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Essays on Tail Risks, Asymmetries, and Cross-Section of Asset Returns

Dissertation thesis

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Declaration of Authorship

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Prague, February 21, 2024

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Abstract

This thesis consists of three papers investigating asymmetric risk, especially downside risk, in empirical asset pricing. The first paper introduces quantile spectral beta, a measure of risk that quantifies the dependence between stock return and a risk factor in a particular part of the joint distribution over a given horizon. We apply the new measure to study the pricing of tail market risk and extreme volatility risk across asset classes. The second paper proposes a new factor pricing model. Instead of focusing on common factors that explain the cross-sectional mean or variance of stock returns, we propose a model that captures the common structure of cross-sectional quantiles. We show that the downside factors possess information relevant to predicting market returns. Moreover, exposure to the downside factors is compensated for in the cross-section of stock returns for U.S. firms. The third paper examines whether different measures of systematic asymmetric risk lead to risk premia because they represent linear exposure to the common factor structure or because of their nonlinear properties. Using instrumented principal component analysis, we show that these measures can be efficiently combined to generate abnormal returns that other factors cannot explain. However, some measures can also be used to capture linear exposures better.

JEL Classification	C21, C23, C58, G11, G12
Keywords	Cross-section of asset returns, factor structure, asymmetric risk, downside risk, tail risk, frequency, quantile

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Abstrakt

Tato disertační práce se skládá ze tří článků, které zkoumají asymetrické, zejména downside, riziko v kontextu empirického oceňování aktiv. První článek představuje kvantilovou spektrální betu, což je míra rizika, která kvantifikuje závislost mezi výnosem akcií a rizikovým faktorem v konkrétní části společné distribuce v daném horizontu. Tuto novou míru používáme ke studiu ocenění chvostového tržního rizika a extrémního rizika volatility napříč třídami aktiv. Druhý článek navrhuje nový oceňovací faktorový model. Místo zaměření se na běžné faktory, které vysvětlují průřezový průměr nebo rozptyl výnosů akcií, my navrhujeme model, který zachycuje společnou strukturu průřezových kvantilů. Ukazujeme, že dolní faktory obsahují relevantní informace pro predikci výnosů trhu. Navíc je expozice vůči dolním faktorům kompenzována v průřezu výnosů akcií u firem z amerického trhu. Třetí článek zkoumá, zda různé míry systematického asymetrického rizika vedou k rizikovým prémiímm, protože představují lineární expozici k běžné struktuře faktorů nebo kvůli jejich nelineárním vlastnostem. Pomocí instrumentuované analýzy hlavních komponent ukazujeme, že tyto míry lze efektivně kombinovat ke generování abnormálních výnosů, které nelze vysvětlit jinými faktory. Nicméně některé míry lze také využít ke zlepšení zachycení lineárních expozic.

Klasifikace JEL	C21, C23, C58, G11, G12
Klíčová slova	Průřez výnosů aktiv, asymetrické riziko, riziko poklesu, chvostové riziko, frekvence, kvantil

Název práce	Studie o chvostových rizicích, asymetriích a výnosech aktiv

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Chapter 1

Introduction

This dissertation thesis consists of three papers focusing on asset pricing questions with a particular focus on non-linear risks. The first paper introduces a new risk measure that simultaneously captures two important dimensions of dependence risks between a stock return and a risk factor–part of their joint distribution and the horizon over which the dependence manifests itself. The second paper investigates the pricing implications of a quantile factor model that captures the dynamics of the cross-sectional quantiles of stock returns. The last paper regards systematic asymmetric risk measures and their relation to the linear factor models. The first two papers are an outcome of the collaboration with my supervisor, Jozef Baruník, who is also co-author of these papers. The third paper is solo-authored by me. Therefore, in these two papers, I stick to "we" when referring to the authors. On the other hand, in the third paper, I use "I" when referring to the author.

In this chapter, I will proceed as follows: Firstly, I position the presented investigations within the asset pricing setting. Secondly, I provide a short summary of each paper. Finally, I conclude with a brief review of the related literature.

1.1 Asset Prices

Asset pricing aims to clarify why certain assets generate greater returns than others. The basic concept states that assets with higher risk demand higher expected returns and thus possess lower prices for investors to hold them. While this relationship theoretically encompasses a basic model linking risk to the covariance of consumption growth and asset returns, the empirical results do not support this prediction. Many efforts are exercised to reconcile this observation.

A significant portion of asset pricing research is based on two simplifying assumptions. The first assumption states that the representative economic agent maximizes her expected utility. This notion is compelling for various reasons, such as the analytic tractability of model solutions. This assumption yields an intuitive prediction that the price of an asset is equal to its expected discounted payoff using the *stochastic discount factor*. On the other hand, such restriction may understate the significant aversion to losses compared to gains across investors.

The second assumption builds on the first by stating that some set of tradable risk factors can linearly approximate the stochastic discount factor. This statement is very convenient because it predicts that the cross-section of returns can be fully explained by assets' betas – regression coefficients from time-series regression of stock returns on the set of selected pricing factors, such as value-weighted market portfolio in the case of Capital asset pricing model.

Although these assumptions yield valuable insights into the asset prices, they open new related questions. For example, over which horizon should we measure the risk captured by the betas? This question has to be answered empirically. We may ask whether maximization of the expected utility adequately captures market risk preferences. Alternatively, we may question the linearity assumption of the stochastic discount factor and argue that higher-order moments and interactions play an essential role in determining the marginal utility growth that underlies the notion of the stochastic discount factor. Finally, we may investigate whether we should fully exploit potential pricing information in the traditional common factors instead of focusing on finding new pricing factors corresponding to various stock characteristics.

We aim to address these questions in various degrees throughout the thesis. A unifying thread in the presented research is that we try to understand multiple deviations from traditional asset pricing models that impose these simplifying assumptions that the data do not support. Generally, we improve the risk quantification associated with priced information in the included papers, emphasizing asymmetric and non-linear risks.

We tackle these tasks using various non-standard tools. We employ approaches from machine learning, such as dimension reduction techniques. We utilize the current quantile regression-based apparatus for factor modeling and dependence measurement. We even implement instruments from frequency-domain econometrics.

1.2 Paper Summaries

In Chapter 2 – *Quantile Spectral Beta: A Tale of Tail Risks, Investment Horizons, and Asset Prices*[1] – we propose a novel measure of risk capturing two essential features that were not previously investigated in their joint setting. The quantile-spectral (QS) beta determines the dependence between asset return and a risk factor in a specific part of their joint distribution over a specific horizon. Our main objective is to measure extremely negative market and stock behavior over short and long horizons. We motivate the investigation by examining the dependence structure between the market factor and two anomaly portfolios over their lead/lag structure. We show that the dependence is especially significant in the joint left tail. We would miss such a complex joint behavior by looking at the usual covariance.

We utilize the QS betas to analyze two risks: tail market risk and extreme market volatility risk. When investigating the tail market risk, we especially focus on capturing additional information over the assumptions that lead to the classical CAPM beta. Based on that, we work with the relative version of the QS beta that is defined as a difference between freely estimated QS beta and QS beta estimated by imposing the assumption of jointly correlated white noises.

When pricing the cross-section of individual stocks, we show that investors price the short-term component of the tail risk and the long-term component of the extreme volatility risk. Additionally, when pricing various portfolios or asset classes, the results indicate that these risks are priced heterogeneously, providing investors with suggestions for optimal investment decisions based on their risk preferences. Moreover, we show that the pricing implications of the QS betas are robust to the inclusion of other related measures of risk.

We also provide an asymptotic theory for the proposed measure. In simulations, we show estimation properties of the cross-sectional relationship with expected returns. In addition, we relate the QS betas to the model of Nakamura et al. (2013) and argue that QS betas can uncover complex dynamics present in financial markets and thus help decide which model best aligns with reality.

[1] This paper was published in the *Journal of Financial Econometrics* (Baruník and Nevrla 2022).

We conclude that solely focusing on contemporary dependence averaged over the whole distribution when assessing risk leaves out important information regarding asset prices.

In Chapter 3 – *Common Idiosyncratic Quantile Risk*[2] – we introduce common factors that capture quantiles of the cross-sectional distribution of asset returns. We show that these factors, estimated using the Quantile factor analysis of Chen et al. (2021), possess vital implications for expected returns. More specifically, we find that the stock market prices the exposures to the common quantile factors aligning with the cross-sectional left-tail of stock returns. These findings expand upon prior research that establishes a connection between asset prices and the exposure to common factors representing the *average* of cross-sectional stock return distributions.

We provide a battery of robustness checks and control for vast risk characteristics previously discovered to predict the cross-section of stock returns. We account for various general risk measures, such as market beta, idiosyncratic volatility, skewness, etc. We control for the effect of state-of-the-art competing systematic asymmetric risk measures, including tail risk beta of Kelly and Jiang (2014), multivariate crash risk of Chabi-Yo et al. (2022), or predicted coskewness of Langlois (2020). We also include various other stock characteristics, e.g., size, book-to-price, illiquidity, and turnover.

We also relate the quantile factors to the predictability of the market return. We show that the left-tail quantile factors are associated with the marginal utility growth in the economy as these factors reliably predict the equity premium. We provide vast evidence that these results cannot be attributed to other previously discovered phenomenona claiming time-series predictability of the market return. Overall, we show that there are factors that capture common downside risk with robust asset pricing information.

We provide approaches to aggregating information from downside or upside factors in both time-series and cross-sectional predictability cases. The corresponding results provide additional evidence that only the downside factors yield valuable information for asset prices. These findings show that the economic agents especially value their aversion to common adverse events that occur on the market and do not especially care for the common upside potential.

When investigating downside and tail risk in financial markets, it is reasonable to ask whether extreme events cannot be attributed to time-varying volatility. Both papers aim to mitigate this possibility. In Chapter 2, we mea-

[2]This paper is currently in the revise & resubmit phase in the *Review of Finance*.

sure the extreme volatility risk in addition to the tail market risk and show that they provide distinct pricing information. Moreover, we show that regarding the volatility risk, the long-horizon component is priced in the cross-section of stock returns. On the other hand, tail risk possess pricing information for stock returns in its short-term component, only.

In Chapter 3, we demonstrate that the factors that capture cross-sectional variance do not produce a significant risk premium. In addition, we conclude that the upside quantile factors lack any pricing implications, either cross-sectional or time-series. In a simulation study, we show that if it were the case that the common volatility drives the asset prices, both downside and upside quantile factors would deliver symmetrical conclusions. Because in the empirical application we show that this is not the case, we conclude that the volatility does not drive our results.

In Chapter 4 – *Asymmetric Risks: Alphas or Betas?* – I examine how systematic asymmetric risk measures (ARMs) relate to linear factor models that are ubiquitous in the empirical asset pricing literature. I investigate whether a set of systematic risk measures that capture various non-linear features of stock return behavior can be efficiently exploited to yield abnormal risk-adjusted returns. I motivate the task by showing that the significance of the risk premiums associated with these measures vary sizable across research settings.

I propose using the instrumented principal component analysis of Kelly et al. (2019) to focus on the risk premium associated with these measures beyond exposures to common risk factors. I let the model decide whether each risk measure helps capture systematic risk relation or anomaly premium. I form pure-alpha portfolios based on the relation between risk measures and anomaly returns. The resulting portfolios generate an improved performance that cannot be achieved using single measures. Moreover, these portfolios enjoy abnormal returns that other factor models cannot span.

Results also suggest that the pure-alpha returns are related to the momentum factor. This effect does not subsume the corresponding abnormal premium when we use the instrumented principal component analysis to form the portfolios. When employing an alternative approach based on a short estimation window that allows for time-varying risk relations, the loss of efficiency leads to the momentum factor taking over. I show that this fact is related to the observation that the ARMs significantly proxy for the exposure to the momentum factor.

I also provide evidence that some of the investigated ARMs can significantly proxy for the exposure to the common factors in a model that assumes linear factor structure without mispricing. This relationship remains significant even when including 32 variables investigated in Kelly et al. (2019).

1.3 Literature Review

The presented research aligns with multiple strands of the empirical asset pricing literature. Below, I present a brief summary of their current developments. As these streams are currently highly researched, the presented review does not aim to be fully exhaustive. Instead, I highlight just a few essential references that constitute significant results.

The first related research agenda aims to capture investors' preference heterogeneity regarding upside and downside risks and their respective effects on expected returns. From a theoretical point of view, an essential step outside the expected utility paradigm is de Castro and Galvao (2019), who propose a dynamic model of an economic agent that maximizes the stream of quantile of future utilities instead of their expected value.

Based on an intertemporal asset pricing model with disappointment aversion, Farago and Tédongap (2018) show that, besides market return and market volatility, three additional factors related to downside risk are priced in the cross-section of asset returns. Moreover, they show that their model successfully jointly prices various asset classes and provides significant improvements over nested specifications previously discussed in the literature. Bollerslev et al. (2022) decompose the traditional market beta into four semibetas based on the signed covariation. They show that only semibeta measuring dependence between a negative asset and a negative market return predicts higher future returns.

The second stream investigates horizon-specific features of risk with relation to asset prices. Many of these endeavors utilize spectral analysis to explore this task. Dew-Becker and Giglio (2016) investigate frequency-specific prices of risk for various leading theoretical models and provide empirical support for investors' aversion to low-frequency fluctuations present in the equity markets. Neuhierl and Varneskov (2021) employ frequency domain techniques to introduce a model-free framework to decompose the stochastic discount factor into transitory and permanent components to infer their contributions to risk premium. Moreover, their results provide suggestions on how to improve existing asset pricing models in terms of their specification and estimation.

Bandi et al. (2021) utilize tools from frequency-domain econometrics to introduce spectral factor models and show that spectral beta, measuring dependence across business-cycle frequencies, is able to explain some of the cross-sectional anomalies. Thus, they suggest that while focusing on frequencies, we may reduce the dimensionality of the factor space.

The third strand of literature relates asset returns to exposures to common factor structure that affects all assets in the market. Lettau and Pelger (2020) generalize principal component analysis by identifying latent factors that not only explain the time variation of assets but also their risk premium. Daniel et al. (2020) propose a better way how to construct characteristic-based factors by hedging out the unrelated sources of risk. Furthermore, there is still an ongoing debate whether the characteristics predict the cross-sectional returns because they capture linear exposure to the factor structure (Kelly et al. 2019) or whether they represent anomalies in terms of factor literature (Kim et al. 2020).

The fourth related approach utilizes machine learning techniques to understand asset prices better. In recent years, there has been a noticeable increase in the usage of these techniques for multiple reasons, such as their theoretical developments, a better understanding of how to implement them in a setting typical for finance, or an increased computation power required. Critical was the observation that economically motivated restrictions must be introduced to exploit these techniques effectively, as illustrated by Avramov et al. (2023).

Gu et al. (2020) illustrate significant investment benefits related to these methods compared to traditional regression-based approaches. Kozak et al. (2020) introduce dimension reduction techniques to identify critical information that shapes the cross-section of asset returns. Chen et al. (2023) estimate a complex asset pricing model using deep neural networks that take advantage of non-arbitrage condition to construct the most informative test assets. Moreover, Kelly et al. (2024) provide a theoretical justification for using large datasets and machine learning tools to understand and predict asset prices.

Many successful efforts fall into two or more of these categories simultaneously. For example, Daniel et al. (2019) propose a theoretically based factor model that contains long- and short-horizon mispricing factors. Jin et al. (2022) investigate coskewness across investment horizons and propose a theoretical model to estimate long-horizon coskewness from data with the highest frequency of observations. Massacci et al. (2021) introduce a conditional factor model in the presence of downside risk that allows for different factor structures in the good and bad states.

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Chapter 2

Quantile Spectral Beta: A Tale of Tail Risks, Investment Horizons, and Asset Prices[1]

This paper investigates how two important sources of risk – market tail risk and extreme market volatility risk – are priced into the cross-section of asset returns across various investment horizons. To identify such risks, we propose a *quantile spectral beta* representation of risk based on the decomposition of covariance between indicator functions that capture fluctuations over various frequencies. We study the asymptotic behavior of the proposed estimators of such risk. Empirically, we find that tail risk is a short-term phenomenon, whereas extreme volatility risk is priced by investors in the long term when pricing a cross-section of individual stocks. In addition, we study popular industry, size and value, profit, investment or book-to-market portfolios, as well as portfolios constructed from various asset classes, portfolios sorted on cash flow duration and other strategies. These results reveal that tail-dependent and horizon-specific risks are priced heterogeneously across datasets and are

[1]This chapter was co-authored with Jozef Baruník and published in the *Journal of Financial Econometrics*. We appreciate helpful comments from Allan Timmermann, Tobias Kley, Michal Kejak, Evžen Kočenda, Lukáš Vácha, and participants at the 2017, 2018 and 2019 Computational and Financial Econometrics Conferences (London and Pisa), the 2017 Slovak Economic Association Meeting (Košice), the 2018 International Symposium in Computational Economics and Finance (Paris), the 2018 SoFiE Summer School (Brussels), the 2021 Frontiers of Factor Investing (Lancaster), the 2021 STAT of ML (Prague), and the 2018, 2019 and 2020 Haindorf Seminars (Hejnice, Humboldt U. & Charles U. joint seminar). Support from the Czech Science Foundation under the 19-28231X (EXPRO) project is gratefully acknowledged. Matěj Nevrla gratefully acknowledges financial support from the UNCE project (UNCE/HUM/035).

important sources of risk for investors.

2.1 Introduction

The classical conclusion of the asset pricing literature states that the price of an asset should be equal to its expected discounted payoff. In the capital asset pricing model (CAPM) introduced by Sharpe (1964), Lintner (1965), Black (1972), we assume that the stochastic discount factor can be approximated by return on market portfolio; thus, expected excess returns can be fully described by their market betas based on covariance between asset return and market return. While early empirical evidence validated this prediction, decades of consequent research have called the ability of the traditional market beta to explain cross-sectional variation in returns into question. We aim to show that to understand the formation of expected returns, one has to look deeper into the features of asset returns that are crucial in terms of the preferences of a representative investor. We argue that two important, risk related features are tail events and frequency-specific (spectral) risk capturing behavior at different investment horizons. To characterize such general risks, we derive a novel *quantile spectral* representation of beta that captures covariation between indicator functions capturing fluctuations of different parts of joint risky asset and market return distributions over various frequencies. Nesting the traditional beta as well as recently introduced spectral beta (Bandi et al. 2021), the new representation captures *tail*-specific as well as horizon- or frequency-specific *spectral* risks.

Intuitively, covariation stemming from (extremely) negative returns of risky assets and (extremely) negative returns of the market that are known as downside risk in the literature should be positively compensated. While early literature (Ang et al. 2006) empirically confirms the premium for bearing downside risk, Levi and Welch (2019) concludes that estimated downside betas do not provide superior predictions compared to standard betas. More recently, Bollerslev et al. (2020) argue that we need to look at finer representations allowing combinations of positive and negative assets and market returns and suggest how such semibetas are priced.

The aim of this paper is to show that there is heterogeneity in the weights that investors assign to the risk for different investment horizons and different parts of the distribution of their future wealth. We argue that previous attempts have failed to fully account for more subtle implications arising from these heterogeneities. An asset drop that covaries with a drop in the market and, at the same time, is a low-frequency event with large persistence should be priced by investors differently than such extreme situations due to high-frequency, transitory events. While in the first situation investors will be pricing a persistent crash resulting in long-term fluctuations in the quantiles of the market's and risky asset's joint distribution, in the latter case the investor cares about the transitory crash resulting in short-run fluctuations. This essentially means that a covariance between the risky asset and discount factor will not only be different across all parts of the joint distribution but will also be different across various investment horizons. Intuitively, these co-occurrences of tail events will have either short-term or long-term effects on the marginal utility of investors. Looking at the beta representation that will capture such information empirically will also be informative for the rare disaster literature (Barro 2006).

Economists have long recognized that decisions under risk are more sensitive to changes in the probability of possible extreme events compared to the probability of a typical event. The expected utility might not reflect this behavior since it weighs the probability of outcomes linearly. Consequently, CAPM beta as an aggregate measure of risk may fail to explain the cross-section of asset returns. Several alternative notions have emerged in the literature. Mao (1970) presents survey evidence showing that decision-makers tend to think of risk in terms of the possibility of outcomes below some target. For an expected utility-maximizing investor, Bawa and Lindenberg (1977) has provided a theoretical rationale for using a lower partial moment as a measure of portfolio risk. Based on the rank-dependent expected utility due to Yaari (1987), Polkovnichenko and Zhao (2013) introduce utility with probability weights and derive the corresponding pricing kernel. As mentioned earlier, Ang et al. (2006); Lettau et al. (2014) argue that downside risk – the risk of negative returns – is priced across asset classes and is not captured by CAPM betas. Furthermore, Farago and Tédongap (2018) extend the results using a general equilibrium model based on generalized disappointment aversion and show that downside risks in terms of market return and market volatility are priced in the cross-section of asset returns.[2]

[2]In addition, it is interesting to note that equity and variance risk premiums are also associated with compensation for jump tail risk (Bollerslev and Todorov 2011). A more general exploration of the asymmetry of stock returns is provided by Ghysels et al. (2016), who propose a quantile-based measure of conditional asymmetry and show that stock returns

The results described above lead us to question the role of expected utility maximizers in asset pricing. A recent strand of literature solves the problem by considering the quantile of utility instead of its expectation. This strand of literature complements the previously described empirical findings focusing on downside risk, as it highlights the notion of economic agents particularly averse to outcomes below some threshold compared to outcomes above this threshold. The concept of a quantile maximizer and its features was pioneered by Manski (1988), and later axiomatized by Rostek (2010). Most recently, de Castro and Galvao (2019) developed a quantile optimizer model in a dynamic setting. A different approach to emphasizing investors' aversion toward less favorable outcomes defines the theory based on Choquet expectations. This approach is based on a distortion function that alters the probability distribution of future outcomes by accentuating probabilities associated with the least desirable outcomes. This approach was utilized in finance, for example, by Bassett Jr et al. (2004).

Whereas aggregating linearly weighted outcomes may not reflect the sensitivity of investors to tail risk, aggregating linearly weighted outcomes over various frequencies or economic cycles also may not reflect risk specific to different investment horizons. One may suspect that an investor cares differently about short-term and long-term risk according to their preferred investment horizon. Distinguishing between long-term and short-term dependence between economic variables has proven to be insightful since the introduction of cointegration (Engle and Granger 1987). The frequency decomposition of risk thus uncovers another important feature of risk that cannot be captured solely by market beta, which captures risk averaged over all frequencies. This recent approach to asset pricing enables us to shed light on asset returns and investor behavior from a different point of view, highlighting heterogeneous preferences. Empirical justification is brought by Boons and Tamoni (2015) and Bandi and Tamoni (2021), who show that exposure in long-term returns to innovations in macroeconomic growth and volatility of the matching half-life is significantly priced in a variety of asset classes. The results are based on the decomposition of time series into components of different persistence proposed by Ortu et al.

from emerging markets are positively skewed. Conrad et al. (2013) use option price data and find a relation between stock returns and their skewness. Another notable approach uses high-frequency data to define realized semivariance as a measure of downside risk (Barndorff-Nielsen et al. 2008). From a risk-measure standpoint, handling negative events, especially rare events, is a highly relevant theme in both practice and academia. The most prominent example is value-at-risk (Adrian and Brunnermeier 2016; Engle and Manganelli 2004).

(2013).

From an empirical asset pricing standpoint, our approach is closely related to Bandi et al. (2021) who introduce spectral beta that measures systematic risk over specific economic cycle. Bandi et al. (2021) show that a single business cycle component of market returns is successful in pricing many anomalous portfolios. Piccotti (2016) further sets the portfolio optimization problem into the frequency domain using matching of the utility frequency structure and portfolio frequency structure, and Chaudhuri and Lo (2016) present an approach to constructing a mean-variance-frequency optimal portfolio. This optimization yields the mean-variance optimal portfolio for a given frequency band, and thus it optimizes the portfolio for a given investment horizon.

From a theoretical point of view, preferences derived by Epstein and Zin (1989) enable the study of frequency aspects of investor preferences and this has quickly become a standard in the asset pricing literature. With the important results of Bansal and Yaron (2004), long-run risk started to enter asset pricing discussions. Dew-Becker and Giglio (2016) investigate frequency-specific prices of risk for various models and conclude that cycles longer than the business cycle are significantly priced in the market. Other papers utilize the frequency domain and Fourier transform to facilitate estimation procedures for parameters hard to estimate using conventional approaches. Berkowitz (2001) generalizes band spectrum regression and enables the estimation of dynamic rational expectation models matching data only in particular ways, for example, forcing estimated residuals to be close to white noise. Dew-Becker (2016) proposes a spectral density estimator of the long-run standard deviation of consumption growth, which is a key component for determining risk premiums under Epstein-Zin preferences and shows superior performance compared to the previous approaches. Crouzet et al. (2017) developed a model of a multifrequency trade set in the frequency domain and showed that restricting trading frequencies reduces price informativeness at those frequencies, reduces liquidity and increases return volatility. One of the rare exceptions that entertains the idea of combining horizon-specific risk with tail events is Barro and Jin (2021), who show that most of the risk premium is attributable to rare event risk, but the long-run risk component contributes to fitting the Sharpe ratio as well.

The debate clearly indicates that the standard assumptions leading to classical asset pricing models do not correspond with reality. In this paper, we suggest that more general pricing models have to be defined and should take into consideration both the asymmetry of the dependence structure among the stock market and the relation of asymmetry to different investor behaviors at various investment horizons.

The main contribution of this paper is threefold. First, based on the frequency decomposition of covariance between indicator functions, we define the *quantile spectral beta* of an asset capturing frequency-specific tail risks and corresponding ways of measuring the beta. The newly defined notion of a beta can be viewed as a disaggregation of a classical beta to a frequency- and tail-specific beta. With this notion, we describe how extreme market risks are priced in the cross-section of asset returns at various horizons. We define frequency-specific tail market risk that captures dependence between extremely low market and asset returns, as well as extreme market volatility risk that is characterized by dependence between extremely high increments of market volatility and extremely low asset returns. Second, we empirically motivate the emergence of such types of risks in the cross-section of asset returns. Third, we estimate models and document these types of risks on a wide number of popular datasets, including Fama-French industry, size and value, profit, investment and book-to-market portfolio, as well as portfolios constructed from various asset classes and sorted on cash flow durations.

The results of this paper suggest that tail risk is consistently priced in the cross-section of asset returns in the short term, while extreme market volatility risk is priced mainly in the long term. The result also holds when we control for popular factors, including moment-based factors that are designed to capture asymmetric features and popular downside risk models (Ang et al. 2006; Lettau et al. 2014; Farago and Tédongap 2018). We also discuss how our new beta representation relates to other risk measures. Finally, we document that the final model capturing tail-specific risks across horizons significantly outperforms the other related models that capture downside risks.

The rest of the paper is structured as follows. Section 2.2 motivates the importance of tail risks across horizons. Section 2.3 introduces the estimation of quantile spectral betas and discusses the asymptotic theory for the estimators, Section 2.4 defines the empirical models used for testing the significance of extreme risks, and Section 2.5 conducts the empirical analysis on individual stocks as well as on various portfolios. Section 2.6 then concludes. In the Appendix, we detail the main technical results regarding the quantile spectral betas, their relation to the rare disaster model, specifications of the related measures of risk, and detailed results from the portfolio estimations. For estimation of quantile spectral betas, we provide package `QSbeta` in R available at `https://github.com/barunik/QSbeta`. Quantile spectral and cross-spectral densities as well as other quantities can be estimated using package `quantspec` in R available at `https://github.com/tobiaskley/quantspec` introduced by Kley (2016).

2.2 Motivation: Why Should We Care About Tail Risks across Horizons

The empirical search for an explanation of why different assets earn different average returns centers around the use of return factor models arising from the Euler equation. With only the assumption of 'no arbitrage', a stochastic discount factor m_{t+1} exists, and under the expected utility maximization framework, for the ith excess return, $r_{i,t+1}$ satisfies $\mathbb{E}[m_{t+1}r_{i,t+1}] = 0$, hence

$$\mathbb{E}[r_{i,t+1}] = \frac{\mathbb{C}ov(m_{t+1}, r_{i,t+1})}{\mathbb{V}ar(m_{t+1})}\left(- \frac{\mathbb{V}ar(m_{t+1})}{\mathbb{E}[m_{t+1}]}\right) = \beta_i \lambda \tag{2.1}$$

where loading β_i can be interpreted as exposure to systematic risk factors and λ as the risk price associated with factors. Equation 2.1 assumes that the risk premium of an asset or a portfolio can be explained by its covariance with some reference economic or financial variable such as consumption growth or return on market portfolio. This simple pricing relation also assumes that independent common sources of systematic risk exist in the economy, and exposure to them can explain the cross-section of asset returns.[3] This leads to the so-called factor fishing phenomenon, which tries to identify other risk factors in addition to the traditional market factors assumed by CAPM using a linear combination of factors that are assumed to have nonzero covariance with a risky asset, and to be independent of each other.

Covariance between the two variables of interest,

$$\gamma_{i,j}^{k} = \mathbb{C}ov\Big(r_{j,t+k}, r_{i,t}\Big) \equiv \mathbb{E}[(r_{j,t+k} - \bar{r}_j)(r_{i,t} - \bar{r}_i)], \tag{2.2}$$

which is central to the asset pricing literature, may not be sufficient in cases in which the investor cares about different parts of the distribution of her future wealth differently or in cases in which an investor cares about specific investment horizons. The empirical literature silently assumes that the risk factors

[3] For example, this is the cornerstone of arbitrage pricing theory (APT) of Ross (1976).

aggregate information over the distribution of returns as well as investment horizons. Part of the literature tracing back to early work by Roy (1952); Markowitz (1952); Hogan and Warren (1974); Bawa and Lindenberg (1977) argues that the reason we do not empirically find support for the above thinking is that the pricing relationship is fundamentally too simplistic. If investors are averse to volatility only when it leads to losses, not gains, the total variance as a relevant measure of risk should be disaggregated.

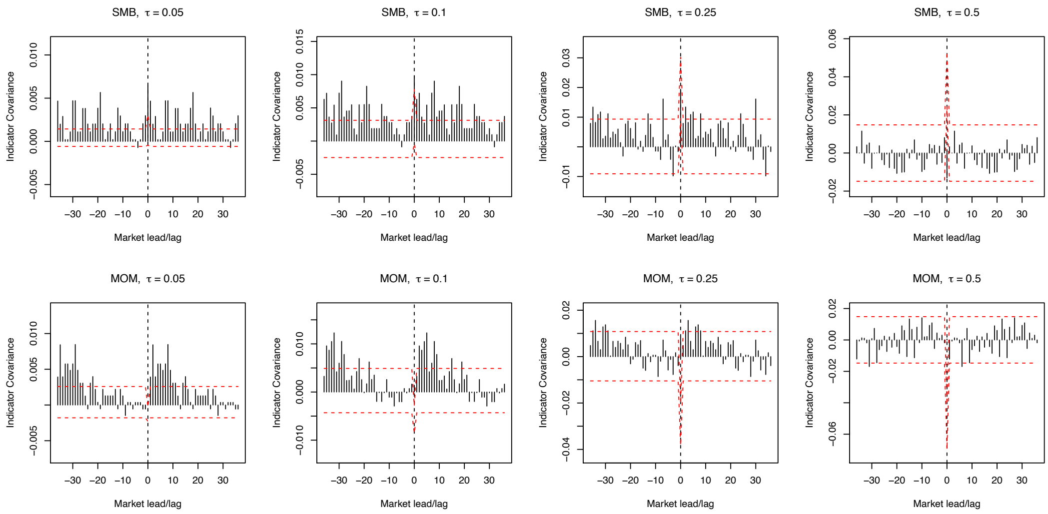
Later work by Ang et al. (2006); Lettau et al. (2014); Farago and Tédongap (2018) shows that investors require an additional premium as compensation for exposures to disappointment-related risk factors called downside risk. Recently, Lu and Murray (2019) argued that bear risk capturing the left tail outcomes is even more important, and Bollerslev et al. (2020) introduced betas based on semicovariances. In contrast to the promising results, Levi and Welch (2019) conclude that estimated downside betas do not provide superior predictions compared to standard aggregated betas, partially due to the difficulties of accurately determining downside betas from daily returns. With a similar argument of an overly simplistic pricing relation, another strand of the literature looks at frequency decomposition and explores the fact that risk factors of claims on the consumption risk should be frequency dependent since consumption has strong cyclical components (Dew-Becker and Giglio 2016).

