Thesis Review

Faculty of Mathematics and Physics
Charles University, Prague

❑ Supervisor's Review	☑ Referee's Review **
❑ BSc. Thesis	☑ MSc. Thesis

Author:	Hryhorii Ovcharenko
Thesis title:	Horizons of type D black holes
Study program:	Physics - Theoretical physics
Submitted:	2024

Supervisor/Referee:	Prof. Hideki Maeda
Affiliation:	Hokkai-Gakuen University, Sapporo, Japan
E-mail:	h-maeda@hgu.jp

Thesis quality (technical field and expertise):
☑ excellent ❑ very good ❑ standard ❑ substandard ❑ nonconforming

Objective accuracy (error appearance):
☑ nearly perfect ❑ standard ❑ frequent, but minor ❑ serious errors

Thesis results:
☑ original ❑ both original and compiled ❑ productive compilation ❑ copied

Thesis size:
❑ large ☑ standard ❑ just acceptable ❑ insufficient

Thesis quality (style and grammar, and graphic arrangement):
☑ excellent ❑ very good ❑ standard ❑ substandard ❑ nonconforming

Misprints:
☑ negligible ❑ acceptable number ❑ very frequent

Overall thesis quality:
☑ excellent ❑ very good ❑ standard ❑ substandard ❑ nonconforming

** Copy and paste this check box, ☑, if it is applicable.

Supervisor's/Referee's Comments:

The main subject of this master's thesis is the Killing horizon in the Petrov type-D exact solution in the four-dimensional Einstein-Maxwell-Lambda system, in particular the Plebanski-Demianski (PD) solution. It is an important class of solutions, including the Kerr-Newman solution describing an asymptotically flat rotating charged black hole. In the coordinate system in which the metric of such a solution is simple, Killing horizons are often coordinate singularities. Examples of such coordinates are the standard diagonal coordinates for the Reissner-Nordstrom solution, or the Boyer-Lindquist coordinates for the Kerr solution. However, in such a singular coordinate system, one cannot correctly calculate physical quantities on the horizon, and regular coordinates covering the horizon must be used to perform a correct analysis. Here it should be noted that both the metric and its inverse must be regular, which has sometimes been overlooked in the literature.

Examples of regular coordinates on the horizon are the single-null, Lemaitre, and Painleve-Gullstrand coordinates for the Schwarzschild solution. For the Kerr solution, the Doran coordinates are regular coordinates and reduce to the Painleve-Gullstrand coordinates in the limit of no rotation. A generalization of the Doran coordinates to the Kerr-Newman solution has been done by Hobson in 2023. However, a further generalization to the more general type-D solutions has been an open problem until now. Such regular coordinate systems on the horizon that describe the metric with elementary functions will be useful in future research. This master's thesis gives a clear answer to this problem.

This master's thesis consists of six chapters, with Chapter 6 being the conclusion. Chapter 1 is an introduction where the author first reviews the Petrov classification and the PD solution. Then he defines the regularity of spacetime in section 1.5 and classifies asymptotic behaviors of a stationary and axisymmetric metric around a regular horizon in a coordinate system where the horizon is singular. Subsequently, the author reviews the Doran coordinates of the Kerr solution in section 1.6 and outlines the thesis in section 1.7.

Chapter 2 reviews the most general Petrov type-D double-aligned solution in the Einstein-Maxwell-Lambda system, namely the Debever-Kamran-McLenaghan (DKM) solution. It is a huge class of exact solutions that consists of the expanding PD solution, non-expanding Kundt solutions, and direct-product solutions, among which the PD solution is of interest in this thesis. Sections 2.3 and 2.4 review the limits of the DKM solution depending on the parameters.

Section 3.1 discusses the regularity of horizons in the PD solution in the singular coordinate system using the asymptotic behaviors of the metric given in Section 1.5. Section 3.2 discusses the problem of conical singularities of this solution. Chapter 4 thoroughly clarifies the parameter dependence of the degeneracy of the horizon of the PD solution using the singular coordinate system at the horizon.

