a different formulation considered in this thesis. They have also performed a number of numerical experiments to attempt to validate the analytical results.

Mathematical level. The mathematical content is excellent and well structured. The first chapter summaries the DG formulation, basic analysis, and bounds on the condition number of the resulting symmetric matrix, for a second order linear partial differential equation with a SPD coefficient matrix with upper and lower bounds. The second chapter then describes a DDM method using this formulation, and then derives the bounds for the condition number of the resulting matrix. The analysis follows existing analysis, but modifications are made to several results for a different formulation. The mathematical content is a good balance of existing work and modifications, and the amount of detail for existing results is good. These chapters do seem to contain a few errors, which are potentially typographical errors. Despite this the mathematical content is very good.

The numerical experiments are fairly good. They contain experiments with linear PDEs which conform to the analysed PDE, as well as nonlinear problems which do not conform but are then linearised in order to be solved iteratively. The main problem with the experiments is that to do not seem to confirm the analytical results, but suitable explanations are given as to why this may have occurred.

Sources. The sources are good and complete.

Formal preparation. The general structure of the thesis is good, although the level of English requires some work. There are several spellings errors (missing letters), missing articles, and some sentences which do not make sense. In general, it is fine, but it appears as if the thesis has not been properly proofread.

COMMENTS AND QUESTIONS

1. In Corollary 12 the denominator contains h_K^4 — should it not be h_K^2? Furthermore, how exactly does this follow from Theorem 11?

2. It seems the thesis just states that *extension operators* R_i^T for mapping from the DG space on the DDM subdomains to the DG space on the whole mesh exist and some of the properties of them. How are this defined in practice; e.g., what is used in the numerical experiments?

3. In the numerical experiments, plots are provided showing the effect of the condition number as the polynomial degree or coarse mesh size changes. For some the rates appear correct; however, the exact growth is not stated or compared to the expected; e.g, for the result in Figure 3.1 the condition number appears to grow with respect to the polynomial degree, but does it rise at the same rate as expected? For the cases where the rate looks correct can you state the actual growth rate and how it compares to the expected rate?

CONCLUSION

I **recommend** this thesis as a Master's Thesis

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