More recently, a new stream of literature led by de Castro and Galvao (2019) assumes agents have quantile preferences. In asset pricing, such an investor prefers future streams of quantiles of utilities leading to $q_{t,\tau}\Big(m_{\tau,t+1}(1+r_{i,t+1})-1\Big)=0$. Assuming quantile preferences, our focus shifts from the search for the best proxy for a discount factor toward the capturing of the general dependence structures that reveal such flexible preferences. Measures we introduce in this paper allow us to identify risks associated with this type of preference.

Recognizing departures from overly simplistic assumptions in the data, we need to examine more general dependence measures since a simple covariance aggregating dependence across distributions as well as investment horizons will not be a sufficient measure of (in)dependence.

To illustrate this discussion, we consider dependence between market returns and a popular small-minus-big portfolio (SMB) as well as momentum portfolio (MOM). While the literature assumes that these factors represent two independent sources of risk with contemporaneous correlation between them and the market being rather small, investigating the dependence in various parts of their joint distribution across different lags and leads reveals interest-

Figure 2.1: *Dependence Structure between the Market and SMB and MOM Factor Portfolios.*



Note: Plots display covariance in the tail and across horizons defined by Eq. 2.3 that measures the general dependence between the market return and the SMB and MOM factors, respectively. Dashed lines represent 95% confidence intervals under the null hypothesis that the two series are jointly normally distributed correlated random variables. Data are sampled with monthly frequency.

ing relations. Instead of aggregate covariance between the market return and a factor portfolio, Figure 2.1 depicts tail- and lead/lag-specific covariation for a threshold value given by τ-quantile of the market return and a given lead/lag k of the following form:

$$\mathbb{C}ov\Big(I\{r_{m,t-k} \leq q_{r_m}(\tau))\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\Big), \tag{2.3}$$

where $r_{m,t}$ is the return of the market factor, $r_{i,t}$ is the return of either the SMB or the MOM portfolio, $I\{.\}$ is an indicator function and q_{r_m} is the quantile function of the market return. This simple measure captures the probability of both returns being below some threshold value in some time interval given by lead/lag k. This can be seen from the fact that $\mathbb{C}ov\Big(I\{r_{m,t-k} \leq q_{r_m}(\tau))\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\Big) = Pr\Big\{r_{m,t-k} \leq q_m(\tau), r_{i,t} \leq q_m(\tau)\Big\} - \tau\tau_i$. Therefore, this dependence essentially measures additional probability over the independence copula of both variables being below some threshold value.

Looking at the median dependence of market return on SMB or MOM portfolio returns (right column of plots for $\tau = 0.5$), we observe that dependence can be fully characterized by rather weak contemporaneous covariation between the market and the SMB and MOM portfolio returns, since no significant relation exists at any lead or lag in the relationship.[4] Moving our attention toward the left tail of the joint distribution, more complicated dependence structures emerge. The departure from the joint Gaussian distribution is strongest in the left tail (left column of plots for $\tau = 0.05$). The co-occurrences of large negative market returns with large negative SMB or MOM portfolio returns are significant and exist at various leads/lags.

For example, if we look at the dependence between the market and SMB in the 5% tail, we can observe that if the market is below this threshold, there is also a significant probability that the SMB portfolio will be below this threshold, with some delay. Similarly, the SMB downturn precedes the market downturn with significant probability.[5] Therefore, instead of arguing that the SMB factor proxies for an independent economic risk, the results suggest that the SMB portfolio captures more complicated market tail risk at some specific horizons.

In other words, the left tail dependence shows that extreme market drop is correlated with extreme negative returns of SMB. This illustrates that large negative market returns are correlated with the situation in which large companies largely outperform small companies in the SMB portfolio. Hence, we document a joint probability of co-occurrence of the market extreme left tail event, and large companies outperform small companies, leading to an increase in default risk in the economy (Chan et al. 1985). An important feature of the dependence not documented by earlier studies is its persistence structure shown by autocorrelations and the same strength for leading one another. At the same time, while momentum is negatively correlated with the market, the second row of Figure 2.1 shows a significant lead-lag relationship of the momentum factor and stock market, pointing us to the intuition that extremely low market returns are cross-correlated with companies with low momentum outperforming those with high momentum.

Note that these observations are closely related to the literature on market frictions, price delays and aggregations and their asset pricing implications.[6] In that sense, we follow a similar vein of thought as Bandi et al. (2021), with the important difference that we focus on the downside risk specifically.

This line of thinking may lead us to the conclusion that such general de-

[4]Note that the dashed lines in the figure represent confidence intervals under the null hypothesis that the two series are jointly normally distributed correlated random variables.

[5]A similar lead/lag investigation regarding business cycle indicators is performed in Backus et al. (2010).

[6]See, e.g., Kamara et al. (2016); Hou and Moskowitz (2005)

pendence structures can hardly be described by traditional contemporaneous correlation-based measures. The illustration suggests that there is no need for many factors to explain the average asset return, as carefully measured exposure to market risk can capture the risk investors care about. A natural way to summarize the dependence across these lead/lag relationships is to employ frequency analysis and precisely summarize this joint structure for specific horizons.

From an economic perspective, it is reasonable to assume that future marginal utility is affected by the realization of low quantile returns today, as this event may lead, for example, to bankruptcy or in other ways significantly shape the behavior of economic agents in the future. In other words, extreme market events can have either short-run or long-run effects on the marginal utility of investors. Previous studies, however, fail to fully account for horizon-specific information in tails, while one of the main reasons turns to the inability to measure such risks. Here, we propose robust methods for the measurement of such risks, and we argue that exploring the risk related to tail events as well as frequency-specific risk is crucial.

To see how tail-specific risks are priced across horizons by investors, we proceed as follows. First, we define a quantile risk measure based on the covariance between indicator functions, which has a natural economic interpretation in terms of probabilities. Second, we introduce its frequency decomposition and combine these two frameworks into the quantile spectral risk measure, which is the building block of our empirical model. This measure enables us to robustly test for the presence of extreme market risks over various horizons in asset prices. The aim is not to convince the reader that the functional form of the preferences precisely follows our model but to show that there is heterogeneity in the weights that investors assign to the risk for different investment horizons and different parts of the distribution of their future wealth. By estimating prices of risk for short- and long-term parts, we are able to identify the horizon that the investor cares most about. Moreover, by estimating prices of risk for various threshold values, we are able to identify the part of the joint distribution toward which the investor is the most risk averse.[7] This is done by controlling for CAPM beta, and the influence of these new measures is mea-

[7]Our investigation complements the work of Delikouras (2017) and Delikouras and Kostakis (2019). These studies investigate the position of the reference point of consumption growth and show that its correct location is crucial for fitting the model based on generalized disappointment aversion.

sured as incremental information over simplifying assumptions that lead to the CAPM beta asset pricing models.

2.3 Measuring the Tail Risks across Horizons: A Quantile Spectral Beta

Here, we formalize the discussion and provide more general measures that will provide a tool for inferring the discussed types of risks from data.

2.3.1 Tail Risk

Let us consider a bivariate, strictly stationary process $\boldsymbol{x}_t = (m_t, r_t)'$ holding some reference economic or financial variable m_t proxying risk and asset returns r_t. The marginal distribution functions of m_t and r_t will be denoted by F_m and F_r respectively, and by $q_m(\tau_m) := F_m^{-1}(\tau_m) := \inf\{q \in \mathbb{R} : \tau_m \leq F_m(q)\}$, and $q_r(\tau_r) := F_r^{-1}(\tau_r) := \inf\{q \in \mathbb{R} : \tau_r \leq F_r(q)\}$, where $\tau_m, \tau_r \in [0, 1]$ denote the corresponding quantile functions.

Since we are interested in pricing extreme negative events, we want to measure dependence and risk in lower quantiles of the joint distribution that can be evaluated by quantile cross-covariance (Kley et al. 2016; Baruník and Kley 2019)

$$\gamma_k^{m,r}(\tau_m, \tau_r) \equiv \mathbb{C}ov\Big(I\{m_{t+k} \leq q_m(\tau_m)\}, I\{r_t \leq q_r(\tau_r)\}\Big), \tag{2.4}$$

$k \in \mathbb{Z}$, and $I\{A\}$ denotes the indicator function of event A. The measure is given by the covariance between two indicator functions and, together with F_m and F_r, can fully describe the joint distribution of the pair of random variables m_t and r_t, that is, provide a measure for their serial and cross-dependency structure. If the distribution functions of the variables are continuous, the quantity is essentially the difference between the copula of the pair m_t and r_t and the independent copula, i.e., $Pr\Big\{m_{t+k} \leq q_m(\tau_m), r_t \leq q_r(\tau_r)\Big\} - \tau_m\tau_r$. Thus, covariance between indicators measures additional information from the copula over an independent copula about how likely it is that the series are jointly less than or equal to a given quantile of the variable m_t. It enables flexible measurement of both the cross-sectional structure and time-series structure of the pair of random variables.

Comparing these new quantities with their traditional counterparts, it can be observed that the covariance and means are essentially replaced by copulas and quantiles. A market beta associated with the tail risk can then be defined using Eq. 2.4. This quantity would be similar to the tail risk measure of Schreindorfer (2019), which is also a function of the τ quantile threshold of consumption growth. The correlation between asset returns and consumption growth is then computed conditional on realizations of consumption growth below the threshold. It is also related to the negative semibetas of Bollerslev et al. (2020), which estimates the dependence between market return and asset return conditional on the co-occurrence of negative events for both market and asset.

2.3.2 Tail Risks across Horizons: A Quantile Spectral Beta

It is natural to further assume that economic agents care not only about different parts of the wealth distribution but also differently about long- and short-term investment horizons in terms of expected returns and associated risks. Investors may be interested in the long-term profitability of their portfolio and may not be concerned with short-term fluctuations. Frequency-dependent features of an asset return then play an important role for an investor. Bandi and Tamoni (2021) argue that covariance between two returns can be decomposed into covariance between disaggregated components evolving over different time scales, and thus the risk on these components can vary. Hence, market beta can be decomposed into a linear combination of betas measuring dependence at various scales, i.e., dependence between fluctuations with various half-lives. Frequency-specific risk at a given time plays an important role in the determination of asset prices, and the price of risk in various frequency bands may differ, which means that the expected return can be decomposed into a linear combination of risks in various frequency bands.

A natural way to decompose covariance between two assets into dependencies over different horizons is in the frequency domain. A frequency domain counterpart of cross-covariance γ_k is obtained as the Fourier transform of the cross-covariance functions $S_{m,r}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_k^{m,r}e^{-\mathrm{i}k\omega}$. Conversely, cross-covariance can be obtained from the inverse Fourier transform of its cross-spectrum as $\gamma_k^{m,r} = \int_{-\pi}^{\pi} S_{m,r}(\omega)e^{\mathrm{i}k\omega}d\omega$, where $S_{m,r}(\omega)$ is the cross-spectral density of random variables m_t and r_t and $\mathrm{i} = \sqrt{-1}$.

This representation of covariation allows us to decompose the covariance and variance into frequency components and disentangle the short-term dependence from the long-term dependence. Using a similar approach, Bandi and Tamoni (2021) estimate the price of risk for different investment horizons and show that investors possess heterogeneous preferences over various economic cycles instead of looking only at averaged quantities over the whole frequency spectrum.

To uncover more general dependence structures, we propose to study the Fourier transform of the covariance of indicator functions $\gamma_k^{m,r}(\tau_m, \tau_r)$ instead. In this way, one can quantify the horizon-specific risk premium across the joint distribution. To define the new beta representation that will allow us to characterize such general risks, we use the so-called quantile cross-spectral densities introduced by Baruník and Kley (2019) as a generalization of quantile spectral densities of Dette et al. (2015).

The cornerstone of this new beta representation lies in quantile cross-spectral density defined as

$$f^{m,r}(\omega;\tau_m,\tau_r) \equiv \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_k^{m,r}(\tau_m,\tau_r)e^{-\mathrm{i}k\omega} \tag{2.5}$$

$$\equiv \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\mathbb{C}ov\Big(I\{m_{t+k}\le q_m(\tau_m)\}, I\{r_t\le q_r(\tau_r)\}\Big)e^{-\mathrm{i}k\omega} \tag{2.6}$$

with $\omega \in \mathbb{R}$ and $\tau_m, \tau_r \in [0,1]$. A quantile cross-spectral density is obtained as a Fourier transform of covariances of indicator functions defined in Equation 2.4, and will allow us to define beta that will capture the tail risks as well as spectral risks.

The *quantile spectral* (QS) betas that characterize horizon- and tail-specific market risk at a given ω, τ_m and τ_r are then defined as

$$\beta^{m,r}(\omega;\tau_m,\tau_r) \equiv \frac{f^{m,r}(\omega;\tau_m,\tau_r)}{f^{m,m}(\omega;\tau_m,\tau_m)}, \tag{2.7}$$

and will be the key quantity in our analysis. To estimate the quantile spectral beta, we use the rank-based copula cross-periodogram introduced by Baruník and Kley (2019)

$$I_{n,R}^{m,r}(\omega;\tau_m,\tau_r) := \frac{1}{2\pi n}d_{n,R}^{m}(\omega;\tau_m)d_{n,R}^{r}(-\omega;\tau_r), \tag{2.8}$$

where $d_{n,R}^m(\omega;\tau_m) := \sum_{t=0}^{n-1} I\{\widehat{F}_{n,m}(m_t)\le\tau_m\}\mathrm{e}^{-\mathrm{i}\omega t}$, and $d_{n,R}^r(\omega;\tau_r) := \sum_{t=0}^{n-1} I\{\widehat{F}_{n,r}(r_t)\le \tau_r\}\mathrm{e}^{-\mathrm{i}\omega t}$ with $\widehat{F}_{n,m}(m_t)$ and $\widehat{F}_{n,r}(r_t)$ being empirical distribution functions of m_t

and r_t, respectively. A consistent estimator of the quantile cross-spectral density is then

$$\widehat{G}^{m,r}_{n,R}(\omega;\tau_m,\tau_r) := \frac{2\pi}{n}\sum_{s=1}^{n-1} W_n\Big(\omega - 2\pi s/n\Big) I^{m,r}_{n,R}(2\pi s/n,\tau_m,\tau_r), \tag{2.9}$$

where W_n denotes a sequence of weight functions, precisely to be defined in the next section studying the asymptotic properties of the proposed estimators. The estimator of the quantile spectral beta is then given by

$$\widehat{\beta}^{m,r}_{n,R}(\omega;\tau_m,\tau_r) := \frac{\widehat{G}^{m,r}_{n,R}(\omega;\tau_m,\tau_r)}{\widehat{G}^{m,m}_{n,R}(\omega;\tau_m,\tau_m)}. \tag{2.10}$$

Before we prove that $\widehat{\beta}^{m,r}_{n,R}(\omega;\tau_m,\tau_r)$ is a legitimate estimate of $\beta^{m,r}(\omega;\tau_m,\tau_r)$, we note that for serially uncorrelated variables (regardless of their joint or marginal distributions), the Freće’het/Hoeffding bounds give the limits that QS beta can attain in the case of a serially independent process as $\frac{\max\{\tau_m+\tau_r-1,0\}-\tau_m\tau_r}{\tau_m(1-\tau_m)} \leq \beta^{m,r}(\omega;\tau_m,\tau_r) \leq \frac{\min\{\tau_m,\tau_r\}-\tau_m\tau_r}{\tau_m(1-\tau_m)}$.

2.3.3 Asymptotic properties of the Quantile Spectral beta

To derive the asymptotic properties of the quantile spectral beta, some assumptions need to be made. Recall (cf. Brillinger (1975), p. 19) that the rth order joint cumulant $\mathrm{cum}(Z_1,\ldots,Z_r)$ of the random vector $(Z_1,\ldots,Z_r)$ is defined as

$$\mathrm{cum}(Z_1,\ldots,Z_r) := \sum_{\{\nu_1,\ldots,\nu_p\}} (-1)^{p-1}(p-1)!E\Big[\prod_{j\in\nu_1} Z_j\Big]\cdots E\Big[\prod_{j\in\nu_p} Z_j\Big],$$

with summation extending over all partitions $\{\nu_1,\ldots,\nu_p\}$, $p=1,\ldots,r$, of $\{1,\ldots,r\}$. Regarding the range of dependence of $\boldsymbol{x}_t \in (m_t,r_t)'$, we make the following assumption:

Assumption 1. *The processes $(\boldsymbol{x}_t)_{t\in\mathbb{Z}}$ are strictly stationary and exponentially α-mixing, that is, there exist constants $K<\infty$ and $\kappa\in(0,1)$, such that*

$$\alpha(n) := \sup_{\substack{A\in\sigma(x_0,x_{-1},\ldots)\\ B\in\sigma(x_n,x_{n+1},\ldots)}} \Big|\mathbb{P}(A\cap B) - \mathbb{P}(A)\mathbb{P}(B)\Big| \leq K\kappa^n, \quad n\in\mathbb{N}. \tag{2.11}$$

Note that the Assumption 1 is a bivariate extension of assumptions made in Kley et al. (2016) and used in Baruník and Kley (2019) to study quantile spectral quantities. It is important to observe that this assumption does not require the existence of any moments, which is in sharp contrast to classical assumptions, where moments up to the order of the respective cumulants must exist, and sets A_j are not required to be general Borel sets, as in classical mixing assumptions. As noted in Baruník and Kley (2019), this assumption holds for a wide range of popular, linear and nonlinear, multivariate and univariate processes that are α-mixing at an exponential rate, including traditional VARMA or vector-ARCH models.

To establish the consistency of the estimates, we further need to consider sequences of weights that asymptotically concentrate around multiples of 2π.

Assumption 2. *The weights are defined as* $W_n(u) := \sum_{j=-\infty}^{\infty} b_n^{-1} W(b_n^{-1}[u + 2\pi j])$*, where* $b_n > 0$*,* $n = 1, 2, \ldots$*, is a sequence of scaling parameters satisfying* $b_n \to 0$ *and* $nb_n \to \infty$*, as* $n \to \infty$*. The weight function* W *is real-valued, even has support* $[-\pi, \pi]$*, bounded variation, and satisfies* $\int_{-\pi}^{\pi} W(u)du = 1$*.*

The main result of this section will legitimize $\widehat{\beta}^{m,r}_{n,R}(\omega; \tau_m, \tau_r)$ as an estimator of the quantile spectral (QS) beta $\beta^{m,r}(\omega; \tau_m, \tau_r)$. The legitimacy of the estimates follows from the fact that the estimators converge weakly in the sense of Hoffman-Jørgensen (cf. Chapter 1 of van der Vaart and Wellner (1996)). We denote this mode of convergence by $\Rightarrow$. The estimators under consideration take values in the space of (elementwise) bounded functions $[0, 1]^2 \to \mathbb{C}^{d\times d}$, which we denote by $\ell^{\infty}_{\mathbb{C}^{d\times d}}([0, 1]^2)$ (Kley et al. 2016). While the results of empirical process theory are typically stated for spaces of real-valued, bounded functions, these results transfer immediately by identifying $\ell^{\infty}_{\mathbb{C}^{d\times d}}([0, 1]^2)$ with $\ell^{\infty}([0, 1]^2)^{2d^2}$.

Using Proposition 1 in Appendix 3.A and following Kley et al. (2016) and Baruník and Kley (2019), we quantify uncertainty in estimating $f^{m,r}(\omega; \tau_m, \tau_r)$ by $\widehat{G}^{m,r}_{n,R}(\omega; \tau_m, \tau_r)$ asymptotically in the following theorem.

Theorem 1. *(Baruník and Kley (2019)) Let Assumptions 1 and 2 hold. Assume that the marginal distribution functions* F_m *and* F_r *are continuous and that constants* $\kappa > 0$ *and* $k \in \mathbb{N}$ *exist, such that* $b_n = o(n^{-1/(2k+1)})$ *and* $b_n n^{1-\kappa} \to \infty$*. Then, for any fixed* $\omega \in \mathbb{R}$*,*

$$\sqrt{nb_n}\Big(\widehat{G}^{m,r}_{n,R}(\omega; \tau_m, \tau_r) - f^{m,r}(\omega; \tau_m, \tau_r) - B^{m,r,(k)}_n(\omega; \tau_m, \tau_r)\Big)_{\tau_m, \tau_r \in [0,1]} \Rightarrow \mathbb{H}^{m,r}(\omega; \cdot, \cdot), \tag{2.12}$$

where the bias is given by $B^{m,r,(k)}_n(\omega; \tau_m, \tau_r) := \sum_{\ell=2}^{k} \frac{b_n^{\ell}}{\ell!} \int_{-\pi}^{\pi} v^{\ell} W(v)dv \frac{\mathrm{d}^{\ell}}{\mathrm{d}\omega^{\ell}} \mathfrak{f}^{m,r}(\omega; \tau_m, \tau_r)$*. The process* $\mathbb{H}^{m,r}(\omega; \cdot, \cdot)$ *is a centered,* $\mathbb{C}$*-valued Gaussian process characterized* *by*

$$
\begin{aligned}
&\operatorname{Cov}\Big(\mathbb{H}^{j_1,j_2}(\omega;u_1,v_1\Big),\mathbb{H}^{k_1,k_2}(\lambda;u_2,v_2)\Big)\\
&\quad=2\pi\Big(\int_{-\pi}^{\pi}W^2(\alpha)\mathrm{d}\alpha\Big)\Big(\mathfrak{f}^{j_1,k_1}(\omega;u_1,u_2)\mathfrak{f}^{j_2,k_2}(-\omega;v_1,v_2)\eta(\omega-\lambda)\\
&\qquad\qquad+\mathfrak{f}^{j_1,k_2}(\omega;u_1,v_2)\mathfrak{f}^{j_2,k_1}(-\omega;v_1,u_2)\eta(\omega+\lambda)\Big),
\end{aligned}
\tag{2.13}
$$

where $\eta(x) := I\{x = 0(\mod 2\pi)\}$ *[cf. (Brillinger 1975, p. 148)] is the* 2π*-periodic extension of Kronecker's delta function. The family* $\{\mathbb{H}(\omega;\cdot,\cdot),\ \omega \in [0,\pi]\}$ *is a collection of independent processes.*

It is important to note that in sharp contrast to classical spectral analysis, where higher-order moments are required to obtain smoothness of the spectral density [cf. Brillinger (1975), p. 27], Assumption 1 guarantees that the quantile cross-spectral density is an analytical function of ω. Assume that W is a kernel of order p; i. e., for some p, that satisfies $\int_{-\pi}^{\pi} v^j W(v)\mathrm{d}v = 0$, for all $j < p$, and $0 < \int_{-\pi}^{\pi} v^p W(v)\mathrm{d}v < \infty$; e. g., the Epanechnikov kernel is a kernel of order $p = 2$. Then, the bias is of order b_n^p. As the variance is of order $(nb_n)^{-1}$, the mean squared error is minimal if $b_n \asymp n^{-1/(2p+1)}$. This optimal bandwidth fulfills the assumptions of Theorem 1. A detailed discussion of how Theorem 1 can be used to construct asymptotically valid confidence intervals can be found in Baruník and Kley (2019).

The independence of the limit $\{\mathbb{H}(\omega;\cdot,\cdot),\ \omega \in [0,\pi]\}$ has two important implications. On the one hand, the weak convergence (2.12) holds jointly for any finite fixed collection of frequencies ω. Furthermore, fixing j_1, j_2 and τ_1, τ_2, the CCR periodogram $\widehat{G}^{j_1,j_2}_{n,R}(\omega;\tau_1,\tau_2)$ and traditional smoothed cross-periodogram determined from the unobservable, bivariate time series

$$
\Big(I\{F_{j_1}(X_{t,j_1}) \le \tau_1\}, I\{F_{j_1}(X_{t,j_2}) \le \tau_2\}\Big), \quad t = 0,\dots,n-1, \tag{2.14}
$$

are asymptotically equivalent. Theorem 1 thus reveals that in the context of the estimation of the quantile cross-spectral density, the estimation of the marginal distribution has no impact on the limit distribution (cf. comment after Remark 3.5 in Kley et al. (2016)).

We are now ready to state the main result of this section.

Theorem 2. *Let Assumptions 1 and 2 hold. Assume that the marginal distribution functions* F_m *and* F_r *are continuous and that constants* $\kappa > 0$ *and* $k \in \mathbb{N}$ *exist, such that* $b_n = o(n^{-1/(2k+1)})$ *and* $b_n n^{1-\kappa} \to \infty$*. Assume that for some* $\varepsilon \in$

$(0, 1/2)$, *we have* $\inf_{\tau\in[\varepsilon,1-\varepsilon]} \mathfrak{f}^{m,m}(\omega; \tau_m, \tau_m) > 0$, *and* $\inf_{\tau\in[\varepsilon,1-\varepsilon]} \mathfrak{f}^{r,r}(\omega; \tau_r, \tau_r) > 0$. *Then, for any fixed* $\omega \in \mathbb{R}$,

$$\sqrt{nb_n}\Big(\widehat{\beta}^{m,r}_{n,R}(\omega;\tau_m,\tau_r)-\beta^{m,r}(\omega;\tau_m,\tau_r)-B^{m,r,(k)}_n(\omega;\tau_m,\tau_r)\Big)_{(\tau_m,\tau_r)\in[\varepsilon,1-\varepsilon]^2} \Rightarrow \frac{1}{\mathfrak{f}_{m,m}}\Big(\mathbb{H}_{m,m}-\frac{\mathfrak{f}_{m,r}}{\mathfrak{f}_{m,m}}\mathbb{H}_{m,r} \tag{2.15}$$

where

$$B^{m,r,(k)}_n(\omega;\tau_m,\tau_r) := \frac{1}{\mathfrak{f}_{m,m}}\Big(B_{m,m} - \frac{\mathfrak{f}_{m,r}}{\mathfrak{f}_{m,m}}B_{m,r}\Big) \tag{2.16}$$

and we have written $\mathfrak{f}_{a,b}$ *for the quantile cross-spectral density* $\mathfrak{f}^{a,b}(\omega; \tau_a, \tau_b)$ *as defined in* (2.5), $B_{a,b} := \sum_{\ell=2}^{k} \frac{b_n^\ell}{\ell!} \int_{-\pi}^{\pi} v^\ell W(v) dv \frac{\mathrm{d}^\ell}{\mathrm{d}\omega^\ell} \mathfrak{f}^{a,b}(\omega; \tau_a, \tau_b)$, *and* $\mathbb{H}_{a,b}$ *for* $\mathbb{H}^{a,b}(\omega; \tau_a, \tau_b\big)$ *defined as a centered,* $\mathbb{C}$*-valued Gaussian process characterized by*

$$\begin{aligned}\mathrm{Cov}\,\Big(\mathbb{H}^{j_1,j_2}(\omega;u_1,v_1\Big),\mathbb{H}^{k_1,k_2}(\lambda;u_2,v_2)\Big) \\ = 2\pi\Big(\int_{-\pi}^{\pi} W^2(\alpha)\mathrm{d}\alpha\Big)\Big(&\mathfrak{f}^{j_1,k_1}(\omega;u_1,u_2)\mathfrak{f}^{j_2,k_2}(-\omega;v_1,v_2)\eta(\omega-\lambda) \\ &+\mathfrak{f}^{j_1,k_2}(\omega;u_1,v_2)\mathfrak{f}^{j_2,k_1}(-\omega;v_1,u_2)\eta(\omega+\lambda)\Big),\end{aligned} \tag{2.17}$$

where $\eta(x) := I\{x = 0(\mod 2\pi)\}$ *[cf. (Brillinger 1975, p. 148)] is the* 2π*-periodic extension of Kronecker's delta function. The family* $\{\mathbb{H}(\omega; \cdot, \cdot),\ \omega \in [0, \pi]\}$ *is a collection of independent processes.*

Proof. The proof is lengthy and technical, and it is therefore deferred to Appendix 2.A.1. □

Convergence to a Gaussian process can be employed to obtain asymptotically valid pointwise confidence bands. A more detailed discussion on how to conduct inference is given in Appendix 2.A.2.

If W is a kernel of order $p \geq 1$, we have that the bias is of order b_n^p. Thus, if we choose the mean square error minimizing bandwidth $b_n \asymp n^{-1/(2p+1)}$, the bias will be of order $n^{-p/(2p+1)}$.

Regarding the restriction $\varepsilon > 0$, note that the convergence (2.15) cannot hold if (τ_1, τ_2) is on the border of the unit square, as the quantile coherency $\boldsymbol{\beta}(\omega; \tau_1, \tau_2)$ is not defined if $\tau_j \in \{0, 1\}$, as this implies that $\mathrm{Var}(I\{F_j(X_{t,j}) \leq \tau_j\}) = 0$.

2.4 Pricing Model for Extreme Risks across the Frequency Domain

Quantile spectral betas defined in the previous sections will be the cornerstone of our empirical models. We assume that QS betas for low threshold values will be significant determinants of risk priced heterogeneously across investment horizons. We will employ QS betas to study two kinds of risk related to the market return. First, we will investigate *tail market risk* (TR), a risk representing dependence between extreme negative events of both market and asset returns at a given horizon. In case the stochastic discount factor is linear in factors and we consider the market return as a risk factor, we further look at the dependence between asset returns and market returns and the threshold values are based on quantiles of market returns.

It is useful to connect our notion of risks to a well-established rare disaster model of Nakamura et al. (2013). QS betas between consumption growth and equity returns can be directly connected to permanent and transitory disasters that moreover unfold over multiple years or just one period. QS beta can be used to clearly distinguish between the dependence structures of these types that are otherwise invisible to investors. The detailed discussion with simulations is relegated to Appendix 2.B due to the limited space of the paper.

Our notion of tail risk also relates to the downside risk of Ang et al. (2006); Lettau et al. (2014). While downside risk stems from covariation of asset returns and market return under some threshold, our notion stems from joint probability of the co-occurrence of extreme negative returns in both asset and market returns. This is more in line with the approach of semibetas (Bollerslev et al. 2020) but with an important feature of the persistence structure of such risks across investment horizons.

Second, we will examine *extreme market volatility risk* (EVR), a risk capturing unpleasant situations in which extremely high levels of market volatility are linked with extremely low asset returns, again with respect to the investment horizon. We argue that both of these concepts capture important features of risk of an asset faced by the investor and thus should be priced in a cross-section of asset returns.

In each of the models defined in the paper, we control for CAPM beta as a baseline measure of risk. This ensures that if the QS betas are proven to be significant determinants of risk premium, they do not simply duplicate the information contained in the CAPM beta. Moreover, in the case of tail market risk, we define relative betas that explicitly capture the additional information over the CAPM beta only.

2.4.1 Tail Market Risk

For better interpretability, we construct a quantile spectral beta for a given frequency band corresponding to reasonable economic cycles. This definition is important since it allows us to define short-run or long-run bands covering corresponding frequencies and hence disaggregate beta based on the specific demands of a researcher.

We expect the dependence between market return and asset return during extreme negative joint events to be positively priced across assets. The stronger the relationship is, the higher the risk premium required by investors. In addition, we expect this risk to be priced heterogeneously across different investment horizons.

To capture the tail market risk measuring the probability of co-occurrence between (extreme) negative events of both market and asset returns at a given horizon, we define

$$\beta_{\text{TR}}^{r_m,r_i}(\Omega;\tau) \equiv \sum_{\Omega\equiv[\omega_1,\omega_2)} \left(\frac{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{r_{m,t+k} \leq q_{r_m}(\tau)\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\Big)e^{-\mathrm{i}k\omega}}{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{r_{m,t+k} \leq q_{r_m}(\tau), I\{r_{m,t} \leq q_{r_m}(\tau)\}\Big)e^{-\mathrm{i}k\omega}} \right). \tag{2.18}$$

The numerator of Eq. (2.18) captures the probability of co-occurrence of the negative events at a given horizon, and the denominator captures information related to the probability of market tail events at a given horizon, which is related to the variation in market returns.