Chapter 5 is the main result of this thesis, which generalizes the Doran coordinates for the Kerr-Newman to the PD solution in several ways. The Doran coordinates for Kerr-Newman correspond to the motion of a geodesically moving massive particle, called a raindrop, and in particular, the time coordinate coincides with the proper time of the raindrop. The author generalized the derivation of the Doran coordinates for Kerr in a clever way and successfully obtained a regular coordinate system on the horizon for the PD solution. The Doran-type regular coordinates for the Kerr-Newman-Taub-NUT-(A)dS solution (5.1) are given by (5.76). In addition, the author derived two more coordinates (5.93) and (5.122) corresponding to the more general massive particle motion. For the most general PD solution (5.94) with acceleration, the regular coordinates on the horizon are given by (5.161), which reduces to (5.76) in the limit of zero acceleration.

This master's thesis provides horizon-penetrating coordinates for the most general PD solution, which is a significant extension of the previous result for Kerr-Newman by Hobson. In this thesis, all the complicated coordinate transformations are presented, which allows the reader to follow each step completely. The review sections are well organized and the bibliography is sufficient. It might be easier for the reader to find the main results of this thesis if the author uses appendices to put some of the complicated equations and show only the most essential equations of the transformations in the main text. In summary, this thesis is well written and produces very non-trivial results, which make a solid contribution to the study of general relativity. The standard of this master's thesis is very high and entirely sufficient to award a master's degree.

Questions raised (and to be answered by the author during the Thesis Defence):

In section 1.5.1, the author defines the regularity of spacetime by the following two conditions:
1) Finiteness of curvature invariants, and
2) Finiteness of the components of the Riemann tensor in an orthonormal frame that coincides with a free-falling (geodesic) particle.

The condition 1 is equivalent to the absence of scalar polynomial (s.p.) curvature singularities. Different from s.p. singularities, a parallelly propagated (p.p.) curvature singularity is defined by the divergence of the components of the Riemann tensor in a parallely propagated (p.p.) frame along a curve (not necessarily a geodesic). (See Hawking-Ellis p.260 for the definitions.) Because a s.p. singularity is a p.p. singularity, by its contraposition, if a spacetime point is not a p.p. singular, then it is not s.p. singular neither. As the author's condition 2 does not require a p.p. frame, it is not equivalent to the absence of p.p. curvature singularities. My first question is whether the orthonormal frames in section 1.5.2 are parallely propagated or not along the corresponding geodesics under consideration.

The author studied the regularity of the horizon in section 1.5.2 in the coordinate system which is singular there. However, I think it would be better to check the correctness using the regular coordinate system just to be sure because the regularity can be shown in a simple and mathematically rigorous way in such a coordinate system. In a second-derivative gravitation theory such as general relativity, a spacetime point is regular if the metric is $C^{1,1}$ there (often written as C^{2-} in physics), namely the metric is at least C^1 and the second derivative may be discontinuous and allows a finite jump. If so, no divergence appears in the field equations, metric, curvature tensors, and matter fields. At the end of section 3.1, the author claims that all horizons (defined by Q=0) of the PD solution (3.6) are regular, independent of the specific expressions of the Q and P functions. This claim seems to be too strong. For example, for $Q(r)=(r-r_h)^{1/2}$, the first derivative of the metric diverges at $r=r_h$ which must cause a curvature singularity there. It clearly shows that differentiability of the metric is important to define regularity.

Based on this observation, my second question is the differentiability of the PD metric on the horizon in the Doran-type regular coordinate systems (5.76) and (5.161). I am sure that it is at least C^2, and it could be analytic (C^w, the Taylor expansion is possible).

Incidentally, I found the following typos:

- In (5.33) and (5.34), $\delta(r)$ should be $\delta(\theta)$.
- $\rho^2(\theta)$ in (5.45) and (5.46) (and many others after that) should be $\rho^2(r,\theta)$. (I understand that $\rho^2(T)$ there means $\rho^2(r_s(T),\theta_s(T))$.)
- The references [7] and [23] are identical.

Supervisor's/Reviewer's recommendation on Thesis rating:

☑ excellent ☐ very good ☐ standard ☐ reject

Done in ....Sapporo, Japan........

Date ..1st June, 2024.....

Name .... Hideki Maeda.........

Signature ... 前田秀基 ...