Similar to Ang et al. (2006) and Lettau et al. (2014), we define relative betas that capture additional information not contained in the classical CAPM beta. In this way, we can test the significance of tail market risk decomposed into the long- and short-term components to obtain their prices of risk separately. Because we want to quantify risk that is not captured by the CAPM beta, we propose to test the significance of tail market risk via differences in the QS beta and QS beta implied by the Gaussian white noise assumption. We call this *relative* QS beta, and we compute it for a given frequency band Ω_j and given market τ-quantile level as

$$\beta^{r_m,r_i}_{\text{rel}}(\Omega_j;\tau) \equiv \beta^{r_m,r_i}_{\text{TR}}(\Omega_j;\tau) - \beta^{r_i}_{\text{Gauss}}(\Omega_j,\tau), \tag{2.19}$$

where $\beta^{r_i}_{\text{Gauss}}(\Omega_j,\tau) = \frac{C_{\text{Gauss}}(\tau,\tau_i;\rho)-\tau\tau_i}{\tau(1-\tau)}$ with C_{Gauss} being a Gaussian copula with correlation ρ between market return and an asset's return.[8]

Assuming that all the relevant pricing information is contained in the CAPM beta, contemporaneous covariance between two time series should capture all the priced information. Moreover, if the series are jointly normally distributed and independent through time, the CAPM beta contains all the available information regarding the dependence. Hence, under the hypothesis that market and asset returns are correlated Gaussian noise, $\beta^{r_m,r_i}_{\text{rel}}(\Omega_j;\tau)$ will not carry any additional information, and CAPM characterizes the risks well. Note that $\beta^{r_i}_{\text{Gauss}}(\Omega_j,\tau)$ is constant across frequencies and depends only on the chosen quantile and correlation coefficient. On the other hand, if investors price information not captured by the CAPM beta, the QS beta estimated without any restriction may identify an additional dimension of risk not contained in the CAPM beta. More specifically, we can identify whether dependence in a specific part of the joint distribution and/or over a specific horizon is significantly priced.

If the CAPM beta captures all the risk information priced in the cross-section, the risk premium corresponding to the relative QS beta will be insignificant. Moreover, if the returns are Gaussian, the relative QS beta will be zero at all frequencies and quantiles.[9]

Our first model is hence a tail market risk (TR) model, which is defined as

$$\mathbb{E}[r^e_{i,t+1}] = \sum_{j=1}^{2} \beta^{r_m,r_i}_{\text{rel}}(\Omega_j;\tau)\lambda_{\text{TR}}(\Omega_j;\tau) + \beta^{r_m,r_i}_{\text{CAPM}}\lambda_{\text{CAPM}}, \tag{2.20}$$

[8] This stems from the fact that quantile cross-spectral density corresponds to a difference of probabilities $Pr\{r_{i,t} \leq_{r_m} (\tau), r_{m,t} \leq q_{r_m}(\tau)\} - \tau\tau_i$, where τ and τ_i are probability levels under a Gaussian distribution, and τ_i is obtained as $\tau_i = F_{r_i}\{q_m(\tau)\}$.

[9] Here, we briefly note that we set the threshold values in the covariance between indicators' measure of dependence as a τ quantile of market return. In the case of TR betas, the thresholds for market and asset returns are the same and are given by the τ quantile of market return. In the case of EVR betas, the threshold for increments of market volatility is given by the τ quantile of the series of increments of market volatility, and the threshold for asset return is given by the τ quantile of market return. Note that one could flexibly choose the thresholds based on the best model fit specific to our datasets. For example, we may choose the threshold value to be asset specific by corresponding to the τ quantile of the asset return. We do not follow this approach because we do not explicitly care about dependence between quantiles in the cross-section. Rather, we care about dependence in extreme market situations.

where $r^e_{i,t+1}$ is the excess return of asset i,[10] $\beta^{r_m,r_i}_{\text{CAPM}}$ is an aggregate CAPM beta, λ_{CAPM} is the price of aggregate risk of the market captured by the classical beta, and $\lambda_{\text{TR}}(\Omega_j, \tau)$ is the price of tail risk (TR) for a given quantile and horizon (frequency band). We specify our models to include the disaggregation of risk into two horizons – long and short. Long horizon is defined by corresponding frequencies of cycles of 3 years and longer, and short horizon by frequencies of cycles shorter than 3 years.[11] The procedure for obtaining these betas is explained below.

The intuition behind the TR model defined in 2.20 is that the relative TR betas will be zero in the case of Gaussian data, and no association between tail risk and the risk premium should be documented since risk is perfectly described by variance. On the other hand, if the data distribution is not Gaussian, the relative TR betas will be significantly different from zero, and the significance of the estimated price of risk captures the pricing effect of the TR over the conventional measure of dependence based on the contemporaneous correlation. We explicitly wish to investigate whether the dependence information over the classical assumptions is a significant determinant of the excess returns, so it is not important whether the CAPM model is true or not.

This specification also relates to the models recently proposed in the literature.[12] First, model of Bandi and Tamoni (2021) builds on the *consumption* CAPM model and thus use consumption as their proxy for risk when evaluating pricing implications of the frequency-dependent risk. Second, Bandi et al. (2021) use the market factor for their analysis of the cross section of asset returns using spectral decomposed factors. In contrast to these attempts, we consider horizon-specific risk in *in tails*.

From the TR perspective, the proposed model also relates to the model of Bollerslev et al. (2020), who investigate the pricing implications of the co-occurrence of the downside events of both market and asset returns. In contrast to our model, Bollerslev et al. (2020) does not consider the horizon over which these risks unfold.

[10]Note that all the risk measures (in line with the literature) present in the paper are calculated using excess returns.

[11]In Appendix 2.D, we perform a robustness check by defining the horizons using 1.5 years as a threshold and the results do not qualitatively differ. Different specifications are available upon request.

[12]Baruník and Kley (2019) features a toy example of TR risk estimated on asset returns as well, but they do not investigate any asset pricing implications of the estimated risk.

2.4.2 Extreme Volatility Risk

Assets with high sensitivities to innovations in aggregate volatility have low average returns (Ang et al. 2006). We further focus on extreme events in volatility and investigate whether dependence between extreme market volatility and tail events of assets is priced across assets. Because time of high volatility within the economy is perceived as time with high uncertainty, investors are willing to pay more for the assets that yield high returns during these tumultuous periods and thus positively covary with innovations in market volatility. This drives the prices of these assets up and decreases expected returns. This notion is formally anchored in the intertemporal pricing model, such as the intertemporal CAPM model of Merton (1973) or Campbell (1993). According to these models, market volatility is stochastic and causes changes in the investment opportunity set by changing the expected market returns or by changing the risk-return trade-off. Market volatility thus determines systematic risk and should determine the expected returns of individual assets or portfolios. Moreover, we assume that extreme events in market volatility play a significant role in the perception of systematic risk and that exposure to them affects the risk premium of assets.

In addition, decomposition of volatility into the short run and long run when determining asset premiums was proven to be useful (Adrian and Rosenberg 2008). Moreover, Bollerslev et al. (2020) incorporated the notion of downside risk into the concept of volatility risk and showed that stocks with high negative realized semivariance yield higher returns. Farago and Tédongap (2018) examine downside volatility risk in their five-factor model and obtain a model with negative prices of risk of the volatility downside factor, yielding low returns for assets that positively covary with innovations of market volatility during disappointing events. Thus, we want to investigate which horizon and part of the joint distribution of market volatility and asset returns generate these findings.

We assume that assets that yield highly negative returns during times of large innovations of volatility are less desirable for investors, and thus, holding these assets should be rewarded by higher risk premiums. In addition, we assume that such risk will be horizon specific. To measure the extreme volatility risk, we define the beta that will capture the joint probability of co-occurrences of negative asset returns and the extreme increment of market volatility across horizons. Because of the nature of covariance between indicator functions, we work with negative market volatility innovations $-\Delta\sigma_t^2 = -(\sigma_t^2 - \sigma_{t-1}^2)$, where we estimate σ_t with a popular GARCH(1,1). Then, the high volatility increments correspond to low quantiles of the distribution of the negative differences. If an asset positively covaries with increments of market volatility, the extreme volatility risk beta will be small, and vice versa. This is in contrast to most of the measures employed in similar analyses. We define the beta that captures extreme volatility risk across horizons as

$$\beta^{r_i}_{\Delta\sigma^2}(\Omega;\tau) \equiv \sum_{\Omega\equiv[\omega_1,\omega_2)} \left(\frac{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{-\Delta\sigma^2_{t+k} \leq q_{-\Delta\sigma^2_t}(\tau)\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\Big)e^{-\mathrm{i}k\omega}}{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{-\Delta\sigma^2_{t+k} \leq q_{-\Delta\sigma^2_t}(\tau), I\{-\Delta\sigma^2_t \leq q_{-\Delta\sigma^2_t}(\tau)\}\Big)e^{-\mathrm{i}k\omega}} \right) \tag{2.21}$$

Threshold values for asset returns are obtained in the same manner as for tail market risk and are derived from the distribution of the market returns, which means that $q_{r_m}(\tau)$ is used as an asset threshold value. For example, for model with $\tau = 0.05$, when computing extreme market volatility beta, as a threshold for negative innovations of market squared volatility, we use the 5% quantile of its distribution (corresponding to the 95% quantile of the original distribution), and the threshold for asset return is set to the 5% quantile of the distribution of market returns.

Our second model, the extreme volatility risk (EVR) model, will test the significance of EVR betas and is defined as

$$\mathbb{E}[r^e_{i,t+1}] = \sum_{j=1}^{2} \beta^{r_i}_{\Delta\sigma^2}(\Omega_j;\tau)\lambda_{\mathrm{EV}}(\Omega_j;\tau) + \beta^{r_i}_{\mathrm{CAPM}}\lambda_{\mathrm{CAPM}}, \tag{2.22}$$

where, as in the case of the TR model, we include the CAPM beta to control for the corresponding risk premium. In line with the results of the current literature (e.g., Boons and Tamoni (2015), Boguth and Kuehn (2013), or Adrian and Rosenberg (2008)), we expect positive prices of risk corresponding to EVR betas. This is because EVR betas measure the dependence between extremely high increments of market volatility (i.e., low values of negative innovations of market volatility) and low values of asset returns. Therefore, if an asset yields low returns in times of high market volatility, investors will require high premiums to hold it. Note that our EVR model closely relates to the model of Farago and Tédongap (2018), who introduces downside volatility betas without the frequency aspect of the risk.

Unlike the TR model, the EVR model does not take into consideration the Gaussianity of the data. The estimated price of EVR will directly measure the pricing implication of extreme dependence between market increments of volatility and asset returns.

2.4.3 Full model

Finally, to show the independence of the two horizon-specific tail risks, we also combine them into the third model that includes both tail market risk and extreme market volatility risk for both short- and long-run horizons, again controlling for a traditional CAPM beta. The model possesses the following form

$$\mathbb{E}[r^e_{i,t+1}] = \sum_{j=1}^{2} \beta^{r_m,r_i}_{rel}(\Omega_j;\tau)\lambda_{\mathrm{TR}}(\Omega_j;\tau) + \sum_{j=1}^{2} \beta^{r_i}_{\Delta\sigma^2}(\Omega_j;\tau)\lambda_{\mathrm{EV}}(\Omega_j;\tau) + \beta^{r_m,r_i}_{\mathrm{CAPM}}\lambda_{\mathrm{CAPM}}. \tag{2.23}$$

We denote this model as the *full model*. Assuming that TR and EVR are priced, using this model, we will investigate whether these risks are subsumed by each other or whether they describe independent dimensions of priced risk.

Throughout the paper, we focus on results for τ equal to 1%, 5%, 10%, 15%, 20%, and 25%. The choice of 1%, 5% and 10% quantiles is natural and arises in many economic and finance applications. Most likely, the most prominent example is value-at-risk, which is a benchmark measure of risk widely used in practice and studied among academics. Remaining values of τ, i.e., 15%, 20%, and 25% capture general downside risk and thus more probable negative joint events.

2.4.4 Estimation

To test our models, we use the standard Fama and MacBeth (1973) cross-sectional regressions. In the first stage, we estimate all required QS betas, relative QS betas, and CAPM betas for all assets. We define two nonoverlapping horizons: short and long. Horizon is specified by the corresponding frequency band. We specify the long horizon by frequencies with corresponding cycles of 3 years and longer, whereas short horizon indicate frequencies with corresponding cycles below 3 years.[13]. QS betas for these horizons are obtained by averaging QS betas over corresponding frequency bands.

In the second stage, we use these betas as explanatory variables and regress average asset returns on them and obtain the model fit. We assess the significance of a given risk by the significance of its corresponding estimated price [14]. In the case of the full model, we obtain the statistical inference on the estimated prices of risk by repeating cross-sectional regression at every time point, i.e., in every month $t = 1, \ldots, T$, we estimate the model of the following form:

$$r^e_{i,t} = \sum_{j=1}^{2} \widehat{\beta}^{r_m,r_i}_{rel}(\Omega_j;\tau)\lambda_{t,\mathrm{TR}}(\Omega_j;\tau) + \sum_{j=1}^{2} \widehat{\beta}^{r_i}_{\Delta\sigma^2}(\Omega_j;\tau)\lambda_{t,\mathrm{EV}}(\Omega_j;\tau) + \widehat{\beta}^{r_m,r_i}_{\mathrm{CAPM}}\lambda_{t,\mathrm{CAPM}}. \tag{2.24}$$

We obtain T cross-sectional estimates of lambdas for each of the corresponding betas. Then, we estimate the prices of risk by time-series averages of the lambdas over the whole period

$$\widehat{\lambda}_k(\Omega_j;\tau) = \frac{1}{T}\sum_{t=1}^{T} \widehat{\lambda}_{t,k}(\Omega_j;\tau), \quad j = 1, 2, \quad k = \mathrm{TR}, \mathrm{EVR}. \tag{2.25}$$

Standard errors and corresponding t-statistics are computed from $\sigma^2\Big(\widehat{\lambda}_k(\Omega_j;\tau)\Big) = \frac{1}{T^2}\sum_{t=1}^{T}\Big(\widehat{\lambda}_{t,k}\Omega_j;\tau) - \widehat{\lambda}_k(\Omega_j;\tau)\Big)^2$ for both horizons $j = \{1, 2\}$ and risks $k = \{\mathrm{TR}, \mathrm{EVR}\}$.

The same estimation logic applies to other studied models. To take into account multiple hypothesis testing, we follow Harvey et al. (2016) and report t -statistics of estimated parameters (below the actual estimates). The overall fit of the model is measured from the OLS regression of the average returns of the assets on their betas. Throughout the paper, we use the root mean squared pricing error (RMSPE) metric, which is a widely used metric for assessing model fit in the asset pricing literature, to assess the overall model performance.

As mentioned earlier, we estimate our models for various threshold values given by the τ quantile of market return. Furthermore, in Appendix 2.C.2, we compare our newly proposed measures with i) classical CAPM ii) downside risk model of Ang et al. (2006) (DR1), iii) downside risk model of Lettau et al. (2014) (DR2), iv) 3-factor model of Fama and French (1993), v) GDA3

[13]For a robustness check using 1.5 years as a threshold value, see Appendix 2.D

[14]As shown in Shanken (1992), if the betas are estimated over the whole period, the second-stage regression is T-consistent.

and GDA5 models of Farago and Tédongap (2018), and vi) coskewness and cokurtosis measures. Details regarding the estimation of the risk measures of the related models are summarized in Appendix 2.E. All the models are estimated for comparison purposes without any restrictions in two stages, similar to our three- and five-factor models. Thus, GDA3 and GDA5 are, despite their theoretical background, estimated without setting any restriction to their coefficients and are also estimated in two stages.

2.4.5 Finite Sample Size Properties of the Testing Approach

Naturally, there is a question of how the 2-stage procedure with estimates on frequency bands performs in typical (small) samples, which we encounter in finance. To give the reader a notion of these properties, we present a simulation exercise to investigate the statistical size of our testing approach. In each run, we simulate returns on either 300 or 30 assets to mirror the settings of our empirical investigation of individual stocks and portfolio returns. Each asset possesses a length of 720 observations using either the classical CAPM model or white noise as a data generating process. First, we simulate time series of returns on the market from the normal distribution $N(\mu, \sigma)$ with $\mu = 0.06/12$ and $\sigma = 0.2/\sqrt{12}$. Second, in the case of the CAPM model, we generate time series of asset returns by randomly drawing the CAPM beta from the normal distribution $N(\bar{\beta}, \sigma_\beta)$, where $\bar{\beta} = 1$ and $\sigma_\beta = 0.5$, and then create the return as

$$R_{it} = \beta_i R_{mt} + \epsilon_{it},\ i = 1, \ldots, N,\ t = 1, \ldots, T. \tag{2.26}$$

In the case of the white noise model, we set all the CAPM betas equal to 0. In the third stage, for every stock, using the simulated data, we estimate their CAPM betas and QS betas (both TR and EVR) and regress the average returns on them using specifications of the TR model, EVR model and full model. We determine the number of cases where we incorrectly reject the null hypothesis that a given QS beta in a given model is a significant determinant of average returns. We set the significance level at $\alpha = 0.05$. Ideally, we would like to observe the rejection rates of approximately 5%. The results are summarized in Table 2.1, which show that the rejection rates typically correspond to the chosen significance level α. This shows the validity of our approach; even for low values of τ and long horizons, there is no significant bias in the rejection rates.

Table 2.1: *Finite Sample Size Properties of the Testing Approach.*

DGP	# of assets	τ	Tail market risk		Extreme volatility risk		Full model			
			λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}
CAPM	$N = 300$	0.01	0.052	0.046	0.062	0.072	0.060	0.048	0.070	0.080
		0.05	0.066	0.062	0.072	0.058	0.066	0.062	0.072	0.058
		0.10	0.056	0.046	0.048	0.088	0.068	0.048	0.066	0.088
		0.15	0.056	0.046	0.050	0.042	0.048	0.046	0.048	0.038
		0.25	0.046	0.054	0.068	0.032	0.056	0.054	0.064	0.030
	$N = 30$	0.010	0.054	0.040	0.056	0.066	0.074	0.052	0.062	0.060
		0.05	0.028	0.060	0.042	0.048	0.026	0.054	0.042	0.060
		0.10	0.044	0.058	0.048	0.058	0.050	0.058	0.044	0.052
		0.15	0.044	0.044	0.048	0.058	0.044	0.038	0.048	0.048
		0.25	0.062	0.054	0.068	0.044	0.056	0.060	0.058	0.054
White noise	$N = 300$	0.01	0.058	0.050	0.064	0.058	0.054	0.056	0.062	0.056
		0.05	0.040	0.064	0.068	0.040	0.036	0.064	0.064	0.042
		0.10	0.044	0.044	0.054	0.046	0.042	0.042	0.066	0.054
		0.15	0.044	0.042	0.060	0.054	0.046	0.046	0.072	0.050
		0.25	0.066	0.040	0.040	0.068	0.062	0.038	0.050	0.064
	$N = 30$	0.01	0.054	0.038	0.074	0.060	0.040	0.040	0.066	0.058
		0.05	0.050	0.060	0.036	0.038	0.048	0.064	0.038	0.040
		0.10	0.046	0.048	0.032	0.048	0.048	0.040	0.034	0.048
		0.15	0.052	0.048	0.042	0.060	0.048	0.040	0.038	0.052
		0.25	0.044	0.072	0.052	0.050	0.036	0.066	0.060	0.036

Note: Here we report rejection rates of the 2-stage estimation procedure when the assets are generated using either the CAPM model or white noise. The significance level is set to $\alpha = 0.05$. The number of simulations is 500.

2.5 Quantile Spectral Risk and the Cross-Sections of Expected Returns

Here, we discuss how extreme risks are priced in the cross-section of asset returns across horizons. We focus on the results from the standard Fama and MacBeth (1973) cross-sectional predictive regressions of the three main models and use various cross-sections of asset returns. We show that the quantile spectral risks are priced heterogeneously across various asset classes. This provides a great opportunity for investors who prefer to avoid certain risks. By choosing a specific asset class in which a specific risk is not associated with a risk premium (i.e., assets with high exposure to this risk do not yield an extra premium and vice versa), investors can avoid this risk without paying extra money for it.

First, we investigate returns on individual stocks from the U.S. market. Next, we use standard Fama-French portfolios sorted on various characteristics. More specifically, we use 30 industry portfolios, 25 portfolios sorted on size and value and decile portfolios sorted either on operating profit, investment or book-to-market. Finally, we use three datasets previously introduced in the literature to illustrate some specific phenomena. First, we analyze the dataset of Lettau et al. (2014), which contains portfolios constructed from various asset classes. Second, we analyze equity portfolios sorted by cash flow duration of Weber (2018). Third, we investigate data on investment strategies constructed across various asset classes from Ilmanen et al. (2021).

We report models estimated for various threshold values given by the τ quantile of the market return. We report models estimated for the 1%, 5%, 10%, 15%, 20% and 25% quantiles.[15] Throughout the paper, market return is computed using the value-weight average return on all CRSP stocks. As a risk-free rate, we use the Treasury bill rate from Ibbotson Associates.[16]

2.5.1 Individual Stocks

We collect our data from the Center for Research in Securities Prices (CRSP) database on a monthly basis. The sample spans from July 1926 to December 2015; we select stocks with a long enough history to obtain precise estimates of our measures of risk. While the main results are presented with a sample of stocks with an available history of 60 years, to study the robustness of our results on a larger cross-section of data, we also report results based on stocks with a shorter history of 50 years. On the other hand, one can argue that the precision of the estimated measures of risk relies on the number of observations available in the tail; hence, we also report results based on stocks with 70 years of available history. We report estimation results in Table 2.2.

Models are estimated for different values of the threshold value given by the τ market quantile to capture the different probabilities of event co-occurrences. The results of the TR model show that the relative TR beta for short horizons is more significant for low values of τ, corresponding to 0.01, 0.05 and 0.10, while for $\tau \geq 0.15$, the relative TR beta becomes significant for long horizons. This pattern is observed across all three samples, but it is weaker among stocks with a history of 50 years, especially regarding the prices of risk corresponding to the long relative TR betas. This result may be caused by the fact that long relative TR betas require a longer history of data to obtain precise estimates in comparison to the short TR betas.

Signs of the estimated prices of risk are intuitive. More extreme dependence between market and asset returns in both horizons leads to a higher risk pre-

[15] We had to rescale the data of Lettau et al. (2014) and Weber (2018) to be comparable to the market return.

[16] All the data were obtained from Kenneth French's online data library.

Table 2.2: *Individual stocks.*

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
	0.01	-0.059	0.657	0.754	26.683	-0.086	-0.275	0.960	28.542	0.125	0.620	-0.210	-0.168	0.834	26.387
		-0.660	3.309	3.911		-0.972	-0.887	5.001		0.901	3.087	-1.485	-0.539	4.196	
	0.05	0.102	1.319	0.717	26.807	0.232	0.380	0.682	28.192	0.080	1.295	0.009	0.303	0.721	26.757
		0.500	3.520	3.871		1.394	0.753	3.028		0.221	3.534	0.030	0.595	2.978	
	0.1	0.368	1.203	0.739	27.121	0.474	0.472	0.558	27.212	-0.064	1.037	0.422	0.311	0.555	26.555
70 years		1.483	2.304	4.144		2.532	0.677	2.533		-0.158	1.973	1.384	0.448	2.312	
(142 assets)	0.15	0.544	0.895	0.733	26.672	0.509	0.552	0.602	27.016	0.242	0.882	0.326	0.538	0.618	26.289
		2.327	1.511	4.075		2.783	0.778	2.903		0.730	1.487	1.223	0.752	2.875	
	0.2	0.665	0.279	0.784	26.995	0.702	-0.272	0.605	25.796	-0.041	0.848	0.693	-0.094	0.586	25.431
		2.566	0.400	4.454		3.665	-0.348	3.070		-0.116	1.250	2.601	-0.120	2.868	
	0.25	0.746	-0.132	0.812	27.244	0.823	-0.543	0.648	25.768	0.009	0.444	0.805	-0.397	0.643	25.676
		2.805	-0.181	4.662		3.816	-0.678	3.366		0.026	0.616	2.810	-0.498	3.284	
	0.01	-0.044	0.439	0.759	29.725	-0.090	0.271	0.939	30.494	0.170	0.387	-0.241	0.255	0.865	29.442
		-0.633	2.693	4.126		-1.293	1.007	5.144		1.431	2.391	-2.010	0.939	4.659	
	0.05	0.189	1.219	0.660	28.674	0.243	0.614	0.645	29.945	0.115	1.271	0.027	0.653	0.661	28.600
		1.068	3.653	3.573		1.471	1.505	2.850		0.379	3.903	0.103	1.594	2.759	
	0.1	0.315	1.000	0.718	29.243	0.503	0.420	0.511	29.201	-0.161	0.939	0.509	0.443	0.493	28.676
60 years		1.388	2.450	4.022		2.471	0.830	2.281		-0.489	2.282	1.737	0.875	2.058	
(267 assets)	0.15	0.500	0.779	0.709	29.257	0.441	0.550	0.603	29.608	0.297	0.819	0.233	0.443	0.626	29.099
		2.188	1.546	3.961		2.248	1.004	2.889		1.042	1.630	0.958	0.805	2.972	
	0.2	0.484	0.287	0.765	29.764	0.630	-0.198	0.595	28.769	-0.234	0.735	0.742	-0.253	0.561	28.620
		2.002	0.537	4.359		3.153	-0.338	3.024		-0.824	1.438	3.007	-0.430	2.808	
	0.25	0.487	0.242	0.785	30.045	0.619	-0.463	0.661	29.389	-0.053	0.574	0.629	-0.443	0.657	29.341
		2.017	0.427	4.513		3.084	-0.764	3.505		-0.203	1.037	2.768	-0.737	3.481	
	0.01	-0.089	0.441	0.823	29.727	-0.099	0.478	0.970	30.281	0.001	0.410	-0.096	0.439	0.873	29.683
		-1.655	2.953	4.631		-1.783	2.508	5.420		0.009	2.787	-1.077	2.337	4.816	
	0.05	-0.022	1.185	0.760	29.233	0.061	0.649	0.800	30.059	0.019	1.268	-0.059	0.780	0.781	29.039
		-0.152	3.949	4.233		0.484	1.971	3.776		0.067	4.400	-0.258	2.421	3.439	
	0.1	0.153	0.820	0.786	29.762	0.289	0.093	0.680	29.700	-0.104	0.801	0.288	0.182	0.665	29.417
50 years		0.794	2.450	4.436		1.785	0.223	3.224		-0.333	2.450	1.167	0.436	2.971	
(528 assets)	0.15	0.348	0.862	0.767	29.600	0.148	0.023	0.783	30.112	0.476	0.830	-0.149	0.101	0.813	29.570
		1.832	2.034	4.307		0.921	0.051	3.891		1.733	1.984	-0.670	0.222	3.945	
	0.2	0.272	0.683	0.810	29.973	0.346	-0.267	0.735	29.791	-0.002	0.743	0.314	-0.298	0.735	29.704
		1.410	1.437	4.605		2.011	-0.566	3.789		-0.009	1.605	1.374	-0.622	3.695	
	0.25	0.257	0.822	0.821	30.035	0.322	-0.051	0.765	30.004	0.034	0.946	0.281	-0.081	0.764	29.885
		1.316	1.594	4.704		1.885	-0.106	4.082		0.152	1.848	1.388	-0.167	4.070	

Note: Prices of risk estimated on monthly stock data from the CRSP database sampled between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ three samples with varying numbers of minimum years. A long horizon is given by frequencies corresponding to a 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

mium, as we may expect. If an asset is likely to deliver poor performance when the market is in a downturn, this asset is not desirable from the point of view of an investor, and to decide to hold such asset, she would require a significant risk premium. From the magnitude of the coefficients, we infer that investors price tail risk in the short term more than in the long term. Moreover, it is important to note that these features are not subsumed by the CAPM beta, as we explicitly control for it in the model, and report TR betas relative to the CAPM beta, as discussed above.

Estimation results for the EVR model are captured in the middle panel of Table 2.2. In this case, parameters are not significant for low values of τ, but starting with $\tau \geq 0.1$, long EVR becomes significantly priced in the cross-section. On the other hand, short-horizon EVR risk is not significantly priced for any values of τ.

Significant prices of risk corresponding to long-horizon EVR betas for $\tau \geq 0.10$ possess intuitive positive signs, as we expected. The EVR betas capture dependence between extremely high increments of market volatility[17] and extremely low asset returns, and the results are consistent with the current literature (Boons and Tamoni 2015; Boguth and Kuehn 2013; Adrian and Rosenberg 2008). Moreover, these results are in line with the conclusions of long-run risk models. We observe few instances of unintuitive negative signs of prices of risk, but these coefficients are insignificant and observed mostly for low values of τ, which may be caused by the measurement error for the corresponding betas. We may conclude that EVR betas, especially their long-term component, provide priced information regarding risk, which is moreover orthogonal to the information featured in the CAPM beta.

In terms of the RMSPE, the TR model delivers better results than the EVR model for low values of τ, as short TR betas are significantly priced for these values of τ. On the other hand, for higher values of τ, the EVR model delivers improved values of RMSPE, as the long EVR betas for these τ values deliver a significant dimension of risk priced in the cross-section and TR betas possess higher explanatory power for lower values of τ.

Moreover, we identify the fact that there is a complex interplay between the horizons and parts of the joint distribution priced in the cross-section. Extreme TR is mostly a short-run phenomenon, and TR associated with more probable joint events (higher values of τ) is priced with respect to long-term

[17]Note that we work with negative increments of market volatility when we estimate the QS betas.

dependence between the market and assets. On the other hand, EVR is not significantly priced in cases of extreme joint events, but as unpleasant events become more probable, the joint dependence between increments of market volatility and asset return in the long run becomes a significant determinant of risk premiums. In Table 8 in Appendix 2.D, we present the results for 1.5-years being the threshold in the definition of the long horizon. The results are qualitatively very similar, and all the findings from the 3-year horizon hold for this case.

From the results above, we can conclude that tail market and extreme market volatility risks are priced in the cross-section of stock returns across different horizons. A natural question arises whether these risks capture different information or whether one measure can subsume the other. For this purpose, we test the full model, which contains both risks for a given τ level at the same time. Estimated parameters can be found in the right panel of Table 2.2. We observe results mostly consistent with the outcomes of the separate TR and EVR models. Significantly priced determinants of the risk are short-term TR for low values of τ and long-term EVR for the higher values of τ, both priced across assets with expected positive signs. Tail risk is more significant for lower values of τ, meaning that dependence between market return and asset return during extremely negative events is a significant determinant of the risk premium. On the other hand, long-term extreme volatility risk is significant for higher values of τ - approximately 0.2. This finding suggests that investors price downside dependence between asset returns and market volatility but focus on more probable market situations. We can deduce that the price of long-run risk mentioned by Bansal and Yaron (2004) is hidden in this coefficient.

The main deviation of the full model from the results of the separate TR and EVR models is that the long TR betas for higher values of τ become insignificant, in contrast with the conclusions from the TR model. One potential explanation for this result is that only a small fraction of the market return fluctuations are due to its long-term component in comparison to the short-term component, and thus, the risk premium for this risk is only small. Another explanation is that the long-term aspect of the market tail risk may be fully captured by the extreme volatility risk, namely, the long TR betas are subsumed by the long EVR betas. This makes sense since variance is much more persistent than the market return (high portion of variance due to the long-term component) and thus investors fear the fluctuation in long-term variance much more than the variance in the short term.

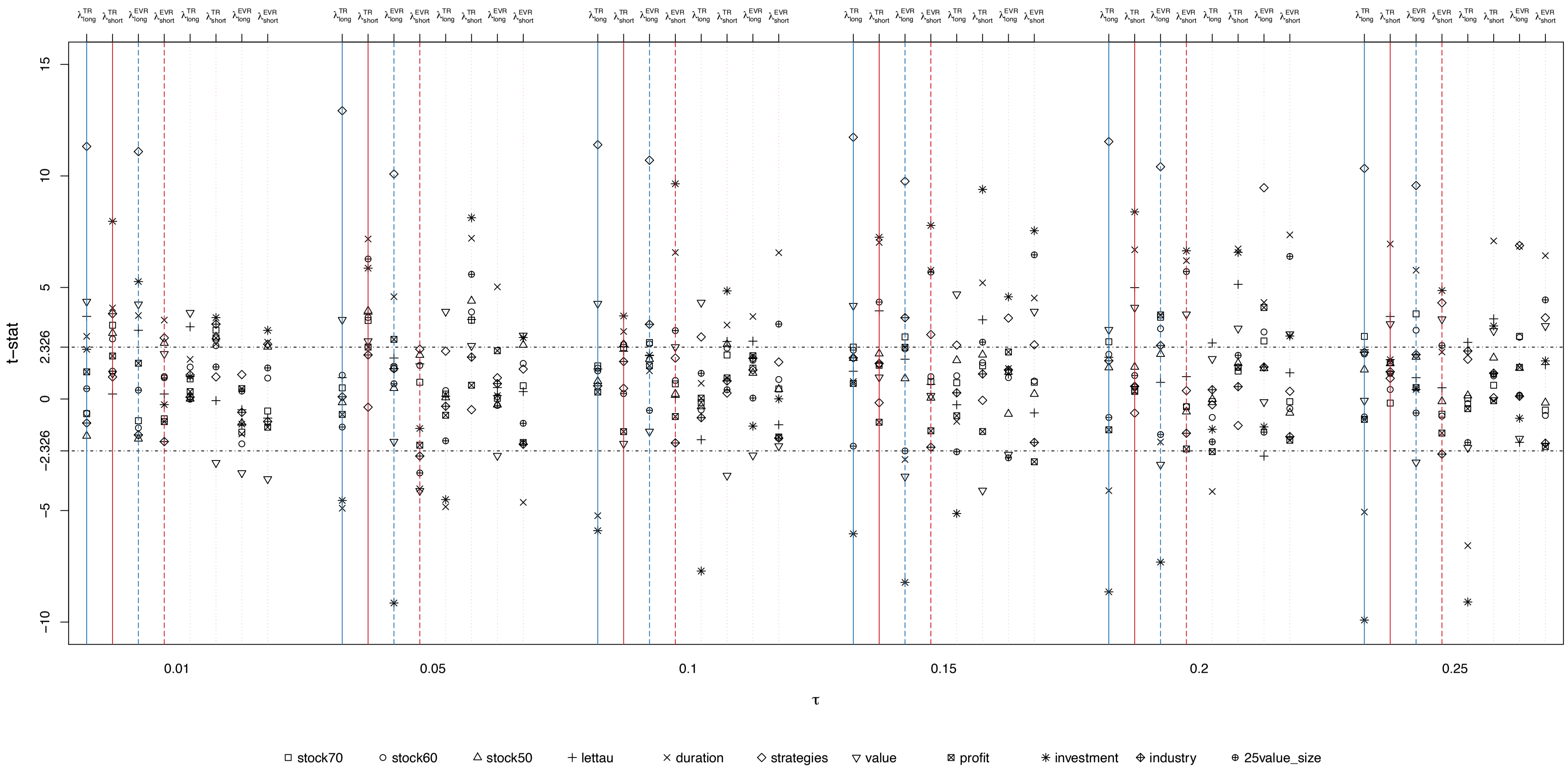
In Appendix 2.C, we use this data sample and show various features of the estimated QS betas. We present distributions of the estimated QS betas to give a notion of their estimated values. Next, we investigate the relation between QS betas and other risk measures previously proposed in the literature. Although the QS measures are correlated with some of the other variables discussed previously in the literature, they do not drive out the QS measures of risk. Moreover, these variables are, in most cases, subsumed by the variables from the full model. Our results are in agreement with recent results of Bollerslev et al. (2020), which show that the dependence characterized by the *co-occurrence* of negative asset and negative market returns possesses the highest explanatory power on the formation of asset returns among all specifications of disaggregated conventional beta. Importantly, we explicitly show that the premium for this risk is generated by the dependence in the extreme left tail and by its short-term component. In addition, we extend the analysis to extreme volatility risk and show that investors focus on more probable joint negative outcomes that unfold over the long horizon.

2.5.2 Other Portfolios

Finally, we investigate the pricing implication across multiple datasets, including popular Fama-French portfolios sorted on various characteristics. We use 30 industry portfolios, 25 portfolios sorted by size and value and decile portfolios sorted by operating profit, investment or book-to-market portfolios of Lettau et al. (2014) constructed from various asset classes, equity portfolios sorted on cash flow duration of Weber (2018) and finally investment strategies constructed across various asset classes from Ilmanen et al. (2021).

Figure 2.2 summarizes the estimation results for all these data. We report $t-$ statistics of estimated prices of QR risks over all portfolios and across tails, which gives a general overview of how tail- and horizon-specific risks are priced across a wide number of portfolios. Appendix 2.F then provides a detailed summary of all results as well as a data description.

Figure 2.2: *Estimated t-statistics of quantile spectral risk for various portfolios.*



Note: For each $\tau \in \{0.01, 0.05, 0.1, 0.15, 0.2, 0.25\}$ quantile, we show Fama-MacBeth t-statistics of estimated prices of risk of the QS models. Vertical line Group 11 portfolios depicted by different symbols. The t-statistics of short-run (red color) and long-run (blue color) risks are shown for tail risk (TR) models by a solid line, extreme volatility risk (EVR) by a dashed line and the full model by dotted lines. Horizontal lines depict 1% and 99% quantiles of the standard normal distribution.

We conclude that a phenomenon of short-term tail risk (TR) is universally priced (although with varying magnitude) across most of the datasets. The results of EVR are slightly more mixed. In the case of individual stocks, it is mostly the long-term part of the EVR that is priced in the cross-section of the expected returns. The same is also true for the aggregated dataset of Lettau et al. (2014). On the other hand, in the case of 25 portfolios sorted on size and value, the short-term part of EVR is priced. There are also datasets in which both components of the EVR risk are priced. These include equity portfolios sorted on cash flow duration of Weber (2018) and investment strategies constructed using various asset classes of Ilmanen et al. (2021). This heterogeneity gives investors the opportunity to follow certain investment strategies according to their aversion to certain risk in a given horizon.

2.6 Conclusion

We introduce a novel approach for isolating the effects of various risk dimensions on the formation of expected returns. Until now, studies have focused either on exploring downside features of risk or on investigating its horizon-specific properties. We define novel measures that estimate risk in a specific part of the joint distribution over a specific horizon, and we show that extreme risks are priced in a cross-section of asset returns heterogeneously across horizons. Furthermore, we argue that it is important to distinguish between tail market risk and extreme volatility risk. Tail market risk is characterized by the dependence between a highly negative market and asset events. Extreme volatility risk is defined as the co-occurrence of extremely high increases in market volatility and highly negative asset returns. Negative events are derived from the distribution of market returns, and their respective quantiles are used to determine threshold values for computing quantile spectral betas.

To consistently estimate the models, data with a sufficiently long history must be employed. However, if these data are available, our measures of risk are able to outperform related measures, and their performance is best for low threshold values, suggesting that investors require a risk premium for holding assets susceptible to extreme risks. Moreover, we show that the state-of-the-art downside risk measures do not capture the information contained in our newly proposed measures. Our results have important implications for asset pricing models. We show that only taking into account contemporaneous dependence averaged over the whole distribution when measuring risk exposure leads to the omitting of important information regarding the risk.

Future work may explore origins of the quantile spectral risk with a particular emphasis on the tail risk. From a data generating process perspective, these attempts could be based on the delayed price adjustment in the spirit of Bandi et al. (2021). From a preference standpoint, one could relate the quantile spectral risk to utility models such as power utility, habits, or non-separable utility specifications and investigate their pricing implications.

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Appendix

2.A Technical Appendix

In this section the proof of the results in Section 2.3.3 is given. Before we begin, note that by a trivial generalisation of Proposition 3.1 in Kley et al. (2016) we have that Assumption 1 implies that there exist constants $\rho \in (0,1)$ and $K < \infty$ such that, for arbitrary intervals $A_m, A_r \subset \mathbb{R}$, arbitrary times $t_m, t_r \in \mathbb{Z}$,

$$|\operatorname{cum}(I\{m_{t_m} \in A_m\}, I\{r_{t_r} \in A_r\})| \leq K\rho^{|t_m - t_r|}. \tag{27}$$

In addition, we will use the following lemma

Lemma .1. *(Baruník and Kley (2019)) Under the assumptions of Theorem 1, the derivative*

$$(\tau_m, \tau_r) \mapsto \frac{\mathrm{d}^k}{\mathrm{d}\omega^k} \mathfrak{f}^{m,r}(\omega; \tau_m, \tau_r)$$

exists and satisfies, for any $k \in \mathbb{N}_0$ and some constants C, d that are independent of $a = (a_m, a_r), b = (b_m, b_r)$, but may depend on k,

$$\sup_{\omega \in \mathbb{R}} \left| \frac{\mathrm{d}^k}{\mathrm{d}\omega^k} \mathfrak{f}^{m,r}(\omega; a_m, a_r) - \frac{\mathrm{d}^k}{\mathrm{d}\omega^k} \mathfrak{f}^{m,r}(\omega; b_m, b_r) \right| \leq C\|a - b\|_1 (1 + |\log \|a - b\|_1|)^D.$$

Following proposition further provides asymptotic properties of $I^{m,r}_{n,R}(\omega; \tau_m, \tau_r)$

Proposition 1. *(Baruník and Kley (2019)) Assume that $(\boldsymbol{x}_t)_{t \in \mathbb{Z}}$ is strictly stationary and satisfies Assumption 1. Further assume that the marginal distribution functions F_m, and F_r are continuous. Then, for every fixed $\omega \neq 0$ mod 2π,*

$$\left(I^{m,r}_{n,R}(\omega; \tau_m, \tau_r) \right)_{(\tau_m, \tau_r) \in [0,1]^2} \Rightarrow \left(\frac{1}{2\pi} \mathbb{D}^m(\omega; \tau_m) \mathbb{D}^r(-\omega; \tau_r) \right)_{(\tau_m, \tau_r) \in [0,1]^2}, \tag{28}$$

where $\mathbb{D}^m(\omega; \tau_m)$ and $\mathbb{D}^r(\omega; \tau_r)$, $\tau \in [0,1]$, $\omega \in \mathbb{R}$ are centered, $\mathbb{C}$-valued Gaus-

sian processes with covariance structure of the following form

$$\mathrm{Cov}(\mathbb{D}^m(\omega;\tau_m),\mathbb{D}^r(\omega;\tau_r)) = 2\pi\mathfrak{f}^{m,r}(\omega;\tau_m,\tau_r).$$

Moreover, the family $\{\mathbb{D}^m(\omega;\,\cdot),\mathbb{D}^r(\omega;\,\cdot)\ :\ \omega\in[0,\pi]\}$ *is a collection of independent processes. In particular, the weak convergence (28) holds jointly for any finite fixed collection of frequencies* ω*.*

For $\omega = 0 \mod 2\pi$ the asymptotic behaviour of the rank-based copula cross-periodogram is as follows: we have $d^j_{n,R}(0;\tau) = n\tau + o_p(n^{1/2})$, where the exact form of the remainder term depends on the number of ties in the process. Therefore, under the assumptions of Proposition 1, we have $I^{m,r}_{n,R}(0;\tau_m,\tau_r) = n(2\pi)^{-1}\tau_m\tau_r 11' + o_p(1)$, where $1 := (1,1)' \in \mathbb{R}^2$.

2.A.1 Proof of the Theorem 2

Proof. By a Taylor expansion we have, for every $y, y_0 > 0$,

$$\frac{1}{y} - \frac{1}{y_0} = -\frac{1}{y_0^2}(y-y_0) + 2\xi^{-3}_{y,y_0}(y-y_0)^2,$$

where ξ_{y,y_0} is between y and y_0. Let $R_n(y,y_0) := 2\xi^{-3}_{y,y_0}(y-y_0)^2$, then

$$\frac{x}{y} - \frac{x_0}{y_0} = \frac{x}{y} - \frac{x}{y_0} + \frac{x}{y_0} - \frac{x_0}{y_0} = \frac{1}{y_0}(y-y_0) - \frac{x_0}{y_0^2}(x-x_0) + r_n, \tag{29}$$

where $r_n = xR_n(y,y_0) + (x-x_0)^2/y_0^2$

Write $\mathfrak{f}_{a,b}$ for $\mathfrak{f}^{a,b}(\omega;\tau_a,\tau_b)$, $G_{a,b}$ for $\widehat{G}^{a,b}_{n,R}(\omega;\tau_a,\tau_b)$, and $B_{a,b}$ for $B^{a,b,(k)}_n(\omega;\tau_a,\tau_b)$ and let

$$\begin{aligned} x &:= G_{a,b} & y &:= G_{a,a} \\ x_0 &:= \mathfrak{f}_{a,b} + B_{a,b} & y_0 &:= \mathfrak{f}_{a,a} + B_{a,a} \end{aligned}$$

By Theorem 1 differences $x - x_0$ and $y - y_0$ are in $O_p((nb_n)^{-1/2})$, uniformly with respect to τ_m, τ_r. Under the assumption that $nb_n \to \infty$, as $n \to \infty$, this entails $G_{a,a} - B_{a,a} \to \mathfrak{f}_{a,a}$, in probability. For $\varepsilon \le \tau_1, \tau_2 \le 1-\varepsilon$, we have $\mathfrak{f}_{a,a} > 0$, such that, by the Continuous Mapping Theorem we have $(G_{a,a} - B_{a,a})^{-3} \to \mathfrak{f}^{-3}_{a,a}$, in probability. As $B_{a,a} = o(1)$, we have $y^{-3} - y_0^{-3} = o_p(1)$. Finally, due to

$$\xi^{-3}_{y,y_0} \le y_n^{-3} \vee y_0^{-3} \le (y_n^{-3} - y_0^{-3}) \vee 0 + y_0^{-3} = o_p(1) + O(1) = O_p(1),$$

we have that $R_n(y, y_0) = O_p((nb_n)^{-1})$.
So we have shown that

$$\widehat{\beta}^{m,r}_{n,R}(\omega;\tau_m,\tau_r) - \frac{\mathfrak{f}_{a,b} + B_{a,b}}{\mathfrak{f}_{a,a} + B_{a,a}} = \frac{1}{\mathfrak{f}_{m,m}}\Big([G_{m,m} - \mathfrak{f}_{m,m} - B_{m,m}] - \frac{\mathfrak{f}_{m,r}}{\mathfrak{f}_{m,m}}[G_{m,r} - \mathfrak{f}_{m,r} - B_{m,r}]\Big) + O_p\Big(1/(nb_n)$$

with the O_p holding uniformly with respect to τ_m, τ_r.Furthermore, note that setting

$$x := \mathfrak{f}_{a,b} + B_{a,b} \qquad y := \mathfrak{f}_{a,a} + B_{a,a}$$
$$x_0 := \mathfrak{f}_{a,b} \qquad y_0 := \mathfrak{f}_{a,a}$$

we have

$$\frac{\mathfrak{f}_{a,b} + B_{a,b}}{\mathfrak{f}_{a,a} + B_{a,a}} - \frac{\mathfrak{f}_{a,b}}{\mathfrak{f}_{a,a}} = \frac{1}{\mathfrak{f}_{a,a}}\Big(B_{a,a} - \frac{\mathfrak{f}_{a,b}}{\mathfrak{f}_{a,a}}B_{a,b}\Big) + O(|\boldsymbol{B}_{a,a}|^2 + |\boldsymbol{B}_{a,b}|^2).$$

By Lemma .1 we have that

$$\sup_{\tau_m,\tau_r\in[\varepsilon,1-\varepsilon]}\Big|\frac{\mathrm{d}^\ell}{\mathrm{d}\omega^\ell}\mathfrak{f}^{m,r}(\omega;\tau_m,\tau_r)\Big| \le C_{\varepsilon,\ell}.$$

Therefore, $B_{m,r}$ satisfies

$$\sup_{\tau_m,\tau_r\in[\varepsilon,1-\varepsilon]}\Big|\sum_{\ell=2}^{k}\frac{b_n^\ell}{\ell!}\int_{-\pi}^{\pi}v^\ell W(v)dv\frac{\mathrm{d}^\ell}{\mathrm{d}\omega^\ell}\mathfrak{f}^{m,r}(\omega;\tau_m,\tau_r)\Big| = o\Big((nb_n)^{-1/4}\Big),$$

which implies that

$$|\boldsymbol{B}_{a,a}|^2 + |\boldsymbol{B}_{a,b}|^2 = o\Big((nb_n)^{-1/2}\Big).$$

Therefore,

$$\sqrt{nb_n}\Bigg(\widehat{\beta}^{m,r}_{n,R}(\omega;\tau_m,\tau_r) - \underbrace{\frac{\mathfrak{f}_{a,b}}{\mathfrak{f}_{a,a}}}_{=:\beta^{m,r}(\omega;\tau_m,\tau_r)} - \underbrace{\frac{1}{\mathfrak{f}_{a,a}}\Big(\boldsymbol{B}_{a,a} - \frac{\mathfrak{f}_{a,b}}{\mathfrak{f}_{a,a}}\boldsymbol{B}_{a,b}\Big)}_{=:B_n^{m,r,(k)}(\omega;\tau_m,\tau_r)}\Bigg)$$

and

$$\sqrt{nb_n}\frac{1}{\mathfrak{f}_{m,m}}\Big([G_{m,r} - \mathfrak{f}_{m,r} - B_{m,r}] - \frac{\mathfrak{f}_{m,r}}{\mathfrak{f}_{m,m}}[G_{m,m} - \mathfrak{f}_{m,m} - B_{m,m}]\Big)$$

are asymptotically equivalent in the sense that if one of the two converges weakly, then so does the other. The assertion then follows by Theorem 1, Slutzky's lemma and the Continuous Mapping Theorem.

□

2.A.2 Construction of pointwise confidence bands for Quantile Spectral Beta

Following Baruník and Kley (2019) and Theorem 2, we construct pointwise asymptotic $(1-\alpha)$ level confidence bands for the real and imaginary parts of $\beta^{m,r}_{n,R}(\omega;\tau_m,\tau_r)$ as follows:

$$C^{(2)}_{\mathrm{r},n}(\omega_{kn};\tau_m,\tau_r) := \Re\widehat{\beta}^{m,r}_{n,R}(\omega_{kn};\tau_m,\tau_r) \pm \Re\sigma^{m,r}_{(2)}(\omega_{kn};\tau_m,\tau_r)\Phi^{-1}(1-\alpha/2),$$

for the real part, and

$$C^{(2)}_{\mathrm{i},n}(\omega_{kn};\tau,\tau_r) := \Im\widehat{\beta}^{m,r}_{n,R}(\omega_{kn};\tau_m,\tau_r) \pm \Im\sigma^{m,r}_{(2)}(\omega_{kn};\tau_m,\tau_r)\Phi^{-1}(1-\alpha/2),$$

for the imaginary part of the quantile spectral beta. Here, Φ stands for the cdf of the standard normal distribution,

$$\Big(\Re\sigma^{m,r}_{(2)}(\omega_{kn};\tau_m,\tau_r)\Big)^2 := 0 \vee \begin{cases} 0 & \text{if } m=r \\ & \quad \text{and } \tau_m=\tau_r, \\ \frac{1}{2}\Big(\mathbb{C}ov(\mathbb{L}_{m,r},\mathbb{L}_{m,r}) + \Re\mathbb{C}ov(\mathbb{L}_{m,r},\mathbb{L}_{r,m})\Big) & \text{otherwise,} \end{cases}$$

and

$$\Big(\Im\sigma^{m,r}_{(2)}(\omega_{kn};\tau_m,\tau_r)\Big)^2 := 0 \vee \begin{cases} 0 & \text{if } m=r \\ & \quad \text{and } \tau_m=\tau_r, \\ \frac{1}{2}\Big(\mathbb{C}ov(\mathbb{L}_{m,r},\mathbb{L}_{m,r}) - \Re\mathbb{C}ov(\mathbb{L}_{m,r},\mathbb{L}_{r,m})\Big) & \text{otherwise.} \end{cases}$$

where $\mathbb{L}_{a,b} = \frac{1}{\mathfrak{f}_{a,a}}\Big(\mathbb{H}_{a,a} - \frac{\mathfrak{f}_{a,b}}{\mathfrak{f}_{a,a}}\mathbb{H}_{a,b}\Big)$. The definition of $\sigma^{m,r}_{(2)}(\omega_{kn};\tau_m,\tau_r)$ is motivated by noting that for any complex-valued random variable Z, with complex conjugate $\bar{Z}$,

$$\mathrm{Var}(\Re Z) = \frac{1}{2}\Big(\mathrm{Var}(Z) + \Re\mathbb{C}ov(Z,\bar{Z})\Big); \quad \mathrm{Var}(\Im Z) = \frac{1}{2}\Big(\mathrm{Var}(Z) - \Re\mathbb{C}ov(Z,\bar{Z})\Big), \tag{30}$$

and we have $\overline{\mathbb{L}_{m,r}} = \mathbb{L}_{r,m}$. Furthermore, note that $\widehat{\beta}^{m,r}_{n,R}(\omega_{kn};\tau_m,\tau_r) = 1$, if $m = r$ and $\tau_m = \tau_r$. We have used $\mathbb{C}ov(\mathbb{L}_{a,b},\mathbb{L}_{c,d})$ to denote an estimator for

$$\mathbb{C}ov\Big(\mathbb{L}^{a,b}(\omega_{kn};\tau_a,\tau_b\Big),\mathbb{L}^{c,d}(\omega_{kn};\tau_c,\tau_d)\Big).$$

Recalling the definition of the limit process in Theorem 2 we derive the following expression:

$$\begin{aligned}\mathbb{C}ov(\mathbb{L}_{a,b},\mathbb{L}_{c,d}) &= \frac{1}{\mathfrak{f}_{a,a}\mathfrak{f}_{c,c}}\mathbb{C}ov\Big(\mathbb{H}_{a,a}-\frac{\mathfrak{f}_{a,b}}{\mathfrak{f}_{a,a}}\mathbb{H}_{a,b},\mathbb{H}_{c,c}-\frac{\mathfrak{f}_{c,d}}{\mathfrak{f}_{c,c}}\mathbb{H}_{c,d}\Big)\\ &= \frac{\mathbb{C}ov(\mathbb{H}_{a,a},\mathbb{H}_{c,c})}{\mathfrak{f}_{a,a}\mathfrak{f}_{c,c}} - \frac{\overline{\mathfrak{f}_{c,d}}\mathbb{C}ov(\mathbb{H}_{a,a},\mathbb{H}_{c,d})}{\mathfrak{f}_{a,a}\mathfrak{f}^2_{c,c}}\\ &\quad - \frac{\mathfrak{f}_{a,b}\mathbb{C}ov(\mathbb{H}_{a,b},\mathbb{H}_{c,c})}{\mathfrak{f}^2_{a,a}\mathfrak{f}_{c,c}} + \frac{\mathfrak{f}_{a,b}\overline{\mathfrak{f}_{c,d}}\mathbb{C}ov(\mathbb{H}_{a,b},\mathbb{H}_{c,d})}{\mathfrak{f}^2_{a,a}\mathfrak{f}^2_{c,c}},\end{aligned}$$

where we have written $\mathfrak{f}_{i,j}$ for the quantile spectral density $\mathfrak{f}^{i,j}(\omega_{kn};\tau_i,\tau_j)$, and $\mathbb{H}_{i,j}$ for the limit distribution $\mathbb{H}^{i,j}(\omega_{kn};\tau_i,\tau_j\Big)$ for any $i,j = a,b,c,d$.

Thus, considering the special case where $a = c = m$ and $b = d = r$, we have

$$\begin{aligned}\mathbb{C}ov(\mathbb{L}_{m,r},\mathbb{L}_{m,r}) &= \frac{1}{\mathfrak{f}^2_{m,m}}\mathbb{C}ov(\mathbb{H}_{m,m},\mathbb{H}_{m,m}) - \frac{\mathfrak{f}_{r,m}}{\mathfrak{f}^3_{m,m}}\mathbb{C}ov(\mathbb{H}_{m,m},\mathbb{H}_{m,r})\\ &\quad - \frac{\mathfrak{f}_{m,r}}{\mathfrak{f}^3_{m,m}}\mathbb{C}ov(\mathbb{H}_{m,r},\mathbb{H}_{m,m}) + \frac{|\mathfrak{f}_{m,r}|^2}{\mathfrak{f}^4_{m,m}}\mathbb{C}ov(\mathbb{H}_{m,r},\mathbb{H}_{m,r}).\end{aligned}$$

and for the special case where $a = d = m$ and $c = b = r$ we have

$$\begin{aligned}\mathbb{C}ov(\mathbb{L}_{m,r},\mathbb{L}_{r,m}) &= \frac{1}{\mathfrak{f}_{m,m}\mathfrak{f}_{r,r}}\mathbb{C}ov(\mathbb{H}_{m,m},\mathbb{H}_{r,r}) - \frac{\mathfrak{f}_{m,r}}{\mathfrak{f}_{m,m}\mathfrak{f}^2_{r,r}}\mathbb{C}ov(\mathbb{H}_{m,m},\mathbb{H}_{r,m})\\ &\quad - \frac{\mathfrak{f}_{m,r}}{\mathfrak{f}^2_{m,m}\mathfrak{f}_{r,r}}\mathbb{C}ov(\mathbb{H}_{m,r},\mathbb{H}_{r,r}) + \frac{\mathfrak{f}^2_{m,r}}{\mathfrak{f}^2_{m,m}\mathfrak{f}^2_{r,r}}\mathbb{C}ov(\mathbb{H}_{m,r},\mathbb{H}_{r,m}).\end{aligned}$$

Finally, we substitute consistent estimators for the unknown quantities. To do so we abuse notation using $\mathfrak{f}_{a,b}$ to denote $\tilde{G}^{a,b}_{n,R}(\omega_{kn};\tau_a,\tau_b)$ and motivated by Theorem 7.4.3 in Brillinger (1975), we use

$$\begin{aligned}&\Big(\frac{2\pi}{n\cdot W^k_n}\Big)\times\Bigg[\sum_{s=1}^{n-1} W_n\Big(2\pi(k-s)/n\Big)W_n\Big(2\pi(k-s)/n\Big)\tilde{G}^{a,c}_{n,R}(\tau_a,\tau_c;2\pi s/n)\tilde{G}^{b,d}_{n,R}(\tau_b,\tau_d;-2\pi s/n)\\ &+\sum_{s=1}^{n-1} W_n\Big(2\pi(k-s)/n\Big)W_n\Big(2\pi(k+s)/n\Big)\tilde{G}^{a,d}_{n,R}(\tau_a,\tau_d;2\pi s/n)\tilde{G}^{b,c}_{n,R}(\tau_b,\tau_c;-2\pi s/n)\Bigg]\end{aligned} \tag{31}$$

to estimate $\mathrm{Cov}(\mathbb{H}_{a,b}, \mathbb{H}_{c,d})$.

2.B Rare Disaster Risk Model and QS Betas

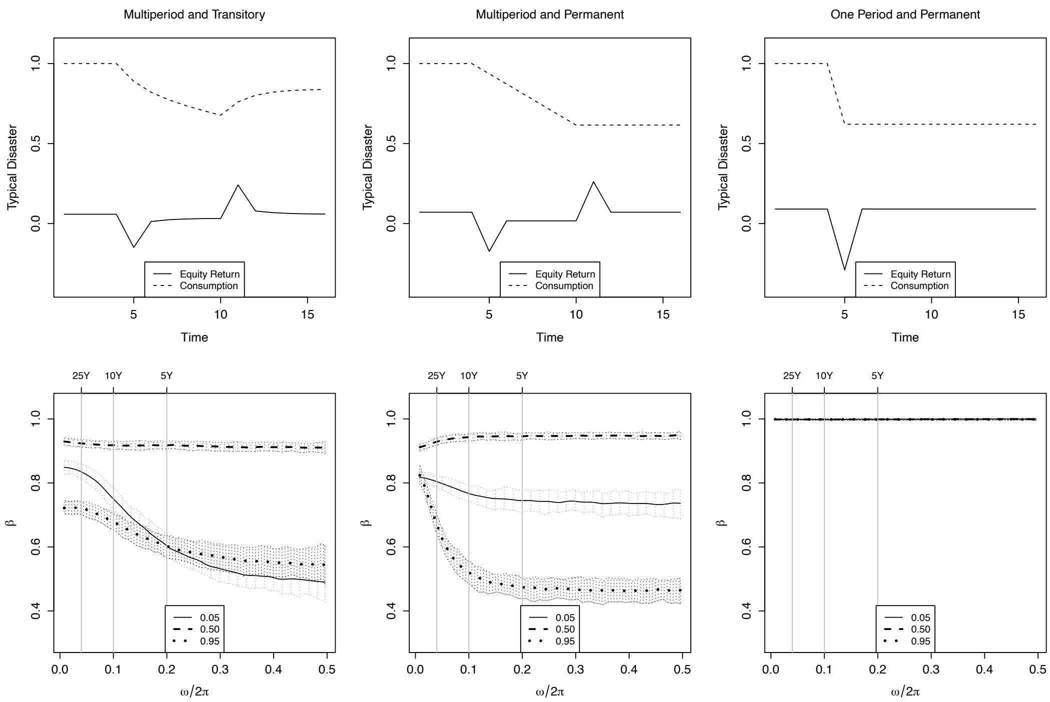
We show how the QS betas relate to the asset pricing model of Nakamura et al. (2013). Their extension of disaster risk model originally proposed by Rietz (1988) and Barro (2006) enables disasters to unfold over multiple periods and partially recover after the disaster. We argue that the QS betas can capture the complex joint dynamics between consumption growth and equity return. To do that, we simulate consumption growth and solve for equity return from three specification of the rare disaster model: 1) Model in which a disaster unfolds over multiple periods, and then a partial recovery occurs. 2) Model with unfolding disaster over multiple periods, but the disaster is permanent. 3) Model with one period disaster which is permanent. We assume preferences of Epstein and Zin (1989) and Weil (1990) and follow Nakamura et al. (2013) in the estimation procedure using their dataset, solution procedure and values of preference parameters.[18] Namely, we set the CRRA, $\gamma = 6.5$, the IES, $\psi = 2$ and the discount factor, $\beta = \exp(-0.034)$.

Figure 3 presents the main results. The first row of the figure contains courses of typical disasters with respect to the detrended consumption and equity return (return on unleveraged consumption claim). We observe that at the onset of the disaster (first drop of the consumption), there is a visible contemporaneous drop at equity return, as well. In case of unfolding disasters, after the end of the disaster period, there is a noticeable positive jump in the equity return. The lower panel of the figure contains QS betas and their 90% confidence intervals simulated from the respective models. Each model is simulated 100 times and each simulation produces a time series of length 50,000 years (we simulate yearly observations).

We can see that the dependence in the median (given by the line corresponding to $\tau = 0.50$) does not dramatically differ across the specifications and is constant over horizons. This implies that using a simple covariance based measure, we cannot distinguish between joint dynamics across different specifications. The most important part of the joint structure contain the tails of the joint distribution over specific horizons. We may think of one period and permanent specification as a benchmark specification. In this case, on average, the extreme events occur contemporaneously and thus the beta across horizons is flat. If we look at the cases with unfolding disasters, the QS betas for the

[18]The code supplementing Nakamura et al. (2013) can be downloaded from `https://eml.berkeley.edu/~enakamura/papers.html`

Figure 3: *QS Betas between Consumption Growth and Equity Return.*



Note: First row depicts typical disasters for various specifications of rare disaster risk model as specified in Nakamura et al. (2013). Second row captures QS betas and their 90% confidence intervals for those specifications. For each specification, QS betas are estimated using 100 simulations of consumption growth series and equity return of length 50,000. Models and parameter values follow Nakamura et al. (2013).

left tail due to the persistency of the disaster posses its peak at the longer horizons. For the case of multiperiod and transitory disaster, the QS betas for the upper tail are very similar to the QS betas for the lower tail, because after the end of the disaster, consumption partially recovers over multiple periods, which mirrors the joint dynamics at the onset of the disaster. On the other hand, in case of multiperiod permanent disaster, at the end of the disaster, there is a positive jump in equity return, but there is no recovery in the consumption. This makes the QS betas peaking at the longer horizons, as there is typically no contemporaneous positive jump in consumption growth and equity return at the end of the disaster.

2.C Features of QS Betas

2.C.1 Summary Statistics about Quantile Spectral Betas

Table 3: *Descriptive Statistics.*

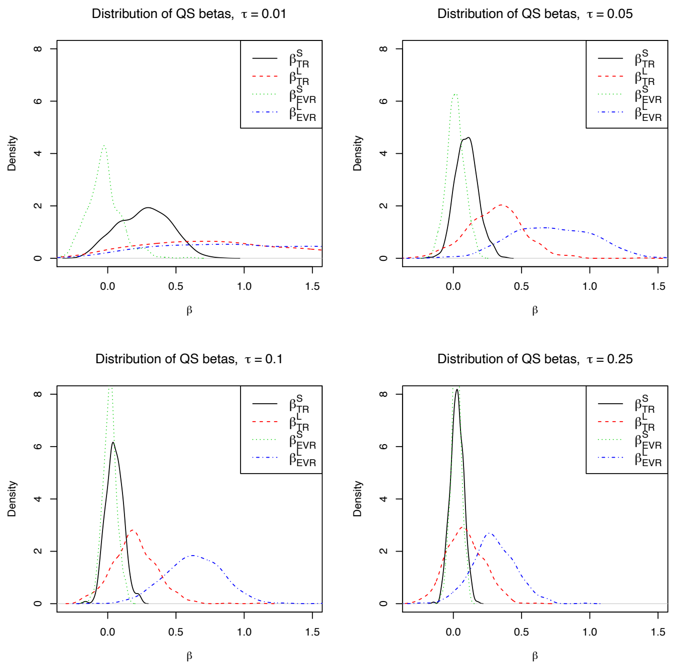
	$\tau = 0.05$					$\tau = 0.10$				
	β^{CAPM}	β^{rel}_{long}	β^{rel}_{short}	β^{EVR}_{long}	β^{EVR}_{short}	β^{CAPM}	β^{rel}_{long}	β^{rel}_{short}	β^{EVR}_{long}	β^{EVR}_{short}
Mean	1.068	0.310	0.098	0.726	0.016	1.068	0.197	0.051	0.632	0.015
Median	1.084	0.324	0.096	0.715	0.016	1.084	0.191	0.048	0.634	0.016
St. Dev.	0.372	0.208	0.083	0.296	0.065	0.372	0.164	0.064	0.212	0.051
β^{CAPM}	1.000	0.234	-0.188	0.595	0.041	1.000	-0.040	-0.100	0.435	0.066
β^{rel}_{long}	0.234	1.000	0.147	0.688	0.032	-0.040	1.000	0.275	0.595	0.055
β^{rel}_{short}	-0.188	0.147	1.000	-0.062	-0.053	-0.100	0.275	1.000	0.104	-0.073
β^{EV}_{long}	0.595	0.688	-0.062	1.000	0.025	0.435	0.595	0.104	1.000	0.112
β^{EV}_{short}	0.041	0.032	-0.053	0.025	1.000	0.066	0.055	-0.073	0.112	1.000

Note: Table summarizes basic descriptive statistics and correlation structure for all betas from our Full model for the two choices of the quantile levels. Betas are computed using CRSP database sampled between July 1926 and December 2015. Presented results are computed on our largest sample, i.e., using stocks with at least 50 years of history. Long horizon is given by frequencies corresponding to 3-year cycle and longer.

We are interested to see what distributions of estimated quantile spectral betas reveal, and so we display the unconditional distribution of the estimated betas used in the TR, EVR and Full models. Table 3 summarizes descriptive statistics for all estimated betas. We focus on two values of τ - 0.05, and 0.10, and present cross-sectional means, medians and standard deviations of the estimated parameters in the top panel. We observe that all the betas are on average positive. This is particularly interesting for relative TR betas, which means that, roughly speaking, average stock posses higher tail dependence with market than suggested by the simple covariance based measures. Bottom panel of Table 3 presents correlation structure of TR, EVR and CAPM betas. We observe higher values of correlation between long-term betas, and also between long-term EVR and CAPM betas. Nevertheless, all these correlation are far below 1, which suggests that all the variables may posses different and potentially important information regarding the risk associated with the assets. Another interesting observations is that the relative TR betas, both long- and short-term, are almost uncorrelated with the CAPM betas, which is exactly what we want to see given their definition.

To further visualize the distributional features, Figure 4 presents unconditional distributions of the betas for four different threshold value for quantile levels. We observe the highest dispersion of betas for the lowest values of τ corresponding the the most extreme case. As we move to higher values of τ, the distributions exhibit less and less variance. Moreover, the distribution of long-term betas is wider than the distribution of the short-term betas for the respective risks.

Figure 4: *Distribution of TR and EVR betas at different tails.*



Note: Plots display kernel density estimates of the unconditional distribution of the short-term and long-term TR and EVR betas. Presented results are computed on our largest cross-section, i.e., using stocks with at least 50 years of history. Long horizon is given by frequencies corresponding to 3-year cycle and longer.

2.C.2 Robustness Checks: Tail Risk across Horizons and Other Risk Factors

Large number of other risk factors and firm characteristics have been documented by the literature as significant drivers of the cross-sectional variation in equity returns (Harvey et al. 2016). While we do not attempt to include the whole exhaustive set of all controls, we would like to see if our newly defined risk factors are not subsumed by a subset of prominent variables, as well as variables related to the tails and moments of the return distribution. Hence we naturally focus on the downside measures and we use downside risks proposed by Ang et al. (2006), downside risk beta specification of Lettau et al. (2014) as well as recently proposed five factor generalized disappointment aversion (GDA5) model by Farago and Tédongap (2018). Further we use coskewness and cokurtotis measures, as well as size, book-to-market and momentum factors used by Fama and French (1993).

To investigate whether our newly proposed measures of risk can be driven out by other determinants of risk proposed earlier in the literature, we include these risks as control variables in the previous regressions. First, we focus on the GDA5 model proposed by Farago and Tédongap (2018) as these are the risks most closely related to ours. It contains two measures of tail market risk as well as two measures of extreme volatility risk, but focuses on various specifications of downside dependence and does not take into consideration frequency aspect of the risks. Based on these related measures, we compare risk measures associated with market return, and market volatility increments separately. The aim of this analysis is to decide which measures of risk better capture the notion of extreme risks associated with risk premium. The detailed specification of the corresponding betas can be found in Appendix 2.E.

Table 4 reports the risk premium of our quantile spectral risks controlled for the GDA5 risks. In case of tail market risk presented in left panel, we see that GDA5 measures of risk (λ_D and λ_{WD}) do not drive out our measures for any value of τ and remain insignificant when we include our TR measures. Moreover, the pattern of prices of risk corresponding to TR betas remain the same as in the TR and Full model specifications. This clearly suggests that our measures captures the asymmetric features of risk priced in the cross-section of assets.

In case of extreme volatility risk, we see from the right panel of Table 4 that the situation is similar. Especially, the price of risk for long-term EVR betas stays significantly strong for higher values of quantile. In addition, short-term EVR betas emerge as a significant predictors for the lower values of τ. On the other hand, GDA5 measures of volatility risk remain insignificant in all of the cases. All the results suggest that our model brings an improvement in terms of identifying form of asymmetric risk which is priced in the cross-section of asset returns.

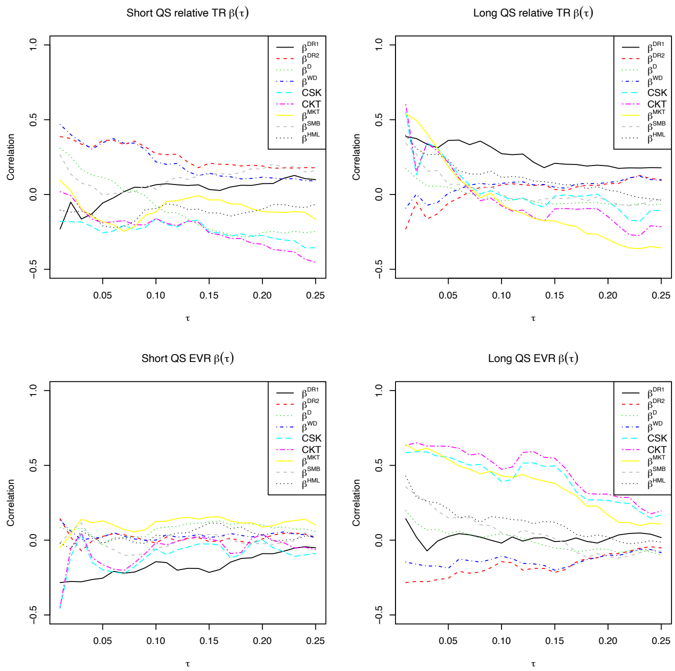
From these results, we can infer that our QS measures may potentially provide an additional information not captured by other risk measures. To further investigate this hypothesis, we present correlation structure of our QS measures with all other highly discussed asset pricing risk measures in Figure 5. Details regarding their specifications are contained in the Applendix 2.E. We plot dependence between them and the QS measures with respect to the value of quantile of the threshold value. Generally, our measures posses the

Table 4: *Estimated Coefficients of the TR and EVR Models Controlled for GDA5 Measures.*

		Tail market risk						Extreme volatility risk					
	τ	λ_D	λ_{WD}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	λ^{CAPM}	RMSPE	λ_X	λ_{XD}	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE
	0.01	-0.027	0.118	-0.034	0.628	0.760	26.377	0.848	0.775	-0.096	0.773	0.735	28.313
		-0.987	0.597	-0.364	3.231	3.879		1.326	1.450	-1.155	3.892	3.781	
	0.05	-0.024	0.207	0.050	1.149	0.779	26.434	0.108	0.285	0.122	1.294	0.704	27.698
		-0.908	1.037	0.234	3.066	4.153		0.169	0.526	0.612	3.431	3.805	
	0.1	-0.017	0.253	0.305	0.795	0.799	26.688	0.157	0.343	0.388	1.220	0.724	26.560
70 years		-0.622	1.279	1.243	1.488	4.436		0.246	0.633	1.580	2.344	4.066	
(142 assets)	0.15	-0.010	0.214	0.449	0.746	0.779	26.326	0.219	0.367	0.572	0.786	0.720	26.627
		-0.357	1.090	1.992	1.210	4.297		0.344	0.678	2.422	1.325	4.018	
	0.2	-0.010	0.244	0.526	0.301	0.824	26.532	0.302	0.492	0.723	0.176	0.766	25.345
		-0.324	1.258	2.115	0.401	4.630		0.473	0.900	2.732	0.257	4.374	
	0.25	-0.018	0.268	0.587	-0.064	0.856	26.693	0.377	0.554	0.850	-0.332	0.793	25.364
		-0.587	1.423	2.300	-0.086	4.878		0.587	1.007	3.089	-0.468	4.576	
	0.01	-0.014	0.141	-0.011	0.338	0.765	29.578	0.367	0.032	-0.060	0.495	0.753	30.288
		-0.584	0.809	-0.151	2.269	4.168		0.631	0.068	-0.950	3.104	4.069	
	0.05	-0.009	0.110	0.187	1.094	0.681	28.606	-0.050	-0.139	0.166	1.218	0.663	29.764
		-0.364	0.627	1.030	3.437	3.719		-0.086	-0.297	0.949	3.679	3.608	
	0.1	0.000	0.164	0.283	0.850	0.740	29.013	-0.044	-0.192	0.284	1.004	0.721	28.928
60 years		0.002	0.946	1.277	2.166	4.166		-0.076	-0.408	1.253	2.453	4.071	
(267 assets)	0.15	0.001	0.182	0.464	0.627	0.733	28.962	-0.076	-0.234	0.479	0.774	0.713	29.459
		0.022	1.067	2.097	1.279	4.108		-0.131	-0.500	2.104	1.570	4.020	
	0.2	-0.007	0.244	0.449	0.092	0.796	29.317	-0.032	-0.147	0.473	0.240	0.765	28.616
		-0.259	1.414	1.906	0.173	4.541		-0.056	-0.314	1.966	0.458	4.401	
	0.25	-0.005	0.237	0.425	0.199	0.813	29.566	-0.028	-0.143	0.477	0.182	0.784	29.268
		-0.205	1.399	1.814	0.367	4.691		-0.049	-0.305	1.995	0.337	4.557	
	0.01	-0.028	0.114	-0.054	0.396	0.823	29.452	0.254	-0.016	-0.108	0.489	0.822	29.913
		-1.399	0.799	-0.968	3.126	4.643		0.465	-0.037	-2.154	3.272	4.611	
	0.05	-0.028	0.118	0.009	1.097	0.778	29.023	0.016	-0.098	-0.049	1.191	0.762	29.791
		-1.389	0.781	0.061	4.134	4.373		0.030	-0.229	-0.359	4.120	4.279	
	0.1	-0.026	0.204	0.161	0.503	0.826	29.567	-0.056	-0.165	0.120	0.843	0.788	29.443
50 years		-1.261	1.338	0.821	1.614	4.726		-0.104	-0.390	0.652	2.559	4.499	
(528 assets)	0.15	-0.025	0.224	0.347	0.452	0.809	29.393	-0.122	-0.219	0.331	0.868	0.769	29.895
		-1.172	1.490	1.827	1.077	4.610		-0.227	-0.518	1.772	2.131	4.378	
	0.2	-0.031	0.268	0.279	0.153	0.860	29.664	-0.107	-0.203	0.260	0.679	0.811	29.577
		-1.357	1.786	1.450	0.307	4.945		-0.200	-0.476	1.350	1.516	4.685	
	0.25	-0.024	0.246	0.250	0.478	0.863	29.723	-0.110	-0.203	0.251	0.804	0.822	29.759
		-1.125	1.673	1.287	0.922	5.015		-0.204	-0.472	1.284	1.672	4.795	

Note: Table reports coefficients and their t-statistics from the horse race estimations. Displayed are prices of risk of three-factor models also including the GDA5 measures for corresponding risks. We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

Figure 5: *Correlations with Other Risk Measures.*



Note: Plots display correlations between the QS betas and various other risk measures widely used in the asset pricing literature using CRSP database between July 1926 and December 2015. Presented results are computed on our largest sample, i.e., using stocks with at least 50 years of history. Long horizon is given by frequencies corresponding to 3-year cycle and longer.

highest correlation with coskewness and cokurtosis and market beta (computed using FF3 specification) in the extreme left tail and long horizon, while they show high correlation with downside risk measures in extreme left tail at short horizon. This suggests that downside risk measures capture short-term risk while moment-based risk measures are more related to the extreme volatility in the long-term. Although the correlations in few cases exceed 0.5 in absolute value, all the values are well below 1 suggesting potentially important additional information regarding the risk.

Next, we check whether these measures can drive out our QS measures in the cross-sectional estimation. Table 5 reports the results of risk prices controlled for coskewness and cokurtosis risks. We first include coskewness into our Full model and check whether it can drive out our risk measures. We can see that although the coskewness is significant, it does not drive out our QS measures, which follow the same pattern as in the case of previous specifications of the models. Table 5 also reports in the right panel horse race regression including cokurtosis. We observe that cokurtosis does not bring any new explanatory information when included in our full model, as the corresponding estimated coefficients for cokurtosis are insignificant for all specifications.

In addition, Table 6 reports the results controlled for the two specification of relative downside betas. In the left panel, we report results with downside risk specification of Ang et al. (2006). We observe that the downside risk beta does not capture any additional important dimension of risk when included in our full model specification. The same is true for the downside risk model of Lettau et al. (2014), which is captured in the right panel.

Finally, Table 7 reports regressions including additional betas from the three-factor model of Fama and French (1993).[19] This model is not explicitly related to the asymmetric features of market or volatility risk, but as we show in the Section 2.3, these factors may be just capturing market risk in different horizons in specific parts of the joint distribution of market and asset returns, so we should check whether they are not superior in describing these kind of risks. As in the case of other horse race regressions, the additional risk factors do not drive out the QS measures, which repeat the same pattern as in the cases without the additional variables.

2.D Different Definition of Long horizon - 1.5 years

[19]We have to include only 2 additional betas as the market beta is already included in our full model.

Table 5: *Estimated Coefficients of the Full Models Controlled for Coskewness and Cokurtosis.*

		Coskewness							Cokurtosis						
	τ	λ^{CSK}	$\lambda^{\mathrm{TR}}_{\mathrm{long}}$	$\lambda^{\mathrm{TR}}_{\mathrm{short}}$	$\lambda^{\mathrm{EV}}_{\mathrm{long}}$	$\lambda^{\mathrm{EV}}_{\mathrm{short}}$	λ^{CAPM}	RMSPE	λ^{CKT}	$\lambda^{\mathrm{TR}}_{\mathrm{long}}$	$\lambda^{\mathrm{TR}}_{\mathrm{short}}$	$\lambda^{\mathrm{EV}}_{\mathrm{long}}$	$\lambda^{\mathrm{EV}}_{\mathrm{short}}$	λ^{CAPM}	RMSPE
	0.01	-0.255	0.122	0.493	-0.122	-0.320	0.790	25.630	-0.020	0.113	0.676	-0.197	-0.241	0.928	26.332
		-1.321	0.879	2.420	-0.854	-1.021	4.152		-0.942	0.820	3.374	-1.390	-0.777	4.217	
	0.05	-0.316	0.016	0.983	0.093	-0.179	0.747	25.911	-0.009	0.017	1.288	0.094	0.236	0.725	26.756
		-1.701	0.044	2.674	0.327	-0.343	3.279		-0.452	0.049	3.492	0.330	0.467	2.874	
	0.1	-0.345	-0.147	0.633	0.470	-0.453	0.610	25.532	-0.022	-0.294	0.952	0.642	0.165	0.575	26.478
70 years		-1.833	-0.368	1.183	1.582	-0.642	2.684		-0.988	-0.709	1.817	1.921	0.242	2.310	
(142 assets)	0.15	-0.344	-0.028	0.301	0.584	-0.205	0.616	25.214	-0.021	0.106	0.647	0.541	0.425	0.663	26.262
		-1.644	-0.084	0.475	2.165	-0.280	3.040		-0.903	0.319	1.062	1.926	0.595	2.721	
	0.2	-0.281	-0.152	0.327	0.743	-0.395	0.630	24.616	-0.023	-0.135	0.509	0.845	-0.052	0.668	25.353
		-1.429	-0.443	0.466	2.925	-0.510	3.234		-0.993	-0.389	0.720	3.111	-0.067	2.735	
	0.25	-0.279	0.001	-0.310	0.754	-0.761	0.709	24.895	-0.017	-0.005	0.021	0.896	-0.477	0.721	25.653
		-1.417	0.004	-0.402	2.835	-0.960	3.752		-0.738	-0.015	0.027	3.089	-0.607	2.921	
	0.01	-0.342	0.159	0.218	-0.091	0.021	0.776	28.728	-0.011	0.169	0.409	-0.229	0.222	0.908	29.442
		-1.988	1.343	1.308	-0.751	0.074	4.427		-0.531	1.424	2.462	-1.857	0.798	4.486	
	0.05	-0.340	-0.071	0.767	0.252	0.144	0.636	28.063	-0.014	0.026	1.225	0.152	0.519	0.672	28.600
		-2.095	-0.233	2.466	0.950	0.338	2.772		-0.749	0.091	3.818	0.629	1.236	2.697	
	0.1	-0.377	-0.385	0.559	0.597	-0.272	0.545	27.857	-0.034	-0.499	0.805	0.812	0.163	0.544	28.572
60 years		-2.406	-1.164	1.441	2.017	-0.522	2.338		-1.632	-1.518	2.029	2.625	0.325	2.188	
(267 assets)	0.15	-0.368	-0.026	0.067	0.505	-0.079	0.624	28.124	-0.018	0.176	0.618	0.397	0.281	0.668	29.093
		-2.246	-0.092	0.139	1.963	-0.139	3.121		-0.860	0.645	1.259	1.707	0.514	2.848	
	0.2	-0.350	-0.266	-0.058	0.715	-0.562	0.639	27.744	-0.025	-0.321	0.360	0.876	-0.254	0.661	28.553
		-2.185	-0.941	-0.114	2.973	-0.954	3.313		-1.211	-1.161	0.701	3.568	-0.433	2.815	
	0.25	-0.361	-0.034	-0.471	0.553	-1.001	0.747	28.311	-0.014	-0.065	0.252	0.681	-0.524	0.725	29.336
		-2.241	-0.132	-0.845	2.564	-1.665	4.073		-0.691	-0.252	0.448	2.990	-0.889	3.127	
	0.01	-0.406	0.046	0.184	-0.014	0.116	0.820	29.347	-0.020	-0.003	0.438	-0.070	0.365	0.940	29.666
		-2.735	0.522	1.259	-0.156	0.599	4.720		-0.973	-0.035	2.858	-0.757	1.906	4.605	
	0.05	-0.398	-0.112	0.754	0.159	0.273	0.746	28.705	-0.026	-0.100	1.215	0.123	0.569	0.815	29.028
		-2.693	-0.403	2.862	0.686	0.837	3.383		-1.404	-0.396	4.236	0.602	1.721	3.413	
	0.1	-0.447	-0.292	0.320	0.417	-0.181	0.682	28.936	-0.039	-0.412	0.656	0.603	0.009	0.716	29.371
50 years		-3.130	-0.933	1.042	1.660	-0.433	3.115		-1.955	-1.420	2.093	2.378	0.022	3.058	
(528 assets)	0.15	-0.419	0.169	0.139	0.143	-0.323	0.793	29.185	-0.018	0.362	0.660	0.008	-0.067	0.852	29.531
		-2.872	0.619	0.363	0.620	-0.697	4.066		-0.920	1.416	1.691	0.036	-0.148	3.734	
	0.2	-0.412	-0.077	-0.056	0.365	-0.522	0.776	29.196	-0.030	-0.120	0.332	0.498	-0.382	0.828	29.700
		-2.871	-0.309	-0.129	1.636	-1.091	4.030		-1.516	-0.509	0.772	2.256	-0.798	3.596	
	0.25	-0.414	0.043	-0.123	0.263	-0.607	0.828	29.387	-0.023	0.020	0.475	0.367	-0.263	0.854	29.880
		-2.905	0.191	-0.259	1.361	-1.262	4.514		-1.142	0.089	1.016	1.872	-0.547	3.746	

Note: Displayed are prices of risk of full models also including either coskewness or cokurtosis. We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

Table 6: *Estimated Coefficients of the Full Models controlled for Downside Risk Betas.*

		DR beta of Ang et al. (2006)							DR of Lettau et al. (2014)						
	τ	λ^{DR1}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE	λ^{DR2}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE
	0.01	-0.017	0.129	0.629	-0.210	-0.194	0.826	26.387	-0.017	0.129	0.629	-0.210	-0.194	0.826	26.387
		-0.044	0.922	3.240	-1.484	-0.631	4.104		-0.044	0.922	3.240	-1.484	-0.631	4.104	
	0.05	0.149	0.042	1.250	-0.005	0.285	0.754	26.736	0.149	0.042	1.250	-0.005	0.285	0.754	26.736
		0.410	0.119	3.448	-0.017	0.559	3.216		0.410	0.119	3.448	-0.017	0.559	3.216	
	0.1	0.082	-0.086	0.997	0.412	0.212	0.573	26.548	0.082	-0.086	0.997	0.412	0.212	0.573	26.548
70 years		0.235	-0.216	1.895	1.414	0.306	2.451		0.235	-0.216	1.895	1.414	0.306	2.451	
(142 assets)	0.15	0.128	0.211	0.864	0.325	0.367	0.636	26.259	0.128	0.211	0.864	0.325	0.367	0.636	26.259
		0.363	0.638	1.433	1.272	0.512	3.036		0.363	0.638	1.433	1.272	0.512	3.036	
	0.2	0.121	-0.098	0.840	0.717	-0.207	0.594	25.397	0.121	-0.098	0.840	0.717	-0.207	0.594	25.397
		0.352	-0.283	1.242	2.780	-0.269	2.962		0.352	-0.283	1.242	2.780	-0.269	2.962	
	0.25	0.135	-0.030	0.454	0.802	-0.475	0.656	25.627	0.135	-0.030	0.454	0.802	-0.475	0.656	25.627
		0.391	-0.090	0.631	2.895	-0.598	3.399		0.391	-0.090	0.631	2.895	-0.598	3.399	
	0.01	0.107	0.161	0.345	-0.211	0.244	0.852	29.418	0.107	0.161	0.345	-0.211	0.244	0.852	29.418
		0.368	1.350	2.368	-1.760	0.912	4.611		0.368	1.350	2.368	-1.760	0.912	4.611	
	0.05	0.135	0.093	1.171	0.041	0.612	0.671	28.588	0.135	0.093	1.171	0.041	0.612	0.671	28.588
		0.486	0.308	3.716	0.167	1.493	2.934		0.486	0.308	3.716	0.167	1.493	2.934	
	0.1	0.192	-0.205	0.839	0.516	0.345	0.509	28.620	0.192	-0.205	0.839	0.516	0.345	0.509	28.620
60 years		0.711	-0.637	2.096	1.836	0.674	2.183		0.711	-0.637	2.096	1.836	0.674	2.183	
(267 assets)	0.15	0.240	0.252	0.657	0.257	0.362	0.638	28.943	0.240	0.252	0.657	0.257	0.362	0.638	28.943
		0.867	0.888	1.313	1.101	0.656	3.119		0.867	0.888	1.313	1.101	0.656	3.119	
	0.2	0.296	-0.236	0.536	0.721	-0.344	0.588	28.443	0.296	-0.236	0.536	0.721	-0.344	0.588	28.443
		1.055	-0.827	1.053	3.062	-0.589	3.021		1.055	-0.827	1.053	3.062	-0.589	3.021	
	0.25	0.313	-0.060	0.404	0.598	-0.605	0.686	29.091	0.313	-0.060	0.404	0.598	-0.605	0.686	29.091
		1.105	-0.233	0.733	2.763	-1.035	3.705		1.105	-0.233	0.733	2.763	-1.035	3.705	
	0.01	0.110	0.003	0.367	-0.086	0.418	0.873	29.683	0.110	0.003	0.367	-0.086	0.418	0.873	29.683
		0.471	0.038	2.938	-0.951	2.285	4.883		0.471	0.038	2.938	-0.951	2.285	4.883	
	0.05	0.142	0.005	1.167	-0.045	0.753	0.790	29.039	0.142	0.005	1.167	-0.045	0.753	0.790	29.039
		0.610	0.018	4.423	-0.206	2.372	3.612		0.610	0.018	4.423	-0.206	2.372	3.612	
	0.1	0.275	-0.120	0.628	0.276	0.164	0.695	29.396	0.275	-0.120	0.628	0.276	0.164	0.695	29.396
50 years		1.192	-0.385	1.978	1.168	0.397	3.189		1.192	-0.385	1.978	1.168	0.397	3.189	
(528 assets)	0.15	0.284	0.446	0.642	-0.134	0.034	0.831	29.516	0.284	0.446	0.642	-0.134	0.034	0.831	29.516
		1.228	1.631	1.568	-0.643	0.076	4.176		1.228	1.631	1.568	-0.643	0.076	4.176	
	0.2	0.304	-0.002	0.546	0.296	-0.332	0.760	29.626	0.304	-0.002	0.546	0.296	-0.332	0.760	29.626
		1.305	-0.007	1.206	1.381	-0.695	3.923		1.305	-0.007	1.206	1.381	-0.695	3.923	
	0.25	0.299	0.033	0.781	0.255	-0.133	0.789	29.797	0.299	0.033	0.781	0.255	-0.133	0.789	29.797
		1.271	0.147	1.549	1.356	-0.276	4.293		1.271	0.147	1.549	1.356	-0.276	4.293	

Note: Displayed are prices of risk of full models also including either downside risk beta of Ang et al. (2006) or downside risk beta specification of Lettau et al. (2014). We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

Table 7: *Estimated Coefficients of the Full Models Controlled for Fama and French (1993) Factors.*

	τ	λ^{SMB}	λ^{HML}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE
70 years (142 assets)	0.01	0.035	-0.050	0.133	0.636	-0.217	-0.174	0.838	26.281
		0.266	-0.280	0.960	3.276	-1.558	-0.569	4.206	
	0.05	-0.073	-0.181	0.401	1.108	-0.238	0.093	0.893	26.349
		-0.550	-1.034	1.238	3.072	-0.989	0.186	4.007	
	0.1	-0.009	-0.200	0.187	0.888	0.276	0.076	0.674	26.119
		-0.066	-1.188	0.562	1.777	1.101	0.112	3.093	
	0.15	0.001	-0.165	0.376	0.612	0.231	0.289	0.702	25.969
		0.010	-0.949	1.232	1.068	1.044	0.416	3.504	
	0.2	0.090	-0.150	0.016	0.535	0.712	-0.259	0.609	25.052
		0.685	-0.869	0.050	0.827	2.965	-0.341	3.107	
	0.25	0.067	-0.150	0.118	0.171	0.753	-0.731	0.691	25.349
		0.517	-0.873	0.359	0.251	2.759	-0.962	3.604	
60 years (267 assets)	0.01	-0.149	0.040	0.198	0.389	-0.233	0.281	0.851	29.218
		-1.184	0.252	1.657	2.622	-1.979	1.053	4.481	
	0.05	-0.175	-0.016	0.291	1.216	-0.204	0.552	0.827	28.377
		-1.434	-0.101	1.035	3.859	-1.018	1.410	3.858	
	0.1	-0.121	-0.051	0.063	0.996	0.255	0.341	0.650	28.578
		-0.978	-0.333	0.216	2.589	1.152	0.676	3.045	
	0.15	-0.143	-0.052	0.414	0.837	0.025	0.442	0.751	28.833
		-1.182	-0.340	1.561	1.831	0.137	0.821	3.926	
	0.2	-0.060	-0.054	-0.151	0.709	0.640	-0.261	0.618	28.464
		-0.492	-0.359	-0.559	1.550	3.120	-0.448	3.258	
	0.25	-0.102	-0.059	0.014	0.623	0.505	-0.534	0.726	29.051
		-0.850	-0.388	0.055	1.243	2.506	-0.940	4.000	
50 years (528 assets)	0.01	-0.087	0.006	-0.005	0.457	-0.081	0.493	0.874	29.354
		-0.761	0.041	-0.063	3.577	-0.921	2.582	4.805	
	0.05	-0.088	-0.051	0.135	1.172	-0.213	0.708	0.905	28.909
		-0.806	-0.361	0.604	4.502	-1.349	2.251	4.504	
	0.1	-0.044	-0.096	0.022	0.791	0.137	0.120	0.774	29.182
		-0.394	-0.697	0.092	2.507	0.781	0.289	3.917	
	0.15	-0.095	-0.126	0.612	0.730	-0.347	0.153	0.950	29.086
		-0.854	-0.919	2.645	1.926	-2.262	0.344	5.144	
	0.2	-0.042	-0.084	0.091	0.682	0.201	-0.306	0.804	29.402
		-0.376	-0.613	0.422	1.710	1.202	-0.652	4.429	
	0.25	-0.055	-0.090	0.101	0.914	0.163	-0.166	0.835	29.519
		-0.499	-0.662	0.476	2.004	1.033	-0.354	4.793	

Note: Displayed are prices of risk of full models also including either HML and SMB betas of Fama and French (1993). We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

Table 8: *Estimated Coefficients of the TR, EVR and Full Models.*

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
	0.01	-0.045	0.644	0.751	26.672	-0.084	-0.258	0.946	28.576	0.135	0.610	-0.213	-0.150	0.832	26.436
		-0.476	3.303	3.910		-0.883	-0.853	4.885		0.896	3.085	-1.366	-0.495	4.151	
	0.05	0.149	1.272	0.716	26.801	0.242	0.378	0.689	28.212	0.160	1.258	-0.024	0.297	0.738	26.755
		0.699	3.485	3.861		1.413	0.774	3.047		0.431	3.519	-0.081	0.602	3.046	
	0.1	0.432	1.153	0.735	27.109	0.519	0.441	0.549	27.226	-0.005	1.004	0.442	0.287	0.553	26.567
70 years		1.673	2.265	4.119		2.588	0.649	2.461		-0.011	1.964	1.357	0.425	2.269	
(142 assets)	0.15	0.611	0.842	0.730	26.651	0.557	0.522	0.592	27.025	0.312	0.832	0.344	0.501	0.615	26.280
		2.493	1.453	4.050		2.836	0.756	2.817		0.898	1.436	1.210	0.719	2.816	
	0.2	0.732	0.224	0.780	26.944	0.750	-0.313	0.590	25.765	0.050	0.753	0.709	-0.139	0.580	25.417
		2.726	0.326	4.432		3.604	-0.411	2.950		0.137	1.127	2.463	-0.183	2.784	
	0.25	0.786	-0.165	0.808	27.241	0.869	-0.563	0.631	25.703	0.045	0.418	0.844	-0.420	0.628	25.608
		2.898	-0.230	4.631		3.695	-0.727	3.238		0.130	0.589	2.751	-0.544	3.165	
	0.01	-0.036	0.430	0.759	29.723	-0.083	0.263	0.934	30.514	0.168	0.380	-0.227	0.250	0.865	29.496
		-0.491	2.685	4.130		-1.105	1.002	5.093		1.342	2.388	-1.784	0.948	4.630	
	0.05	0.225	1.187	0.661	28.681	0.256	0.611	0.655	29.996	0.175	1.241	0.016	0.646	0.675	28.617
		1.211	3.659	3.574		1.493	1.545	2.878		0.562	3.921	0.060	1.626	2.819	
	0.1	0.368	0.958	0.715	29.239	0.550	0.371	0.501	29.193	-0.109	0.906	0.538	0.402	0.490	28.678
60 years		1.549	2.407	3.999		2.516	0.753	2.200		-0.318	2.265	1.730	0.815	2.015	
(267 assets)	0.15	0.559	0.729	0.706	29.244	0.488	0.495	0.593	29.614	0.358	0.770	0.250	0.398	0.622	29.089
		2.326	1.482	3.940		2.349	0.923	2.805		1.191	1.573	0.968	0.739	2.918	
	0.2	0.538	0.236	0.762	29.736	0.677	-0.251	0.581	28.726	-0.166	0.657	0.757	-0.296	0.555	28.607
		2.139	0.450	4.343		3.146	-0.440	2.913		-0.560	1.309	2.870	-0.517	2.739	
	0.25	0.523	0.208	0.782	30.035	0.669	-0.526	0.645	29.286	-0.041	0.539	0.680	-0.506	0.641	29.244
		2.114	0.375	4.491		3.065	-0.895	3.382		-0.154	0.996	2.755	-0.870	3.356	
	0.01	-0.087	0.436	0.825	29.726	-0.088	0.463	0.969	30.286	-0.007	0.407	-0.073	0.422	0.871	29.689
		-1.535	2.981	4.634		-1.507	2.500	5.396		-0.082	2.829	-0.798	2.305	4.787	
	0.05	0.001	1.162	0.762	29.233	0.071	0.636	0.807	30.075	0.060	1.242	-0.055	0.765	0.790	29.054
		0.004	3.980	4.238		0.542	1.991	3.795		0.209	4.422	-0.236	2.446	3.474	
	0.1	0.195	0.783	0.784	29.756	0.316	0.069	0.672	29.669	-0.060	0.770	0.302	0.155	0.662	29.400
50 years		0.972	2.401	4.418		1.836	0.168	3.152		-0.187	2.416	1.165	0.381	2.932	
(528 assets)	0.15	0.404	0.811	0.765	29.583	0.168	-0.003	0.776	30.089	0.532	0.782	-0.150	0.084	0.810	29.552
		2.010	1.968	4.288		0.991	-0.006	3.817		1.840	1.923	-0.637	0.187	3.893	
	0.2	0.324	0.631	0.808	29.957	0.377	-0.316	0.725	29.730	0.038	0.692	0.332	-0.342	0.727	29.650
		1.599	1.363	4.594		2.045	-0.687	3.694		0.144	1.535	1.359	-0.731	3.611	
	0.25	0.303	0.775	0.819	30.024	0.373	-0.116	0.752	29.926	0.046	0.912	0.335	-0.136	0.750	29.817
		1.509	1.546	4.690		2.019	-0.247	3.980		0.199	1.833	1.536	-0.287	3.958	

Note: Prices of risk estimated on monthly stock data from CRSP database sampled between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ three samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 1.5-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

2.E Specification of the Related Models

In this section, we briefly describe the specification of the models we use in the Appendix 2.C.2. We denote market excess return as r_m and its mean and variance as μ_m and σ_m^2, respectively. Excess return of an asset is denoted as r_i with mean μ_i and variance σ_i^2.

We present how we estimate betas in the first-stage regression. The second-stage regression is the same for all the models and is performed via OLS by regressing the average asset returns on their betas. This then leads to the estimated values of RMSPE.

2.E.1 Downside Risk Models

We follow two specifications of the downside risk models. First, we use specification of Ang et al. (2006) and estimate their relative downside risk betas as

$$\beta_i^{DR1} \equiv \beta_{i,\mu_m}^{-} - \beta_i = \frac{\mathbb{C}ov(r_i, r_m | r_m < \mu_m)}{\mathbb{V}ar(r_m | r_m < \mu_m)} - \frac{\mathbb{C}ov(r_i, r_m)}{\mathbb{V}ar(r_m)}. \tag{32}$$

Downside risk beta specification of Lettau et al. (2014) is then obtained as

$$\beta_i^{DR2} \equiv \beta_{i,\delta}^{-} - \beta_i = \frac{\mathbb{C}ov(r_i, r_m | r_m < \delta)}{\mathbb{V}ar(r_m | r_m < \delta)} - \frac{\mathbb{C}ov(r_i, r_m)}{\mathbb{V}ar(r_m)} \tag{33}$$

where we define the threshold value as $\delta \equiv \mu_m - \sigma_m$.

2.E.2 Generalized Disappointment Aversion Models

We employ specification of Generalized Disappointment Aversion (GDA) models of Farago and Tédongap (2018) and estimate two main versions of their cross-sectional models. Their models are based on disappointment events $\mathcal{D}_t$.

GDA3

First model is their three-factor model, which does not contain volatility-related factors. The betas posses the following form

$$\beta_{i,m} \equiv \frac{\mathbb{C}ov(r_i, r_m)}{\mathbb{V}ar(r_m)} \tag{34}$$

$$\beta_{i,\mathcal{D}} \equiv \frac{\mathbb{C}ov(r_i, I(\mathcal{D}))}{\mathbb{V}ar(I(\mathcal{D}))} \tag{35}$$

$$\beta_{i,m\mathcal{D}} \equiv \frac{\mathbb{C}ov(r_i, r_m I(\mathcal{D}))}{\mathbb{V}ar(r_m I(\mathcal{D}))} \tag{36}$$

where we follow the specification and set $\mathcal{D}_t = \{r_{m,t} < b\}$ where $b = -0.03$ and I is an indicator function.

GDA5

Five-factor specification of the GDA model contains, in addition to the betas from the three-factor model, the following betas

$$\beta_{i,X} \equiv \frac{\mathbb{C}ov(r_i, \Delta\sigma_m^2)}{\mathbb{V}ar(\Delta\sigma_m^2)} \tag{37}$$

$$\beta_{i,X\mathcal{D}} \equiv \frac{\mathbb{C}ov(r_i, \Delta\sigma_m^2 I(\mathcal{D}))}{\mathbb{V}ar(\Delta\sigma_m^2 I(\mathcal{D}))} \tag{38}$$

where the disappointment events are given by $\mathcal{D}_t = \left\{ r_{m,t} - a\frac{\sigma_m}{\sigma_X}\Delta\sigma_{m,t}^2 < b \right\}$ where $\Delta\sigma_{m,t}^2$ are increments of market volatility, $\sigma_X^2 = \mathbb{V}ar(\Delta\sigma_m^2)$, $a = 0.5$ and $b = -0.03$.

2.E.3 Coskewness and Cokurtosis

Following work of Kraus and Litzenberger (1976); Harvey and Siddique (2000); Dittmar (2002); Ang et al. (2006), we estimate the coskewness and cokurtosis as

$$CSK_i \equiv \frac{\mathbb{E}[(r_i - \mu_i)(r_m - \mu_m)^2]}{\sqrt{\mathbb{E}[(r_i - \mu_i)^2]}\mathbb{E}[(r_m - \mu_m)^2]}, \tag{39}$$

$$CKT_i \equiv \frac{\mathbb{E}[(r_i - \mu_i)(r_m - \mu_m)^3]}{\sqrt{\mathbb{E}[(r_i - \mu_i)^2]}\mathbb{E}[(r_m - \mu_m)^{3/2}]}\,. \tag{40}$$

2.E.4 Fama-French Three-Factor Model

Betas of the three-factor model of Fama and French (1993) are estimated via time-series regression of excess asset return on three factors: SMB (obtained by sorting stocks based on their size), HML (obtained by sorting stocks based on their book-to-market vale) and MKT (market factor)

$$r_{i,t} = \alpha_i + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \beta_i^{MKT} MKT_t + e_{i,t}. \tag{41}$$

Factor data were obtained from Kenneth French's online data library.

2.F Detailed Description of the Portfolio Results

2.F.1 Fama-French Portfolios

In this section, we employ two sets of Fama-French portfolios. First set contains two samples: 25 portfolios double-sorted on size and value and 30 industry portfolios. These two datasets were chosen because they posses the longest history available across all the Fama-French portfolios. Their time span ranges between July 1926 and April 2020. Second set contains three datasets of portfolios sorted on the following characteristics: operating profit, investment and book-to-market. Portfolios sorted on operating profit and investment posses significantly shorter history of observations between July 1963 and March 2020.

Regarding the first dataset, the results are summarized in Table 9. In the case of portfolios double sorted on size and value, the short component of QS and short component of EVR risks are priced. Regarding the industry sorted portfolios, only the short term TR is consistently priced across the model specifications. For those investors who fear the high volatility states, these results suggest that the more appropriate strategy involves investing based on the industries rather than size and value, as you do not have to pay a premium for portfolios that posses low EVR betas - portfolios whose extreme negative returns are less probable to co-occur with extreme positive increments of market volatility.

The second set of portfolios include equities sorted on operating profit, investments and book-to-market. The results are given in the Table 10. Generally, short TR is priced across these portfolios with the expected sign. On the other hand, using the portfolios sorted on investment, there is a strong negative relation between long TR and asset returns, which may seem unintuitive. Regarding the EVR, its short term part is priced across investment portfolios and book-to-market portfolios.

2.F.2 Other Portfolios

In this section, we provide analysis of QS risk performed on other widely used datasets. The estimated models are reported in Table 11. First, we focus on portfolios employed in Lettau et al. (2014). This dataset contains portfolios formed across multiple asset classes. First, the dataset contains 6 currency portfolios sorted on interest rate differential (we exclude high inflation currencies similar to the approach of Lettau et al. (2014)). Second, we have 5

Table 9: *Fama-French Long History Portfolios.*

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
	0.01	0.060	0.502	0.645	2.557	0.047	0.425	0.707	2.602	-0.002	0.483	0.076	0.320	0.615	2.545
		0.461	1.249	3.618		0.395	0.950	3.251		-0.010	1.438	0.354	1.393	2.722	
	0.05	-0.486	3.170	0.704	2.258	0.231	-2.263	0.586	2.520	-0.569	3.041	-0.093	-0.848	0.831	2.230
		-1.257	6.276	3.990		0.685	-3.315	1.460		-1.875	5.590	-0.291	-1.088	2.491	
	0.1	0.597	0.270	0.607	2.535	-0.252	5.020	0.843	2.496	0.505	0.445	0.016	3.783	0.523	2.422
25 portfolios		1.259	0.243	3.536		-0.511	3.067	1.771		1.152	0.415	0.043	3.358	1.496	
sorted on size and value	0.15	-0.954	3.584	0.826	2.473	-0.892	6.249	1.142	2.011	-1.015	2.112	-0.946	6.342	1.318	1.866
		-2.112	4.345	4.607		-2.326	5.690	3.224		-2.366	2.549	-2.636	6.463	4.177	
	0.2	-0.349	1.421	0.739	2.615	-0.606	9.894	0.858	2.001	-0.850	2.598	-0.606	10.489	0.860	1.922
		-0.827	1.054	4.140		-1.592	5.712	2.824		-1.920	1.958	-1.486	6.393	2.935	
	0.25	-0.271	1.630	0.717	2.594	-0.216	5.347	0.770	2.390	-0.932	1.242	0.065	6.623	0.674	2.319
		-0.800	1.143	4.137		-0.626	2.398	2.839		-1.952	1.106	0.172	4.443	2.573	
	0.01	-0.132	0.874	0.712	1.411	-0.208	-0.740	1.007	1.897	0.020	0.823	-0.155	-0.402	0.781	1.373
		-1.077	3.823	3.231		-1.611	-1.909	4.160		0.086	3.356	-0.597	-1.011	2.833	
	0.05	0.036	0.903	0.675	1.898	0.445	-2.141	0.443	1.878	-0.162	0.791	0.390	-1.728	0.447	1.792
		0.101	1.975	3.274		1.357	-2.558	1.296		-0.326	1.878	0.690	-2.027	1.022	
	0.1	0.208	1.044	0.651	1.848	0.722	-2.173	0.261	1.720	-0.504	0.545	0.875	-1.972	0.209	1.683
30 industry		0.592	1.680	3.286		3.348	-1.969	0.986		-0.838	0.809	1.825	-1.751	0.604	
portfolios	0.15	0.561	1.135	0.607	1.704	0.930	-2.461	0.246	1.612	0.147	0.774	0.647	-2.147	0.358	1.564
		1.843	1.594	3.044		3.644	-2.162	0.929		0.278	1.122	1.303	-1.947	1.080	
	0.2	0.602	0.442	0.665	1.917	0.840	-1.770	0.384	1.784	0.177	0.431	0.674	-1.970	0.430	1.767
		1.714	0.568	3.388		2.398	-1.535	1.437		0.414	0.555	1.434	-1.688	1.468	
	0.25	0.823	0.926	0.648	1.734	0.652	-2.961	0.550	1.806	0.879	0.883	0.046	-2.418	0.673	1.611
		2.093	1.221	3.239		1.993	-2.469	2.191		2.152	1.157	0.123	-1.984	2.628	

Note: Prices of risk estimated on monthly return data of 30 industry portfolios and portfolios double sorted on size and book-to-market. Sample period covers time interval between July 1926 and April 2020. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

Table 10: *Fama-French Portfolios.*

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
	0.01	0.266	0.844	0.337	0.417	0.353	-0.329	0.467	0.663	0.097	1.012	0.185	-0.388	0.244	0.265
		1.217	1.929	1.462		1.603	-1.014	1.668		0.333	2.738	0.466	-1.264	0.700	
	0.05	-0.301	1.308	0.669	0.920	0.684	-2.969	0.422	0.723	-0.355	0.306	0.782	-1.790	0.317	0.625
		-0.692	2.328	3.255		2.673	-2.070	1.617		-0.728	0.618	2.164	-1.951	0.987	
	0.1	0.119	-1.008	0.747	0.976	0.844	-1.697	0.142	0.701	0.018	0.394	0.986	-2.376	0.037	0.688
		0.315	-1.461	3.608		1.493	-0.783	0.318		0.037	0.945	1.939	-1.705	0.093	
Operating profit	0.15	0.262	-0.933	0.709	1.041	0.803	-1.173	0.263	0.986	-0.807	-0.974	1.804	-2.295	-0.104	0.820
		0.696	-1.041	3.335		2.318	-1.426	1.008		-0.751	-1.458	2.108	-2.814	-0.255	
	0.2	-1.191	0.226	0.884	0.886	1.284	-4.962	0.147	0.849	-1.646	0.742	1.662	-3.192	0.121	0.579
		-1.379	0.340	4.024		3.769	-2.240	0.577		-2.360	1.415	4.102	-1.850	0.471	
	0.25	-0.947	1.579	0.725	0.914	0.195	-2.627	0.599	0.589	-0.332	-0.049	0.356	-2.294	0.541	0.575
		-0.913	1.638	3.511		0.511	-1.525	2.166		-0.433	-0.075	1.418	-2.138	2.185	
	0.01	0.234	2.347	-0.145	1.219	1.093	-0.047	-0.114	3.332	0.768	2.230	-0.774	0.729	0.196	0.933
		2.228	7.966	-0.626		5.265	-0.251	-0.405		1.009	3.649	-1.126	3.077	0.374	
	0.05	-1.855	5.496	0.668	1.333	-1.674	-1.749	1.995	2.535	-2.088	5.045	0.035	1.831	0.593	1.279
		-4.548	5.866	3.130		-9.143	-1.320	7.719		-4.508	8.124	0.151	2.755	2.487	
	0.1	-4.729	4.393	1.274	2.007	0.950	7.075	-0.330	2.960	-5.437	3.225	-0.552	0.004	1.902	1.984
		-5.902	3.724	5.048		1.964	9.647	-0.699		-7.711	4.848	-1.215	0.007	3.911	
Investment	0.15	-2.556	7.013	0.811	2.068	-2.463	16.677	1.543	2.440	-5.158	8.173	4.022	14.513	-1.794	1.366
		-6.042	7.252	3.908		-8.222	7.779	5.729		-5.133	9.396	4.581	7.545	-3.767	
	0.2	-4.208	14.158	0.559	1.746	-3.982	10.646	2.676	2.933	-1.998	13.989	-2.161	10.860	1.174	1.301
		-8.644	8.388	2.439		-7.310	6.644	6.950		-1.360	6.574	-1.248	2.824	1.387	
	0.25	-5.272	2.109	1.236	1.770	0.216	7.497	0.527	3.296	-4.753	3.109	-0.416	2.632	1.341	1.681
		-9.910	1.765	4.718		0.428	4.862	1.257		-9.098	3.275	-0.863	1.710	3.095	
	0.01	0.783	0.378	0.249	1.537	1.045	0.830	-0.323	2.178	2.016	-1.162	-1.923	-1.355	1.793	1.141
		4.365	1.142	1.220		4.243	2.022	-1.130		3.866	-2.861	-3.316	-3.587	3.971	
	0.05	1.909	2.898	0.061	1.386	-0.682	-6.138	1.661	2.020	2.821	2.012	-0.957	2.715	0.669	1.148
		3.552	2.593	0.274		-1.912	-4.116	3.755		3.917	2.384	-2.543	2.847	1.886	
	0.1	2.997	-1.702	0.276	1.449	-0.672	5.498	1.240	2.791	3.098	-3.078	-1.303	-3.900	1.558	1.159
		4.274	-2.004	1.368		-1.457	2.332	2.571		4.323	-3.434	-2.526	-2.110	3.180	
Book-to-market	0.15	2.979	1.100	0.147	1.504	-1.844	0.164	2.095	2.378	5.562	-6.472	-0.957	8.492	0.252	1.063
		4.180	0.971	0.693		-3.473	0.147	4.427		4.697	-4.100	-2.493	3.918	0.772	
	0.2	1.940	5.670	0.272	2.043	-1.110	8.519	1.166	2.151	1.055	4.724	-0.066	4.433	0.335	1.999
		3.102	4.096	1.333		-2.942	3.797	4.017		1.799	3.161	-0.133	2.907	0.908	
	0.25	-0.046	8.320	0.442	2.212	-0.956	9.493	1.193	2.160	-1.799	6.455	-0.477	8.514	0.893	2.006
		-0.072	3.362	2.283		-2.841	3.578	4.198		-2.185	3.050	-1.778	3.276	3.536	

Note: Prices of risk estimated on monthly return data of portfolios sorted on operating profit, investment and book-to-market. Sample period covers time interval between July 1963 (July 1926 for book-to-market portfolios) and March 2020. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

commodity futures portfolios sorted on basis. Third, we include returns on 5 corporate bond portfolios sorted on credit spread. And fourth, we have equity portfolios sorted on various characteristics (6 double sorted on size and value, 5 on CAPM beta, 5 on industry, 6 double sorted on momentum and size).[20] Here, we present results for the aggregated dataset. This dataset was introduced to show the usefulness of downside risk beta for pricing. From the results we can conclude that the short component of TR for most τ threshold values is priced using the aggregated dataset. Its long component is significant for some medium values of τ. Regarding the EVR, its short term component for lower values of τ is priced as well.

Second, we look at the equity portfolios sorted on cash flow duration proposed in Weber (2018). The results can be found in the second section of Table 11. Similarly as in the previous case, short term part of TR is priced across these portfolios. On the other hand, its long term part is negatively priced across these assets, which may be counterintuitive. The EVR is priced using its both components.

Finally, we use returns on factors constructed from various asset classes from Ilmanen et al. (2021). This dataset was chosen because of its long history and because it spans many asset classes including U.S. and international equities, fixed income assets, currencies and commodities using value, momentum, carry, defensive and multi-style type of investment strategy. We report the results in the third panel of Table 11. We can see that using the TR model, the long term TR is priced, and both parts of EVR are priced. But if we look at the results of the Full model, only the EVR coefficients remain consistently significant.

[20]We do not include option portfolios because they have short history starting in 1986, which is not suitable for our analysis.

Table 11: *Various Portfolios.*

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
Lettau et al. (2014)	0.01	0.744	0.126	0.455	8.739	0.562	0.115	0.112	9.116	0.953	-0.042	-0.135	-0.420	0.600	8.702
		3.703	0.225	1.792		3.080	0.225	0.399		3.238	-0.074	-0.484	-0.855	1.894	
	0.05	0.428	4.432	0.590	8.724	0.738	1.779	0.264	9.634	0.059	4.368	0.268	0.378	0.501	8.708
		0.956	3.801	2.294		1.838	1.575	0.968		0.126	3.565	0.515	0.333	1.603	
	0.1	0.784	3.342	0.602	9.988	1.029	0.109	0.248	9.935	-0.847	4.122	1.468	-1.817	0.202	9.693
		1.344	2.138	2.359		1.945	0.071	0.858		-1.824	2.579	2.593	-1.154	0.668	
	0.15	0.627	7.934	0.578	9.408	1.213	1.165	0.293	10.037	-0.117	8.497	0.906	-1.273	0.392	9.367
		1.243	3.951	2.241		1.783	0.657	0.999		-0.257	3.551	1.116	-0.622	1.263	
	0.2	0.990	11.846	0.487	8.925	0.519	2.222	0.481	10.623	2.392	10.492	-2.392	2.494	0.954	8.525
		1.871	4.994	1.939		0.746	1.004	1.639		2.514	5.140	-2.560	1.176	2.799	
	0.25	1.187	10.080	0.459	9.502	0.700	1.154	0.455	10.705	1.842	9.434	-1.674	3.161	0.767	9.295
		2.208	3.700	1.809		0.956	0.503	1.541		2.538	3.594	-1.944	1.544	2.353	
Weber (2018)	0.01	0.611	1.702	0.008	1.428	0.826	8.684	-0.796	1.460	0.852	1.246	-0.795	6.825	-0.123	0.695
		2.809	4.083	0.033		3.741	3.537	-2.297		1.779	2.466	-1.568	2.545	-0.281	
	0.05	-2.640	4.875	1.027	2.864	1.953	-3.555	-0.249	4.277	-2.562	5.166	2.319	-3.806	-0.211	1.318
		-4.897	7.174	4.416		4.587	-4.026	-0.696		-4.832	7.206	5.026	-4.633	-0.641	
	0.1	-4.297	3.906	1.361	3.989	0.827	9.226	-0.426	3.270	0.316	3.719	2.061	8.453	-1.611	3.190
		-5.230	3.027	5.394		1.262	6.569	-0.669		0.704	3.318	3.698	6.560	-2.939	
	0.15	0.257	8.307	0.155	3.620	-0.829	10.041	0.840	3.099	-0.568	5.073	0.858	11.976	-0.447	2.906
		0.783	7.019	0.696		-2.711	5.768	2.299		-1.005	5.208	1.369	4.526	-0.885	
	0.2	-2.453	9.974	0.580	3.804	-0.838	6.443	1.052	4.413	-2.801	15.421	3.088	13.817	-1.799	2.779
		-4.107	6.689	2.658		-1.926	6.203	2.645		-4.146	6.730	4.330	7.357	-3.280	
	0.25	-3.435	8.407	0.658	3.870	1.904	2.376	-0.463	4.693	-5.417	8.747	4.081	8.589	-1.818	3.097
		-5.066	6.948	2.989		5.773	2.107	-1.514		-6.571	7.086	6.850	6.427	-4.362	
Ilmanen et al. (2021)	0.01	2.401	1.279	-0.528	20.445	2.279	3.100	-0.440	19.912	1.087	1.338	1.086	2.842	-0.424	19.407
		11.322	0.988	-1.714		11.091	2.741	-1.484		1.076	0.991	1.094	2.455	-1.420	
	0.05	6.482	-0.905	0.058	19.846	4.840	5.149	-0.779	20.155	4.160	-1.209	1.709	3.082	-0.283	19.650
		12.922	-0.367	0.196		10.085	2.232	-2.446		2.146	-0.477	0.952	1.342	-0.541	
	0.1	7.660	1.560	0.377	24.118	5.830	6.750	-0.729	22.877	4.715	0.885	1.916	6.173	-0.038	23.208
		11.398	0.480	1.487		10.703	1.834	-2.309		2.781	0.266	1.304	1.657	-0.102	
	0.15	8.689	-0.554	0.438	24.783	6.698	12.464	-0.763	23.232	3.026	-0.202	4.250	10.625	-0.335	23.133
		11.734	-0.167	1.803		9.759	2.889	-2.407		2.416	-0.059	3.628	2.430	-1.045	
	0.2	9.876	-2.295	0.368	27.105	10.214	1.875	-0.756	22.503	-0.278	-4.394	10.922	1.825	-1.075	22.170
		11.540	-0.632	1.487		10.411	0.374	-2.359		-0.259	-1.188	9.470	0.346	-3.447	
	0.25	8.151	3.455	0.557	30.032	10.378	20.757	-0.502	28.317	1.925	0.215	8.146	19.229	-0.268	27.793
		10.336	0.925	2.305		9.569	4.311	-1.594		1.781	0.057	6.879	3.642	-0.969	

Note: Prices of risk estimated on monthly data of various datasets. Models are estimated for various values of thresholds given by τ. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

Chapter 3

Common Idiosyncratic Quantile Risk[1]

We identify a new type of risk that is characterised by commonalities in the quantiles of the cross-sectional distribution of asset returns. Our newly proposed quantile risk factor is associated with a quantile-specific risk premium and provides new insights into how upside and downside risks are priced by investors. In contrast to the previous literature, we recover the common structure in cross-sectional quantiles without making confounding assumptions or aggregating potentially non-linear information. We discuss how the new quantile-based risk factor differs from popular volatility and downside risk factors, and we identify where the quantile-dependent risks deserve greater compensation. Quantile factors also have predictive power for aggregate market returns.

3.1 Introduction

The question of how relevant the information contained in different parts of the return distribution is to an investor has received considerable attention in the recent empirical asset pricing literature (Ang et al. 2006; Van Oordt and Zhou 2016; Chabi-Yo et al. 2018; Lu and Murray 2019), with number of studies focusing on the tails or extremes in the cross-section of returns (Kelly and Jiang 2014; Chabi-Yo et al. 2022). These studies typically rely on assumptions

[1]This chapter was co-authored with Jozef Baruník and is currently R&R in the *Review of Finance*. We appreciate helpful comments from participants at the 2023 Financial Econometrics Conference (Lancaster), the 2022 Haindorf Seminar (Hejnice, Humboldt U. & Charles U. joint seminar), the 2022 STAT of ML (Prague), and University of Sussex (Brighton) seminar. Support from the Czech Science Foundation under the 19-28231X (EXPRO) project is gratefully acknowledged.

about moment conditions as well as the existence of a model that generates returns. In contrast to the literature, our aim is to use conditional quantiles of observed returns to capture set of nonlinear factors that provide finer characterization of risk. In particular, we want to explore the common, possibly non-linear movements in the panel of the firm's idiosyncratic quantiles. In doing so, we remain agnostic about the data generating process. We believe that such structures provide richer information for investors than the information that can be obtained by making assumptions about the moments. In particular, we will identify where quantile-dependent risk exposures deserve greater compensation. Both volatility and downside risk measures hide such details while aggregating information about risk.

The information captured by quantile-dependent factors can be related to the behaviour of investors with quantile preferences (de Castro and Galvao 2019). Quantiles contain rich information because they capture heterogeneity in risk and allow the separation of risk aversion and elasticity of intertemporal substitution. Our main interest is to show that there are strong common factors across quantiles of the cross-sectional distribution of asset returns that are more informative about investors' compensation requirements. We argue that such risk is distinct from other types of risk associated with the distribution of returns, such as downside risk or volatility risk. The quantile-dependent risk premia associated with such factors are then used to generalise notion of upside risk and downside risk.

Just as quantile regression extends classical linear regression, our quantile factor model of asset returns extends the approximate factor models used in the empirical asset pricing literature. In the spirit of the popular Principal Component Analysis, which recovers the conditional mean, we work with more general quantile factor models (QFMs). These are flexible enough to capture quantile-dependent objects that cannot be captured by standard tools. Unlike standard principal component analysis, quantile factor models are able to capture hidden factors that shift distributional properties such as moments or quantiles. Moreover, these factors can vary across the distribution of each unit in the panel, allowing the factors to be properly inferred when the idiosyncratic error distributions have heavy tails. Importantly, such factors differ from the usual mean and volatility factors when we abandon the traditional location and scale shift model structure and allow for more general, possibly unknown, data generating processes. In effect, quantile-dependent risk is treated as constant in factor models based on such assumptions. Downside risk models then aggregate the quantiles, usually under some distributional assumption.

Our main contribution is to investigate the pricing implications of common non-linear factors that are quantile specific for the predictability of aggregate market returns and the cross-section of stock returns. We are interested in factors that identify the risk premium associated with different quantiles of the return distribution in terms of both downside (or tail) risk and upside potential. Our approach will identify new information about risk beyond the usual moments associated with tail risks. To this end, we use the quantile factor model of Chen et al. (2021) and investigate the pricing implications of quantile-dependent factors while controlling for various linear factors and exposures to them. Our objective is also motivated by the increasing evidence of non-linearities in equity markets.[2] We aim to show that the common quantile risk present in the stock return data carries different information from the common volatility and downside risks. Our quantile dependent factors also carry strong information for both the cross-section of asset returns and the time series predictability of the equity premium.

We begin by identifying common factor structures in the idiosyncratic quantiles of stocks in the Center for Research in Security Prices (CRSP) over a sample spanning 1960 to 2018. We discuss the relationship with volatility and downside risk factors and show that quantile factors have predictive power for aggregate market returns. Predictive regressions show that a one standard deviation increase in quantile risk predicts a statistically significant increase in annualised excess market returns of up to 7.05% in the case of the left tail. These results hold out-of-sample, are stronger for the left tail, and are robust to controlling for a wide range of popular predictors studied by Welch and Goyal (2007), as well as tail risk (Kelly and Jiang 2014), common volatility risk (Herskovic et al. 2016), and variance risk premium (Bollerslev et al. 2009). We also document the predictive power of the upper tail factor with a smaller effect of up to 3.50% increase in annualised returns, hence the effect is asymmetric. Moreover, the predictive power of the upper tail factors disappears when looking at the out-of-sample performance.

We also find that idiosyncratic quantile risk has significant predictive power for the cross-section of average returns. We show that stocks with high loadings of past quantile risk in the left tail earn up to an annual six-factor alpha of

[2]E.g., Amengual and Sentana (2020) report a non-linear dependence structure in short-term reversals and momentum. Ma et al. (2021) show that many firm-level characteristics have a complex relationship with returns in terms of quantiles.

8.57% higher than stocks with low tail risk loadings for 0.2 quantiles. This risk premium is not subsumed by other commonly priced factors such as common volatility, tail and downside risk, and other popular risk factors. Investors thus have a strong aversion to tail risk with respect to the common movements in idiosyncratic returns. On the other hand, the absence of the risk premium associated with the factors for the upper quantiles suggests that investors are not upside potential seekers. Both results are consistent with the literature on the impact of asymmetric dependencies on asset prices.

Our work is related to several strands of the literature. The first relates to the factor-based asset pricing models that are very popular in the empirical pricing literature (Ross 1976; Fama and French 1993; Kelly et al. 2019). In sharp contrast to this literature, our approach remains agnostic about the nature of the true data generating process and uses the conditional quantiles of observed returns without imposing moment conditions.

The second strand to which we contribute is the study of idiosyncratic risk that co-moves across assets, thus exploring common trends that are not captured by first moment factors. The bulk of this research is motivated by the introduction of the idiosyncratic volatility puzzle proposed by Ang et al. (2006a). Unfortunately, all existing explanations of the anomaly are based on lottery preferences, market frictions or other factors[3] only for 29-54% of the puzzle using individual stocks Hou and Loh (2016).

The third line of thought that we take into account deals with asymmetric properties of systematic risk and how they are incorporated into asset prices. Interest in this type of model was reignited by Ang et al. (2006) and their introduction of downside beta, which captures the covariance between asset and market returns conditional on the market being below some threshold. Bollerslev et al. (2021) further decompose traditional market beta into semibetas, which are characterised by the signed covariation between market and asset returns. They show that only the semibetas associated with negative market and asset returns predict significantly higher future returns. More recently, Bollerslev et al. (2022) argue that betas are granular and associated with a risk premium that depends on the relevant part of the return distributions.

[3]For a comprehensive list of references belonging to each of these categories, see Hou and Loh (2016). The only exception to this observation is the lottery-based explanation using the highest realised return from the previous month, proposed by Bali et al. (2011) and confirmed in European markets by Annaert et al. (2013). However, Hou and Loh (2016) argue that this explanation is not valid as it is an almost perfect collinear range-based measure of idiosyncratic volatility.

From a theoretical point of view, there are many justifications for the departure from classical common factor pricing theory to the asymmetric forms of the utility function. Probably the most relevant for our work is the dynamic quantile decision maker of de Castro and Galvao (2019), who decides based on quantile dependent preferences. Barro (2006), building on Rietz (1988), introduced the rare disaster model and showed that tail events may have significant ability to explain various asset pricing puzzles, such as the equity premium puzzle. The other popular model that considers asymmetric features of risk is the generalised disappointment aversion model of Routledge and Zin (2010), which inherently assumes that investors are downside averse. Based on these preferences, Farago and Tédongap (2018) introduced an intertemporal equilibrium asset pricing model and showed that the disappointment-related factors should be priced in the cross-section. Moreover, they prove that their model performs well empirically by jointly pricing different asset classes with significant prices for the risk associated with the disappointment factors.

There are also attempts to combine the two or three of these research agendas. Herskovic et al. (2016) introduced a risk factor based on the common volatility of firm-level idiosyncratic returns, and showed its pricing capabilities for the cross section of different asset classes. For example, Kelly and Jiang (2014); Allen et al. (2012); Jondeau et al. (2019) explore the risks associated with skewness, tails and extremes. Giglio et al. (2016) estimate quantile-specific latent factors using systemic risk and financial market distress variables to predict macroeconomic activity. Much of the research investigating common tail risk and its implications for asset pricing relies on options data. They argue that the tail factor identifies additional information beyond the volatility factor. Andersen et al. (2020) show strong predictive power for future equity risk premia in US and European equity index derivatives. Bollerslev and Todorov (2011) combine high-frequency and options data and use a non-parametric approach to conclude that a large part of the equity and variance risk premia is related to jump tail risk.

The rest of the paper is structured as follows. Section 3.2 proposes the quantile factor model for asset returns, discusses the methodology of estimating the quantile-specific factors and the data we use, and provides the link to the volatility factors. Section 3.3 presents the results on the time series predictability of the aggregate market return using the common idiosyncratic quantile factors. Section 3.4 examines the cross-sectional asset pricing implications of the proposed factors. Section 3.5 concludes.

3.2 Common Idiosyncratic Quantile Factors

Researchers usually assume that time variation in equity returns can be captured by relatively small number of common factors with following structure[4]

$$r_{i,t} = \alpha_i + \beta_i^\top f_t + \epsilon_{i,t} \tag{3.1}$$

where $r_{i,t}$ is excess return of an asset $i = 1, \dots, N$ at time $t = 1, \dots, T$, f_t is a $k \times 1$ vector of common factors and β_i is a $k \times 1$ vector of the asset's i exposures to the common factors. Such time-series regressions as the one in (3.1) yielding high R^2 are used to identify factors serving as good proxies for aggregate risks present in the economy. Exposures to the relevant factors captured by β_i coefficients should be compensated in the equilibrium and explain the risk premium of the assets

$$\mathbb{E}_t[r_{i,t+1}] = \beta_i^\top \lambda_t \tag{3.2}$$

where the λ_t is a $k \times 1$ vector of prices of risk associated with factor exposures. Importantly, while the arbitrage pricing theory (APT) of Ross (1976) suggests that any common return factors f_t are valid candidate asset pricing factors, the idiosyncratic return residuals $\epsilon_{i,t}$ are assumed not to be priced. This implication is due to many simplifying assumptions, such that an average investor can perfectly diversify her portfolio or that the linear model (3.1) is correctly specified.

In these models, only common return factors are valid candidate pricing factors, and sensitivities to those factors determine the risk premium associated with an asset (Ross 1976). This strand of literature yields highly successful and popular results focusing on the parsimonious models (Fama and French 1993), as well as exploration of statistically motivated latent factors.[5] Recently, Kelly et al. (2019) introduced instrumented principal component analysis, which enables to flexibly model the latent factors with time-varying loadings using the

[4]Recently, Lettau and Pelger (2020) introduce Risk-Premium Principal Component Analysis that allows for systematic time-series factors incorporating information from the first and second moment.

[5]This approach dates back to Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986). For a comprehensive overview of machine learning methods applied to asset pricing problems such as measuring expected returns, estimating factors, risk premia, or stochastic discount factor, model selection, and corresponding asymptotic theory, see Giglio et al. (2022).

observable characteristics.[6] In addition, Ma et al. (2021) introduced a semi-parametric quantile factor panel model that considers stock-specific characteristics, which may non-linearly affect stock returns in a time-varying manner. They find that many characteristics possess a non-linear effect on stock returns. In contrast to these authors, the approach used in our paper is more general since it allows not only loadings but also factors to be quantile-dependent. Moreover, our approach does not require the loadings to depend on observables and has direct relation of the approximate factor models that are ubiquitous in the finance literature.

While large literature have focused mainly on the diversification assumption, we aim to question linear nature of the factor model, and our focus is on exposure to parts of idiosyncratic return's distribution instead. Recently, Herskovic et al. (2016) documents strong comovement in idiosyncratic volatility that does not arise from omitted factors, and even after saturating the factor regression with up to ten principal components, residuals that are virtually uncorrelated display same co-movement seen in raw returns.

While the exposure to common movements in volatility seem to carry strong pricing implications, we ask if there exist additional structure insufficiently captured by volatilities especially in a non-linear and heavy tailed financial data. In other words, we ask if various parts of the return distributions may have pricing implications for the cross-section of stock returns.[7]

In parallel to simple factor structure in idiosyncratic volatility of a panel of returns recovered commonly by researchers (Ang et al. 2006b; Herskovic et al. 2016), we aim to recover genuine unobserved structure in idiosyncratic quantiles. These quantities will be more informative for investors in case of the heavy-tailed nonlinear data in which the second moment is not sufficient quantity for capturing risk. We will show the relation of quantile factors to volatility under some specific model assumptions, relate the proposed factor model to existing approaches recovering various factor structures from data and also provide a first look at the quantile factor structures in cross-section of

[6]Other notable recent contributions to the factor literature are, e.g., Kozak et al. (2018) and Giglio et al. (2021). The recent availability of high-frequency return data also motivated the development of continuous-time factor models.Aït-Sahalia et al. (2020) proposed a generalization of the classical two-pass Fama-MacBeth regression from the classical discrete-time factor setting to a continuous-time factor model and enables uncovering complex dynamics such as jump risk and its role in the expected returns.

[7]Ando and Bai (2020) document that the common factor structures explaining the upper and lower tails of the asset return distributions in global financial markets have become different since the subprime crisis.

the U.S. stocks. Importantly, we will show that our quantile dependent factors carry different information from the structure recovered using volatility or some popular downside risk measures that require certain moment conditions to be met.

3.2.1 Quantile Factor Model

To formalize the discussion, we assume the panel of returns of length T and width N after elimination of common mean factors from the time-series regression

$$r_{i,t} = \alpha_i + \beta_i^\top f_t + \epsilon_{i,t} \tag{3.3}$$

to have τ-dependent structure $f_t(\tau)$ in idiosyncratic errors that we coin common idiosyncratic quantile – CIQ(τ) – factors, $f_t(\tau)$

$$Q_{\epsilon_{i,t}}\Big[\tau|f_t(\tau)\Big] = \gamma_i^\top(\tau)f_t(\tau), \tag{3.4}$$

that implies

$$\epsilon_{i,t} = \gamma_i^\top(\tau)f_t(\tau) + u_{i,t}(\tau), \tag{3.5}$$

where $f_t(\tau)$ is an $r(\tau) \times 1$ vector of random common factors, and $\gamma_i(\tau)$ is $r(\tau) \times 1$ vector of non-random factor loadings with $r(\tau) \ll N$ and the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ satisfies the quantile restriction $P[u_{i,t}(\tau) < 0|f_t(\tau)] = \tau$ almost surely for all $\tau \in (0,1)$.

To estimate the common factors that capture co-movement of quantile-specific features of distributions of the idiosyncratic parts of the stock returns, we use Quantile Factor Analysis (QFA) introduced by Chen et al. (2021). In contrast to the principal component analysis (PCA), QFA allows to capture hidden factors that may shift more general characteristics such as moments or quantiles of the distribution of returns other than mean. The methodology is also suitable for large panels and requires less strict assumptions about the data generating process as we will discuss in detail here.

The quantile-dependent factors and its loadings can be estimated as

$$\underset{(\gamma_1,\dots,\gamma_N,f_1,\dots,f_T)}{\operatorname{argmin}} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\rho_\tau\left(\epsilon_{it} - \gamma_i^\top f_t\right) \tag{3.6}$$

where $\rho_\tau(u) = (\tau - \mathbf{1}\{u \leq 0\})u$ is the check function while imposing the following normalizations $\frac{1}{T}\sum_{t=1}^{T} f_t f_t^\top = \mathbb{I}_r$, and $\frac{1}{N}\sum_{i=1}^{N} \gamma_i \gamma_i^\top$ is diagonal with non-increasing diagonal elements. A potential problem that may arise in small samples is the so-called quantile crossing, that is, the estimated quantiles are not guaranteed to be monotonic in τ. If this occurs, the approach due to Chernozhukov et al. (2010) can be employed to establish monotonicity of the estimated quantiles. In our empirical applications reported later, quantile crossing never arises.

As discussed in Chen et al. (2021), this estimator is related to the principal component analysis (PCA) estimator studied in Bai and Ng (2002) and Bai (2003) similarly as quantile regression is related to classical least-square regression. Unlike the PCA estimator of Bai (2003), the estimator does not yield an analytical closed form solution. To solve for the stationary points of the objective function, Chen et al. (2021) proposed a computational algorithm called iterative quantile regression. Moreover, they show that the estimator possess same convergence rate as the PCA estimators for approximate factor model. We follow their approach when estimating the quantile factors.[8]

It is important here to make relation to the recent literature that attempts to recover possibly non-linear commonalities and dependence structures in cross-section of returns. For example Pelger and Xiong (2022) allowed factors to be state-dependent, Chen et al. (2009) provided theory for nonlinear factors and Gorodnichenko and Ng (2017) estimated joint level and volatility factors simultaneously. Important strand of the literature is using copulas and documents nonlinear tail dependence, co-skewness, and co-kurtosis in cross-sectional dependence among monthly returns on individual U.S. stocks (Amengual and Sentana 2020) or provides flexible copula factor model (Oh and Patton 2017) .

Different from these studies, our model remains agnostic about the nature of the true data generating process, and use the conditional quantiles of the observed data to capture nonlinearities in factor models. In contrast to the literature, we also do not require the idiosyncratic errors to satisfy certain moment conditions. Hence our approach is more flexible as it estimates factors shifting relevant parts of the return distributions without restricting assumptions, relying on the properties of the density. The approach also departs from existing factor literature in not requiring the loadings to depend on observables and considers the factors to be quantile-dependent objects.

[8] We employ the authors' Matlab codes provided on the Econometrica webpage.

3.2.2 Relation to Common Factors in Volatility

Quantiles of stock returns can be related to variety of quantities as well as distributional characteristics in specific cases. A specifically important quantity in finance that can relate to quantiles of the return distribution for a typically assumed location-scale model is volatility. As discussed by ample literature started by Ang et al. (2006b), there exists genuine factor structure in the idiosyncratic volatility of panel of asset returns. Applying PCA (or cross-sectional averages) to squared residuals, once mean factors have been removed from the returns (a procedure labeled PCA-SQ hereafter) will recover that structure. We will use this approach to study the relation to quantile specific factors on data, but before we do so, let's discuss the relation theoretically.

It is important to note that the volatility structure will be recovered only if the data-generating process were to be known, and well characterized by the first two moments of the distribution. Yet in case of more general, or even unknown data generating processes that will not be well characterized by the first two moments, such approaches will fail to characterize the risks precisely, and quantile factor models will estimate more useful information.

To illustrate the discussion and provide the link between volatility and quantiles in such restrictive models, let's consider the data generating process to be a typical location-scale model with two unrelated factors in the first and second moments. Idiosyncratic returns $\epsilon_{i,t}$ of such model will be zero mean i.i.d. process independent of both factors with cumulative distribution function $F_{\epsilon_{i,t}}$. Further let $Q_{\epsilon_{i,t}}(\tau) = F^{-1}_{\epsilon_{i,t}}(\tau) = \inf\{s : F_{\epsilon_{i,t}}(s) \leq \tau\}$ be a quantile function of $\epsilon_{i,t}$ and assume the median is zero. Then the following model that is typical for finance

$$r_{i,t} = \beta_i f_{1,t} + (\sigma_{i,t}^\top f_{2,t})\epsilon_{i,t}, \tag{3.7}$$

where $\sigma_{i,t}$ is time-varying volatility of an ith stock and $\sigma_{i,t} f_{2,t} > 0$ can be assumed to generate returns. When $f_{1,t}$ and $f_{2,t}$ do not share common elements, then

$$Q_{r_{i,t}}\Big[\tau | f_t(\tau)\Big] = \beta_i f_{1,t} + \sigma_{i,t}^\top f_{2,t} Q_{\epsilon_{i,t}}(\tau) \tag{3.8}$$

for $\tau \neq 0.5$ and $Q_{r_{i,t}}\Big[\tau | f_t(\tau)\Big] = \beta_i f_{1,t}$ for $\tau = 0.5$. Note that here loadings on the factor are the only quantile-dependent objects and structure in the mean and volatility describes well the structure in quantiles. While this is already restrictive example that operates with the assumption on first two moments, even in such case standard PCA will not provide consistent estimates if the distribution of $\epsilon_{i,t}$ is heavy-tailed (Chen et al. 2021).

But what if the data follows more complicated models than the one implied by location-shift models? Consider adding asymmetric dependence such as

$$r_{i,t} = \beta_i f_{1,t} + f_{2,t}\epsilon_{i,t} + f_{3,t}\epsilon_{i,t}^3, \tag{3.9}$$

where $\epsilon_{i,t}$ is standard normal random variable with cumulative distribution function $\Phi(.)$. The quantiles of the returns will then follow

$$Q_{r_{i,t}}\Big[\tau|f_t(\tau)\Big] = \beta_i f_{1,t} + \Phi^{-1}(\tau)\Big[f_{2,t} + f_{3,t}\Phi^{-1}(\tau)^2\Big], \tag{3.10}$$

for $\tau \neq 0.5$ and we can clearly see that second factor in $f(\tau) = [f_{1,t}, f_{2,t} + f_{3,t}\Phi^{-1}(\tau)^2]^\top$ is quantile dependent.

The main benefit of the model proposed is that being agnostic about data generating process and moment conditions, we use conditional quantiles of the observed returns to capture nonlinearities in factor models. In case these factors are different from those obtained on first and second moments, they will also be more informative for investors. In the next section we estimate these quantities and compare them to volatility as well as other downside risk factors to find support that data show such a rich structures.

3.2.3 Common Idiosyncratic Quantile Factor and the US Firms

To estimate the common idiosyncratic quantile – CIQ(τ) – factors, we use returns on stocks from the Center for Research in Securities Prices (CRSP) database sampled between January 1963 and December 2018. We include all stocks with codes 10 and 11 in estimating the CIQ(τ) factors. We adjust the returns for delisting as described in Bali et al. (2016). We follow the standard practice in the literature and exclude all "penny stocks" with prices less than one dollar to avoid biases related to these stocks.[9] We performed the analysis using all the stocks, and the results did not qualitatively change. When not stated otherwise, we use monthly data for both factor estimation and beta calculations.

[9]See, e.g., Amihud (2002).

In the process of the factor estimation, we proceed in a few steps. First, we use a moving window of 60 months of monthly sampled observations. We select the stocks that have all the observations in this window. For all these stocks, we run time-series regression to eliminate the influence of the common (linear) factors

$$\forall i : r_{i,t} = \alpha_i + \beta_i^\top f_t + e_{i,t}, \quad t = 1, \ldots, T \tag{3.11}$$

and save the residuals $e_{i,t}$. For the common factors f_t, which we eliminate from the stock returns, we resort to the three factors of Fama and French (1993).[10] Second, we use the residuals from the first step and, for every τ, estimate common idiosyncratic quantile factors, $f_t(\tau)$

$$\forall \tau : e_{i,t} = \gamma_i(\tau) f_t(\tau) + u_{i,t}(\tau) \tag{3.12}$$

where the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ satisfies the quantile restriction following the methodology discussed in the previous subsection. We use only the first – the most informative – estimated factor for our purposes. In the overwhelming majority of the cases, the algorithms proposed in Chen et al. (2021) select exactly one factor to be the correct number of factors that explain the panels of idiosyncratic returns.

Since we are interested to see how the quantile dependent factors relate to volatility, we estimate an approximate factor model on squared residuals that captures the common volatility factor. More specifically, we use residuals obtained from the Equation 3.11, square them and estimate on them first principal component using PCA. Such factor denoted as PCA-SQ will fail to capture the full factor structure if the distribution of the idiosyncratic returns possess non-normal features (Chen et al. 2021).

While it is one of our main questions to study if quantile dependent risk is present in the markets, and is not subsumed by volatility and downside risk, we first look at the correlations between these risks. Consistent with common volatility factor literature, we also focus on the changes in the CIQ(τ), and we work with ΔCIQ(τ) factors.[11] Intuitively, we will look at how investors price the innovations of these risks rather then levels.

[10] As discussed in Herskovic et al. (2016), there is a little difference between the results obtained using factors of Fama and French (1993) and purely statistically motivated ones estimated using the PCA framework.

[11] If not stated otherwise, in the rest of the paper, we perform all the analyses using ΔCIQ(τ) factors.

Table 3.1: *Correlations between CIQ(τ) and other factors.*

variable / τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
Panel A: *Levels of factors*											
PCA-SQ	-0.76	-0.73	-0.69	-0.56	-0.24	0.15	0.23	0.53	0.70	0.75	0.78
CIV	-0.45	-0.43	-0.39	-0.31	-0.06	-0.05	0.15	0.27	0.36	0.39	0.40
TR	0.13	0.12	0.12	0.07	0.01	-0.11	-0.11	-0.26	-0.27	-0.24	-0.23
VRP	-0.05	-0.04	-0.05	-0.02	0.04	-0.09	-0.03	0.07	0.08	0.08	0.09
VIX	-0.37	-0.34	-0.30	-0.20	0.12	0.11	0.20	0.36	0.40	0.39	0.39
Panel B: *Differences of factors*											
PCA-SQ	-0.53	-0.47	-0.43	-0.30	-0.09	0.21	0.22	0.37	0.53	0.59	0.65
CIV	-0.21	-0.20	-0.18	-0.15	-0.09	0.04	0.07	0.09	0.10	0.11	0.08
TR	0.04	0.03	0.03	-0.01	-0.08	-0.10	-0.15	-0.26	-0.29	-0.27	-0.25
VRP	0.12	0.11	0.11	0.06	0.08	-0.02	-0.04	-0.06	-0.08	-0.09	-0.10
VIX	0.24	0.25	0.27	0.27	0.26	0.04	0.07	0.10	0.02	-0.04	-0.11

Note: The table reports correlations between CIQ(τ) factors and factors related to the asymmetric and variance risk. Data contain the period between January 1963 and December 2018.

Table 3.1 reports correlations between CIQ(τ) factors and factors related to the variance and asymmetric risk. In Panel A, we work with levels of CIQ(τ) factors and other factors, in Panel B, we focus on differences of the factors. First, we look at the dependence between CIQ(τ) factors and PCA-SQ factor. We can see that the correlation is the strongest if we move to the tails with the correlation for CIQ(0.1) and PCA-SQ being equal to -0.76 for the level of the factors but it decreases substantially if we look at the differences with the correlation being equal to -0.53. Moreover, the correlation is stronger for the CIQ(τ) factors with τ above the median.

Next, we look at the correlations with the common idiosyncratic variance factor of Herskovic et al. (2016). In this case, the correlations are slightly higher for τs below the median, with peak correlation at $\tau = 0.1$ being equal to -0.45 for the levels of the factors. On the other hand, if we move to the differences, the correlation decreases to -0.21. Correlations with the tail risk factor (TR) are relatively small with the peak at $\tau = 0.8$ with a -0.29 correlation. Especially low are the correlations between TR factor and CIQ(τ) factors for downside values of τ. Correlations with the variance risk premium (VRP) factor of Bollerslev et al. (2009) are very low as well, with values no higher than 0.12 in absolute value for both levels and differences of the factors. Finally, correlations with the VIX index are symmetrical around the median τ with a peak of 0.39 at $\tau = 0.9$ while there is a stronger correlation between downside τs and the VIX with values around 0.26 in differences.

This preliminary analysis suggests that behavior of idiosyncratic quantiles shocks is in non-negligible part distinct from shocks to volatility and downside risk measures.

In addition, Table 3.2 provides correlations between CIQ(τ) factors at different quantiles. Correlation between CIQ(τ) in levels for the upper and lower part of the distribution are far from perfect, e.g., the correlation between the lower tail factor CIQ(0.1) and upper tail CIQ(0.9) is -0.69. This observation suggests that the factors do not simply duplicate information and are hence not likely to be rescaled information contained in common volatility factor (captured by e.g., PCA-SQ). Moreover, this dependence decreases substantially if we look at the increments of the CIQ(τ) factors – dependence between lower and upper tail factors reduces to -0.32. These results suggest that there is a potential for different pricing information across quantiles and that this information does not simply mirror information contained in the common volatility.

Table 3.2: *Correlations between CIQ(τ) factors.*

τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
0.1	.	0.98	0.95	0.86	0.55	-0.03	-0.05	-0.32	-0.56	-0.63	-0.69
0.15	0.97	.	0.98	0.91	0.63	0.00	0.01	-0.24	-0.50	-0.58	-0.65
0.2	0.93	0.97	.	0.95	0.71	0.05	0.05	-0.16	-0.42	-0.52	-0.60
0.3	0.85	0.91	0.95	.	0.82	0.13	0.15	0.06	-0.22	-0.33	-0.43
0.4	0.68	0.77	0.83	0.93	.	0.23	0.28	0.36	0.12	0.02	-0.08
0.5	0.07	0.12	0.17	0.25	0.34	.	0.75	0.41	0.34	0.30	0.26
0.6	0.12	0.17	0.21	0.29	0.40	0.78	.	0.47	0.40	0.37	0.32
0.7	0.14	0.24	0.31	0.49	0.66	0.47	0.54	.	0.93	0.87	0.79
0.8	-0.10	-0.01	0.07	0.25	0.46	0.41	0.48	0.92	.	0.98	0.94
0.85	-0.21	-0.13	-0.05	0.13	0.35	0.39	0.46	0.85	0.96	.	0.97
0.9	-0.32	-0.25	-0.18	-0.01	0.22	0.33	0.39	0.75	0.90	0.95	.

Note: The table presents unconditional correlations between CIQ(τ) factors in levels (above diagonal) and differences (below diagonal). We estimate the factors using FF3 residuals of the monthly CRSP stocks' returns. Data contain the period between January 1963 and December 2018.

Overall, we can see that the correlations between CIQ(τ) factors and other related factors are far from perfect. The highest degree of comovement is, not surprisingly, seen for levels of CIQ(τ) factors and PCA-SQ factor, which is substantially reduced if we look at the differences of those factors. Moreover, a strong asymmetry in the correlations across τ suggests that the information contained in the downside and upside CIQ factors differ.

3.3 Time-series Predictability of Market Return

We start examining the information content of the CIQ(τ) factors for subsequent short-term market returns. Here we aim to predict the monthly excess return on the market that we approximate by the value-weighted return of all CRSP firms. In the regressions, we also control for popular predictive variables used in Welch and Goyal (2007) as well as three closely related factors – TR factor of Kelly and Jiang (2014), the innovations of common idiosyncratic volatility (ΔCIV) factor of Herskovic et al. (2016), and the VRP factor of Bollerslev et al. (2009).[12] Moreover, we construct the PCA-SQ factor and use its increments to control for the effect of the common volatility. Because the CIQ(τ) factors are estimated using a rolling window, we use the last value of the factors estimated from each rolling window to construct a single series of the CIQ(τ) factors.

First, we report the results from the univariate regressions of the market return on the differences of the CIQ(τ) factors at various τ quantile levels of the form

$$r_{m,t+1} = \gamma_0 + \gamma_1 \times \Delta f_t(\tau) + \epsilon_{t+1} \tag{3.13}$$

in Table 3.3. We report estimated scaled coefficients to capture the effect of one standard deviation increase of the independent variable on the subsequent annualized market return. The corresponding t-statistics are computed using Newey-West robust standard errors using six lags.

The results in Table 3.3 document strong predictive power using the ΔCIQ(τ) factors for the left part of the distribution, with the peak for $\tau = 0.3$, where the increase (decrease) of one standard deviation in the factor predicts subsequent decrease (increase) of 7.05 percents in annualized market return.[13] There is also some predictive power for the upper tail factor when CIQ(0.9), but the effect is much smaller with only 3.50 percent increase in annualized market return accompanied with only less than one-third of the R^2 from the lower tail. From a perspective of an investor, in times of high risk – captured by large negative increments of the left-tail CIQ(τ) factor, she requires a premium for investing. And thus, these risky periods correlate with the high marginal utility states of the investors.

Together with in-sample (IS) R^2, we also report the out-of-sample (OOS) R^2 from expanding window scheme. We use data up to time t to estimate the prediction model and then forecast the $t+1$ return (the first window contains 120 monthly periods to obtain sufficiently reasonable estimates). Then, the

[12] We replicated tail risk factor construction of Kelly and Jiang (2014) by ourself; we acquired data of Herskovic et al. (2016) from Bernard Herskovic's webpage and data of Bollerslev et al. (2009) from Hao Zhou's webpage.

[13] Note that the lower tail factors are on average negative. Increase (decrease) of these factors corresponds to the decrease (increase) of risk, which leads to a decrease (increase) of the required risk premium.

Table 3.3: *Predictive power of the $\Delta CIQ(\tau)$ factors.*

τ	Coeff.	t-stat	R^2 IS	R^2 OOS	R^2 IS CT	R^2 OOS CT
0.1	-6.31	-2.77	1.40	1.09	1.21	1.42
0.15	-6.49	-2.74	1.48	1.17	1.20	1.45
0.2	-6.38	-2.63	1.43	1.13	1.14	1.33
0.3	-7.05	-2.98	1.75	1.21	1.21	1.41
0.4	-6.59	-2.92	1.53	0.58	0.83	0.76
0.5	0.15	0.07	0.00	-0.37	0.00	-0.19
0.6	0.29	0.13	0.00	-0.30	0.00	-0.23
0.7	-0.88	-0.48	0.03	-0.67	0.03	-0.37
0.8	2.09	1.13	0.15	-0.26	0.10	-0.08
0.85	3.05	1.67	0.33	-0.03	0.21	0.31
0.9	3.50	1.88	0.43	0.06	0.29	0.31

Note: The table reports results from the univariate predictive regressions of the value-weighted return of all CRSP firms on the ΔCIQ(τ) factors for various $\tau \in (0, 1)$. Coefficients are scaled to capture the effect of one standard deviation increase in the factor on the annualized market return in percent. The corresponding t-statistics are computed using the Newey-West robust standard errors using six lags. We report both in-sample (IS) and out-of-sample (OOS) R^2s. We also truncate the predictions at zero following Campbell and Thompson (2007) (CT) and report corresponding IS and OOS R^2s. The time span covers the period between January 1960 and December 2018.

window is extended by one observation, the prediction model is re-estimated and a new forecast is obtained. We repeat this procedure until the whole sample is exhausted. The corresponding R^2 is computed by comparing conditional forecast and historical mean computed using the available data up to time t, i.e., $1 - \sum_t (r_{m,t+1} - \widehat{r}_{m,t+1|t})^2 / \sum_t (r_{m,t+1} - \bar{r}_{m,t})^2$ where $\widehat{r}_{m,t+1|t}$ is out-of-sample forecast of the $t+1$ return using data up to time t, and $\bar{r}_{m,t}$ is the historical mean of the market return computed up to date t. Unlike the case of the IS R^2, the OOS R^2 can attain negative values if the conditional forecasts perform worse than the historical mean forecast. The positive values of the OOS R^2 for τ between 0.1 and 0.4 provide strong evidence for the benefits of the ΔCIQ(τ) factors for predicting the market return in the real-world setting. On the other hand, the predictability vanishes for the higher values of τ.

To assess the economic usefulness for the investors, we further follow suggestions from Campbell and Thompson (2007) (hence CT). They propose to truncate the predictions from the estimated model at 0, as the investor would not have used a model to predict a negative premium. This non-linear modification of the model should introduce caution into the models. Based on this modification, we report both IS and OOS R^2s. Naturally, using this transformation, the IS R^2 does not improve for any of the models, but the performance rises for the OOS analysis. Results suggest that the common fluctuations in the lower part of the excess returns distributions robustly predict the subsequent market movement.

Next, we run bivariate regressions to assess whether the proposed quantile factors contain additional information not included in the relevant previously proposed variables

$$r_{m,t+1} = \gamma_0 + \gamma_1 \times \Delta f_t(\tau) + \gamma_2 \times f_t^{Control} + \epsilon_{t+1} \tag{3.14}$$

where we separately control for variables that may contain duplicate information. First, in Table 3.4, we report coefficients and their t-statistics while controlling for differences of the PCA-SQ factor, the ΔCIV of Herskovic et al. (2016), the TR factor of Kelly and Jiang (2014), and the VRP factor of Bollerslev et al. (2009), respectively. For better comparability, we also include results from the univariate predictions using the $\Delta\text{CIQ}(\tau)$ factors only. In the case of PCA-SQ factor, we can see that neither the significance nor the magnitude of the predictive power of the downside CIQ factors is diminished. Moreover, the borderline significance of the upside CIQ factors vanishes. This suggests that the common volatility element is not the driving force of the predictive performance of the quantile factors. In the second case, while controlling for the ΔCIV, the results regarding the $\Delta\text{CIQ}(\tau)$ factors remain the same, and ΔCIV proves not to predict future market returns. In the case of the TR factor, the $\Delta\text{CIQ}(\tau)$ factors mirror the results from the univariate regressions in terms of coefficients and their significance. TR factor is significant across all the specifications, although its effect is smaller and less significant than in the case of $\Delta\text{CIQ}(\tau)$ for the lower tail values of τ. In the third case, the VRP factor appears to be the most closely related in terms of predictability to the $\Delta\text{CIQ}(\tau)$ factors.[14] The VRP is highly significant, and at the same time, it diminishes the effect of the $\Delta\text{CIQ}(\tau)$ factors – the scaled coefficients decreases around 1.7 percentage points, and the corresponding t-statistics are now approximately 1.5. This decrease in significance may be also caused by substantial decrease of the available time period as the VRP starts in 1990.

As a next step, we control for variables discussed in Welch and Goyal (2007).[15] Instead of a large table of coefficients and t-statistics through all variables and quantiles, we summarize the results in the Table 3.5, in which

[14]We acknowledge that there is no clear theoretical link between VRP and $\Delta\text{CIQ}(\tau)$ factors. The VRP is associated with the aggregate S&P 500 composite index (rather than the value-weighted return of all CRSP stocks), which it only significantly predicts over a medium-term horizon. However, we have included it for informational purposes and to potentially stimulate a discussion regarding the relationship between these two phenomena in the future.

[15]For the information regarding the specification of the variables, see Welch and Goyal (2007). We obtained the data from the Iwo Welch's webpage.

Table 3.4: *Bivariate predictive regressions.*

control / τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
CIQ	-6.31	-6.49	-6.38	-7.05	-6.59	0.15	0.29	-0.88	2.09	3.05	3.50
	(-2.77)	(-2.74)	(-2.63)	(-2.98)	(-2.92)	(0.07)	(0.13)	(-0.48)	(1.13)	(1.67)	(1.88)
R^2	1.40	1.48	1.43	1.75	1.53	0.00	0.00	0.03	0.15	0.33	0.43
CIQ	-6.05	-6.12	-5.89	-6.54	-6.33	-0.62	-0.52	-2.56	0.22	1.35	1.93
	(-2.45)	(-2.43)	(-2.35)	(-2.83)	(-2.83)	(-0.31)	(-0.24)	(-1.21)	(0.10)	(0.56)	(0.78)
PCA-SQ	0.48	0.77	1.14	1.74	3.14	3.79	3.77	4.59	3.54	2.86	2.40
	(0.22)	(0.36)	(0.56)	(0.91)	(1.59)	(1.96)	(1.90)	(2.00)	(1.40)	(1.05)	(0.86)
R^2	1.40	1.50	1.47	1.85	1.87	0.48	0.48	0.67	0.47	0.51	0.55
CIQ	-6.72	-6.89	-6.71	-7.29	-6.71	0.18	0.33	-0.84	2.17	3.15	3.56
	(-2.82)	(-2.77)	(-2.66)	(-2.98)	(-2.87)	(0.08)	(0.15)	(-0.47)	(1.14)	(1.68)	(1.87)
ΔCIV	-1.96	-1.96	-1.78	-1.61	-1.19	-0.57	-0.58	-0.49	-0.77	-0.92	-0.84
	(-0.59)	(-0.59)	(-0.54)	(-0.49)	(-0.36)	(-0.16)	(-0.17)	(-0.14)	(-0.22)	(-0.26)	(-0.24)
R^2	1.53	1.61	1.54	1.84	1.58	0.01	0.01	0.04	0.17	0.36	0.45
CIQ	-6.28	-6.44	-6.36	-6.99	-6.52	0.31	0.35	-0.76	2.27	3.12	3.58
	(-2.76)	(-2.72)	(-2.63)	(-2.96)	(-2.88)	(0.15)	(0.16)	(-0.41)	(1.22)	(1.72)	(1.93)
TR	4.67	4.64	4.69	4.62	4.60	4.72	4.71	4.69	4.80	4.76	4.77
	(2.33)	(2.32)	(2.35)	(2.31)	(2.31)	(2.33)	(2.33)	(2.32)	(2.35)	(2.34)	(2.34)
R^2	2.17	2.24	2.20	2.50	2.27	0.78	0.78	0.80	0.96	1.12	1.23
CIQ	-4.63	-4.80	-4.60	-4.67	-4.54	0.41	0.43	-0.92	0.85	2.31	1.67
	(-1.39)	(-1.38)	(-1.38)	(-1.46)	(-1.47)	(0.15)	(0.14)	(-0.35)	(0.33)	(0.87)	(0.67)
VRP	11.83	11.79	11.62	11.55	11.44	11.58	11.56	11.52	11.64	11.73	11.67
	(5.62)	(5.60)	(5.38)	(5.31)	(5.22)	(5.35)	(5.32)	(5.33)	(5.45)	(5.54)	(5.50)
R^2	6.06	6.12	6.05	6.07	6.03	5.23	5.23	5.25	5.25	5.43	5.33

Note: The table reports results from the bivariate predictive regressions of the value-weighted return of all CRSP firms on ΔCIQ(τ) factors for various $\tau \in (0, 1)$ and other control variables. We employ the PCA-SQ factor, innovations of CIV factor of Herskovic et al. (2016), TR factor of Kelly and Jiang (2014), and the VRP factor of Bollerslev et al. (2009), respectively. Coefficients are scaled to capture the effect of one-standard-deviation increase in the factor on the annualized market return in percent. The corresponding t-statistics are computed using the Newey-West robust standard errors using six lags. The time span covers the period between January 1960 and December 2018 except the VRP that starts in January 1990.

we include t-statistics of the ΔCIQ(τ) factors from the bivariate regressions of the form 3.14 while controlling for said variables. We observe that none of the variables drives out the significance of the ΔCIQ(τ) factors. Moreover, the magnitude of the significance remains very close to the ones from the univariate regressions.

3.3.1 Prediction using many CIQ(τ) Factors

Because it is *ex-ante* not clear on which quantile the investor should base her investment strategy on, we perform an out-of-sample prediction exercise which utilizes information from more than one ΔCIQ(τ) factor when constructing a forecast. The results are summarized in Table 3.6. We use either all of the factors when predicting the market return or we use two disjunct subsets of them. Using the first subset, we employ a prior assumption that only the downside factors ($\tau < 0.5$) are significant predictors of the market return. Second subset imposes the premise that the upside factors ($\tau > 0.5$) possess the forecasting power for the aggregate return. To do that, we use various models to exploit the information from the ΔCIQ(τ) factors. We train the

Table 3.5: *Controlled predictive significance of the $\Delta CIQ(\tau)$ factors using Welch and Goyal (2007) variables.*

control/ τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
dp	-2.78	-2.75	-2.65	-3.01	-2.94	0.04	0.11	-0.51	1.11	1.65	1.88
dy	-2.75	-2.72	-2.63	-2.98	-2.92	0.04	0.11	-0.52	1.09	1.61	1.84
ep	-2.77	-2.74	-2.64	-2.99	-2.93	0.06	0.13	-0.52	1.11	1.66	1.87
de	-2.77	-2.74	-2.63	-2.98	-2.91	0.07	0.12	-0.46	1.16	1.71	1.90
svar	-2.81	-2.75	-2.64	-2.96	-2.87	0.10	0.21	-0.23	1.39	1.87	2.06
bm	-2.77	-2.74	-2.63	-2.98	-2.92	0.07	0.13	-0.49	1.12	1.67	1.88
ntis	-2.72	-2.69	-2.59	-2.93	-2.89	0.07	0.13	-0.47	1.12	1.67	1.87
tbl	-2.75	-2.74	-2.62	-2.95	-2.89	0.08	0.16	-0.44	1.15	1.71	1.88
lty	-2.75	-2.73	-2.61	-2.96	-2.89	0.07	0.15	-0.46	1.13	1.68	1.88
ltr	-2.52	-2.52	-2.44	-2.82	-2.79	-0.03	0.08	-0.50	0.94	1.47	1.63
tms	-2.82	-2.79	-2.68	-3.03	-2.97	0.09	0.14	-0.46	1.19	1.71	1.91
dfy	-2.72	-2.69	-2.59	-2.95	-2.92	0.09	0.13	-0.47	1.11	1.62	1.82
infl	-2.63	-2.61	-2.50	-2.85	-2.84	0.14	0.17	-0.45	1.08	1.62	1.79

Note: The table summarizes t-statistics associated with the ΔCIQ(τ) factors from bivariate regressions when controlling for macroeconomic variables discussed in Welch and Goyal (2007). The dependent variable is the value-weighted return of all CRSP firms. The t-statistics are computed using the Newey-West robust standard errors using six lags. The time span covers the period between January 1960 and December 2018.

models on the first 120 monthly observations and then expand the estimation window as discussed before. We report both simple OOS R^2 and OOS R^2 CT to asses the fit. When performing regularization in the coefficient estimation, one has to choose so called tuning parameters. We choose the tuning parameters based on the in-sample leave-one-out full cross-validation procedure. We chose the forecast construction methods following Dong et al. (2022).

The first models that we employ is an OLS model which uses a OLS fitted multivariate regression model (estimated in-sample) to predict one-month-ahead return of the market. We can see that using all the ΔCIQ(τ) factors to predict OOS return yields a negative R^2. This is caused by the overfitting problem when we use many correlated variables and do not impose any parameter regularization. Using only either downside or upside factors and truncating the prediction at zero, yield some marginal gains for the investor.

The LASSO (least absolute shrinkage and selection operator, Tibshirani (1996)) model (estimator) introduces a regularization in the estimation procedure of the predictive coefficients. In the case of LASSO, only a subset of the predictors is chosen to have non-zero coefficients. As we can see, the performances for all τ and downside τ models substantially improve. On the other hand, prediction based on the upside τs do not yield a good fit even after the introduction of a regularization.

Next, we generalize the previous LASSO model and report results based on the elastic net (ENET) estimator (Zou and Hastie 2005). The estimator employs ℓ_1 (LASSO) and ℓ_2 (ridge regression, Hoerl and Kennard (1970)) penalty terms. For simplicity reasons, we chose the penalty weights to be both equal to 0.5 without any tuning procedure. As we can see, the results closely mirror the results from the LASSO estimation.

As a next model, we perform a simple combination forecast. We first obtain univariate forecasts for each $\Delta\text{CIQ}(\tau)$ factor separately and then the final forecast is obtained as a simple average of the univariate forecasts. We can see that the model performs very well for selection of all τs and downside τs, with R^2 being up to 1.26% for downside τs and R^2 CT of 1.39%. On the other hand, upside τs do not lead to any valuable forecasts.

C-LASSO and C-NET follow the same idea as the Combination model but instead of averaging all the univariate forecasts, they run multivariate penalized regression (LASSO and ENET, respectively) of the future market return on the univariate forecasts to select the best combination of them. The resulting forecast is then obtained by plugging the last value of $\Delta\text{CIQ}(\tau)$ from a window into the fitted models. Once again, all τ and downside τ subsets perform both very well, with R^2 of 0.93% and R^2 CT of 1.29% for downside τ C-LASSO. But the models using upside τ yield even negative R^2. This is the case for all the remaining models which use upside τ factors.

PCA model aggregates information and creates the first principal component from all the $\Delta\text{CIQ}(\tau)$ factors and uses it as the prediction variable in the univariate prediction regression. We observe that the downside τ PCA model performs the best across all the specifications.

Finally, the OLS selection model fits univariate prediction models for each $\Delta\text{CIQ}(\tau)$ factor and uses the univariate model 3.13 with the best in-sample fit to predict the future market return. This simple approach yields very solid performance of 0.87% for R^2 and 1.28% for R^2 CT.

To summarize this section, we observed that using the downside $\Delta\text{CIQ}(\tau)$ factors in various multivariate models, we obtain significant positive performance. On the contrary, the upside $\Delta\text{CIQ}(\tau)$ factors do not result into economic gains because they do not outperform the forecasts based on the historical mean. All the results thus suggest that the driving force behind the downside quantile factors' performance is not the common volatility component but the information contained in the left part of the common factor structure.

Table 3.6: *Out-of-sample performance of the forecast combinations.*

	All τ		Downside τ		Upside τ	
model	R^2	R^2 CT	R^2	R^2 CT	R^2	R^2 CT
OLS	-1.53	-0.40	-0.44	0.53	-0.31	0.39
LASSO	0.94	0.95	0.21	0.80	-0.25	0.14
ENET	0.92	1.03	0.07	0.71	-0.11	0.27
Combination	1.10	1.06	1.26	1.39	-0.07	0.07
C-LASSO	0.79	0.92	0.93	1.29	-0.61	-0.23
C-NET	0.86	0.78	0.85	1.22	-0.65	-0.19
PCA	1.17	1.22	1.21	1.46	-0.31	-0.10
OLS selection	0.87	1.28	0.87	1.28	-0.73	-0.10

Note: The table reports performance of various specifications of multivariate predictive models using all ΔCIQ(τ) factors, τ below median ΔCIQ factors (downside), or above median ΔCIQ factors (upside). The time span covers the period between January 1960 and December 2018.

3.4 Pricing the CIQ(τ) Risks in the Cross-Section

In this section, we investigate the pricing implications of the presented common idiosyncratic quantile factors for the cross-section of stock returns. We hypothesize that the stochastic discount factor increases in the CIQ(τ) risk, as the risk-averse investor's marginal utility is high in the states of high CIQ(τ) risk. Based on that hypothesis, we assume that the assets that perform poorly in the states of high CIQ(τ) risk will require a higher risk premium for holding by the investors. On the other hand, assets that perform well during these states serve as a hedging tool and will be traded with higher prices and thus lower expected returns. The stock's sensitivities to the factors capture betas estimated by the linear regression of stocks' returns on the factors. If not explicitly stated otherwise, we use as our predicted variable monthly out-of-sample returns following the estimation window. We also try to predict one-year returns using portfolios to assess the persistence of the CIQ(τ) betas and thus indirectly investigate the transaction costs related to the trading of these factors. Data that we employ cover the usual asset pricing period between January 1963 and December 2018. We exclude “penny stocks" with prices less than one dollar to avoid related biases.

To alleviate the concerns that the quantile factors simply mirror the dynamics of the idiosyncratic volatilities of the single-stock returns, in the case of pricing the cross-section, we perform the estimation of the factors using standardized idiosyncratic returns.[16] Specifically, we estimate time-varying volatility using exponentially weighted moving average model. Then, we use the

[16]In Appendix in Table 16, we report correlations between the CIQ(τ) factors estimated using standardized data. The correlations are generally smaller.

$\Delta f_t(\tau)$ estimates as our risk factors. For all available stocks and and for all τ, we estimate quantile-specific betas

$$r_{i,t} = \alpha_i + \beta_i(\tau)\Delta f_t(\tau) + v_{i,t}(\tau),$$

using the least-square estimator. These betas will be used in the following asset pricing tests as a measure of the exposure to the CIQ(τ) factors. Same as the factors, betas are also estimated using the 60-month rolling window. We include the stocks that possess at least 48 monthly observations. Betas computed up to time t are used to predict returns at time $t+1$ or further – no overlap between estimation and prediction periods. The control variables are estimated using the same procedure as originally proposed.

Later in the analysis, we also control for the effect of the increments of the PCA-SQ factor, ΔCIV factor and many other related variables to show that the effect of the newly proposed quantile factors is not subsumed by the effect of any related factor or stock-specific variable.

3.4.1 Cross-sectional Regressions

As a first step in the investigation of the cross-sectional implications of exposures to the common idiosyncratic quantile risks, we perform two-stage Fama and MacBeth (1973) predictive regressions. We explore the hypothesis that the exposures to the ΔCIQ(τ) factors align with the future excess returns of the stocks. This type of asset pricing test moreover conveniently allows for simultaneous estimation of many risk premiums associated with various risk measures. That means that we can estimate the risk premium associated with the CIQ(τ) risks while controlling for other risk measures previously proposed in the literature. More specifically, for each time $t = 1, \ldots, T-1$ using all of the stocks $i = 1, \ldots, N$ available at time t and $t+1$,[17] we cross-sectionally regress all the returns at time $t+1$ on the betas estimated using only the information available up to time t. This procedure yields estimates of prices of risk $\lambda_{t+1}(\tau)$ while controlling for the most widely used competing measure of risk

$$r_{i,t+1} = \alpha + \beta_{i,t}^{CIQ(\tau)}(\tau)\lambda_{t+1}^{CIQ(\tau)}(\tau) + \beta_{i,t}^{\top Control}\lambda_{t+1}^{Control} + e_{i,t+1} \qquad (3.15)$$

[17]A stock is identified as available, if it possess at least 48 monthly return observations during the last 60-month window up to time t and also an observation at time $t+1$.

where $\beta_{i,t}^{Control}$ is vector of control betas or other stock characteristics and $\lambda_{t+1}^{Control}$ is vector of corresponding prices of risk. Using $T-1$ cross-sectional estimates of the prices of risk, we compute the average price of risk associated with each $\lambda^{CIQ}(\tau)$ as

$$\widehat{\lambda}^{CIQ(\tau)}(\tau) = \frac{1}{T-1}\sum_{t=2}^{T}\widehat{\lambda}_t^{CIQ(\tau)}(\tau) \tag{3.16}$$

and report them along with their t-statistics based on the Newey-West robust standard errors.

We summarize the first set of results in Table 3.7 where we report estimation outcomes of controlling the effect of ΔCIQ(τ) factors by general risk measures. But first, we report results from the univariate regressions on CIQ(τ) betas. We observe similar results to those obtained from the market predictions – the exposure to the common idiosyncratic downside events is significantly compensated in the cross-section of stock returns. For example, CIQ(τ) for $\tau = 0.2$ possess a coefficient of 1.11 (t-stat = 2.57), on the other hand, for $\tau = 0.8$, the estimated coefficient is equal to -0.14 (t-stat = -0.30). This suggests that the exposure to the common idiosyncratic downside events is significantly compensated in the cross-section. On the contrary, to hold assets with high exposure to the upside common movements the investors have to pay a small discount for those stock, although not statistically significant one.

As those results suggest, there is a strong asymmetry in the pricing implications of the ΔCIQ(τ) factors. To further assess it, we perform the following set of bivariate regressions

$$\begin{aligned} r_{i,t+1} = \alpha_{t+1} + \beta_{i,t}^{CIQ}(\tau_{down})\lambda_{t+1}(\tau_{down})^{CIQ} + \beta_{i,t}^{CIQ}(\tau_{up})\lambda_{t+1}(\tau_{up})^{CIQ} + e_{i,t+1}, \\ \tau_{down} = \{0.1, 0.15, 0.2, 0.3, 0.4\}, \tau_{up} = \{0.6, 0.7, 0.8, 0.85, 0.9\} \end{aligned} \tag{3.17}$$

where we assess the joint effect of downside and upside CIQ(τ) factors. We report t-statistics for each pair of $\lambda(\tau_{down})^{CIQ}$ and $\lambda(\tau_{up})^{CIQ}$ in the Figure 3.1. We observe that the prices of risk associated with downside risk remain statistically significant using every combination of downside and upside CIQ factors. On the other hand, the risk prices for the upside potential are in agreement with the previous results – insignificant but negative when controlling for higher values of τ_{down}.

Next, in the rest of the Table 3.7, we present results from bivariate regres-

Table 3.7: *Fama-MacBeth regressions using $\Delta CIQ(\tau)$ factors and general risk measures.*

	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
CIQ(τ)	0.90	0.94	1.11	1.52	2.50	0.55	1.01	0.71	-0.14	-0.22	-0.27
	(2.52)	(2.42)	(2.57)	(2.88)	(2.80)	(0.20)	(0.41)	(1.13)	(-0.30)	(-0.49)	(-0.60)
CIQ(τ)	0.41	0.46	0.59	0.95	1.78	0.55	0.91	1.02	0.39	0.30	0.22
	(1.43)	(1.53)	(1.76)	(2.18)	(2.25)	(0.21)	(0.39)	(1.74)	(0.97)	(0.80)	(0.59)
Mkt	-0.23	-0.23	-0.23	-0.22	-0.23	-0.25	-0.25	-0.24	-0.25	-0.25	-0.24
	(-1.70)	(-1.72)	(-1.71)	(-1.68)	(-1.69)	(-1.88)	(-1.87)	(-1.77)	(-1.83)	(-1.85)	(-1.84)
CIQ(τ)	0.72	0.75	0.87	1.19	2.03	-0.37	-1.11	0.41	-0.08	-0.11	-0.14
	(2.56)	(2.47)	(2.54)	(2.64)	(2.65)	(-0.17)	(-0.52)	(0.80)	(-0.22)	(-0.34)	(-0.42)
Idiosyncratic volatility	-14.12	-14.14	-14.15	-14.17	-14.40	-14.57	-14.61	-14.64	-14.58	-14.24	-13.86
	(-2.18)	(-2.18)	(-2.17)	(-2.20)	(-2.17)	(-2.20)	(-2.20)	(-2.19)	(-2.21)	(-2.18)	(-2.12)
CIQ(τ)	0.87	0.91	1.08	1.47	2.43	0.42	0.91	0.72	-0.13	-0.22	-0.28
	(2.47)	(2.38)	(2.53)	(2.76)	(2.74)	(0.15)	(0.38)	(1.17)	(-0.29)	(-0.50)	(-0.63)
Idiosyncratic skewness	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
	(0.26)	(0.29)	(0.28)	(0.31)	(0.33)	(0.37)	(0.41)	(0.47)	(0.47)	(0.53)	(0.57)
CIQ(τ)	0.86	0.90	1.07	1.47	2.43	0.43	0.91	0.73	-0.12	-0.21	-0.27
	(2.46)	(2.37)	(2.52)	(2.76)	(2.75)	(0.16)	(0.38)	(1.18)	(-0.27)	(-0.48)	(-0.61)
Skewness	-0.38	-0.30	-0.34	-0.25	-0.17	-0.13	-0.03	0.17	0.15	0.32	0.46
	(-0.13)	(-0.10)	(-0.12)	(-0.09)	(-0.06)	(-0.04)	(-0.01)	(0.06)	(0.05)	(0.11)	(0.17)

Note: The table contains estimated prices of risk and t-statistics from the Fama-MacBeth predictive regressions. Each segment contains prices of risk of CIQ(τ) betas while controlling for various risk measures. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$. Note the coefficients are multiplied by 100 for clarity of presentation.

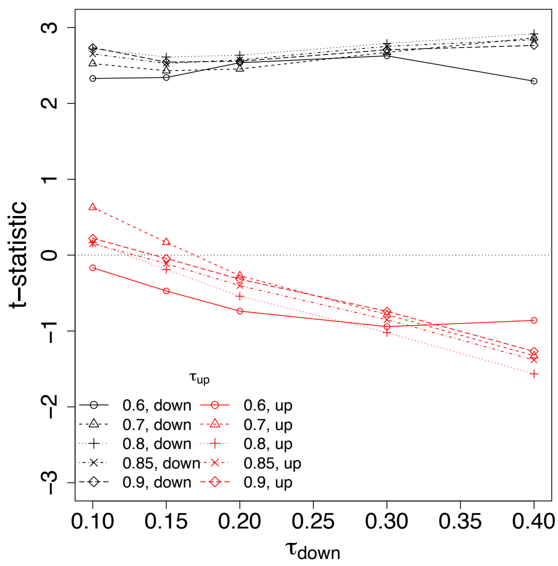
sions when controlling for the effect of general risk measures. We report the results of including CAPM betas by regressing the returns on the market return (Mkt). Interestingly, the effect of the CAPM beta diminishes the pricing relationship for the extreme left τ CIQ factors but the price of risk related to the linear exposure to the market factor possess counterintuitive negative sign – consistent with previous empirical evidence. Next, we control for the effect of the idiosyncratic volatility computed from the residuals of the 3-factor model of Fama and French (1993). The effect of the CIQ exposures remain very close to the one from the univariate regressions. Besides that, we confirm the presence of the idiosyncratic volatility puzzle. Next, we present results when controlling for idiosyncratic and total skewness. Those variables do not possess a significant pricing information for the cross-section, on the other hand, the effect of the CIQ factors remain consistent with the previous results.

Second, we report results from the bivariate[18] regressions in which we include as a control various risk measures based on common volatility or asymmetric dependence. Those measures were previously in the literature proven to be significant predictors of expected returns. We summarize the estimation outcomes in Table 3.8.

To investigate whether the quantile factors provide different priced informa-

[18]Except for the coskewness and cokurtosis, which we include both at the same time in the regression.

Figure 3.1: $\Delta CIQ(\tau)$ *betas – bivariate cross-sectional regressions.*



Note: The figure reports t-statistics of prices of risks from bivariate regressions from the Equation 3.17 of CIQ(τ) betas for downside and upside τs. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

tion beyond conventional approximate factor models, we construct and control for the following factor related to the common volatility. To do that, we proceed similarly as in the case of market prediction and construct a factor based on principal component analysis that captures dynamics in the common volatility. More specifically, as in the construction of the quantile factors – using the 60-month moving window, we extract the standardized idiosyncratic returns and square them. Then, we perform principal component analysis on those squared residuals and take the first principal component that explains the most common time variation across the squared residuals, and we denote it as PCA-SQ. We then difference the factor and use its increments as a control factor. From the results, we can conclude that the quantile factors extract very different information regarding the expected returns, as the specification based on the factor extracted from the squared residuals turn out not to be a significant predictor in the cross-section of stock returns. One has to look deeper into the common distribution if he wants to identify priced information regarding the common distributional movements.

Next, we employ volatility betas computed on differences of the CIV factor of Herskovic et al. (2016). We see that the results regarding CIQ(τ) betas still hold both qualitatively and quantitatively similar to the case of univariate

Table 3.8: *Fama-MacBeth regressions using CIQ(τ) factors and asymmetric risk measures.*

	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
CIQ(τ)	0.87	0.93	1.01	1.31	2.21	0.40	0.79	1.09	0.20	0.20	-0.12
	(2.45)	(2.52)	(2.53)	(2.69)	(2.42)	(0.14)	(0.30)	(1.55)	(0.43)	(0.43)	(-0.26)
PCA-SQ	8.56	8.97	2.57	-9.04	-22.35	-23.82	-26.22	-39.26	-29.58	-30.30	-14.48
	(0.35)	(0.37)	(0.11)	(-0.38)	(-0.92)	(-0.93)	(-1.01)	(-1.38)	(-1.04)	(-1.07)	(-0.54)
CIQ(τ)	0.77	0.81	0.99	1.38	2.34	1.26	2.14	0.89	-0.04	-0.13	-0.18
	(2.09)	(2.10)	(2.38)	(2.66)	(2.68)	(0.49)	(0.91)	(1.61)	(-0.10)	(-0.30)	(-0.41)
CIV	-0.39	-0.40	-0.43	-0.46	-0.50	-0.57	-0.55	-0.55	-0.53	-0.53	-0.51
	(-1.58)	(-1.64)	(-1.75)	(-1.91)	(-2.08)	(-2.46)	(-2.35)	(-2.34)	(-2.21)	(-2.23)	(-2.16)
CIQ(τ)	0.86	0.88	1.03	1.37	2.16	-0.77	-0.24	0.24	-0.43	-0.44	-0.44
	(2.45)	(2.28)	(2.42)	(2.64)	(2.53)	(-0.29)	(-0.10)	(0.41)	(-0.91)	(-1.00)	(-1.00)
TR	0.11	0.11	0.11	0.12	0.12	0.12	0.11	0.12	0.12	0.12	0.12
	(1.33)	(1.30)	(1.32)	(1.40)	(1.41)	(1.47)	(1.42)	(1.38)	(1.43)	(1.42)	(1.36)
CIQ(τ)	0.82	0.87	1.03	1.41	2.25	0.28	0.89	0.82	-0.03	-0.12	-0.18
	(2.40)	(2.39)	(2.47)	(2.83)	(2.76)	(0.11)	(0.39)	(1.38)	(-0.07)	(-0.27)	(-0.41)
Coskew	-0.12	-0.13	-0.14	-0.16	-0.16	-0.16	-0.15	-0.17	-0.17	-0.17	-0.17
	(-0.44)	(-0.46)	(-0.51)	(-0.57)	(-0.57)	(-0.58)	(-0.57)	(-0.61)	(-0.61)	(-0.62)	(-0.60)
Cokurt	-0.11	-0.11	-0.11	-0.11	-0.13	-0.16	-0.16	-0.15	-0.14	-0.14	-0.14
	(-1.50)	(-1.48)	(-1.45)	(-1.51)	(-1.72)	(-2.06)	(-2.07)	(-2.01)	(-1.90)	(-1.88)	(-1.78)
CIQ(τ)	0.68	0.73	0.89	1.30	2.26	0.55	0.88	0.87	0.13	0.03	-0.05
	(2.19)	(2.20)	(2.39)	(2.72)	(2.74)	(0.21)	(0.39)	(1.51)	(0.33)	(0.07)	(-0.13)
β^{DR}	-0.12	-0.12	-0.12	-0.11	-0.12	-0.14	-0.14	-0.13	-0.13	-0.13	-0.12
	(-1.17)	(-1.17)	(-1.15)	(-1.11)	(-1.18)	(-1.40)	(-1.41)	(-1.27)	(-1.29)	(-1.28)	(-1.25)
CIQ(τ)	0.96	1.01	1.18	1.63	2.69	0.51	0.69	0.68	-0.15	-0.24	-0.28
	(2.76)	(2.60)	(2.88)	(3.19)	(3.15)	(0.20)	(0.29)	(1.10)	(-0.34)	(-0.56)	(-0.64)
HTCR	119.53	118.76	118.64	119.47	118.91	111.84	113.06	118.60	118.29	116.58	114.63
	(3.00)	(2.98)	(2.97)	(2.97)	(2.91)	(2.75)	(2.77)	(2.88)	(2.92)	(2.96)	(2.94)
CIQ(τ)	0.80	0.80	0.84	1.06	1.68	-0.75	-0.73	0.15	-0.27	-0.32	-0.38
	(2.41)	(2.43)	(2.32)	(2.22)	(2.00)	(-0.28)	(-0.28)	(0.21)	(-0.55)	(-0.70)	(-0.80)
β^{-}	0.15	0.14	0.13	0.12	0.12	0.11	0.12	0.10	0.11	0.12	0.14
	(0.69)	(0.64)	(0.62)	(0.60)	(0.55)	(0.51)	(0.55)	(0.47)	(0.49)	(0.54)	(0.63)
CIQ(τ)	0.86	0.89	1.05	1.44	2.34	0.33	0.78	0.66	-0.15	-0.22	-0.27
	(2.45)	(2.26)	(2.46)	(2.73)	(2.61)	(0.12)	(0.32)	(1.09)	(-0.33)	(-0.51)	(-0.63)
DOWN ASY	-0.46	-0.46	-0.55	-0.56	-0.54	-0.58	-0.50	-0.43	-0.40	-0.27	-0.20
	(-0.23)	(-0.23)	(-0.26)	(-0.27)	(-0.26)	(-0.27)	(-0.24)	(-0.21)	(-0.19)	(-0.13)	(-0.10)
CIQ(τ)	1.07	1.10	1.26	1.58	2.65	1.26	1.03	1.13	0.13	-0.01	-0.15
	(2.70)	(2.56)	(2.68)	(2.61)	(2.54)	(0.40)	(0.37)	(1.55)	(0.25)	(-0.03)	(-0.32)
MCRASH	2.31	2.27	2.26	2.19	2.17	2.20	2.22	2.12	2.00	1.99	2.01
	(2.59)	(2.57)	(2.55)	(2.48)	(2.43)	(2.41)	(2.46)	(2.25)	(2.07)	(2.05)	(2.08)
CIQ(τ)	0.92	0.96	1.09	1.45	2.19	1.01	1.58	0.83	-0.12	-0.24	-0.33
	(2.70)	(2.62)	(2.66)	(2.72)	(2.40)	(0.39)	(0.66)	(1.40)	(-0.31)	(-0.64)	(-0.89)
COS PRED	-2.19	-2.26	-2.29	-2.40	-2.51	-2.53	-2.57	-2.50	-2.37	-2.40	-2.42
	(-1.30)	(-1.34)	(-1.34)	(-1.40)	(-1.47)	(-1.49)	(-1.52)	(-1.46)	(-1.39)	(-1.43)	(-1.44)

Note: The table contains estimated prices of risk and t-statistics from the Fama-MacBeth predictive regressions. Each segment contains prices of risk of ΔCIQ(τ) betas while controlling for various asymmetric risk measures. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$. Note the coefficients are multiplied by 100 for clarity of presentation.

regressions. Moreover, CIV risk is priced as well; especially strong is the relationship when we control for CIQ(τ) betas with τ from the right part of the distribution. These results suggest that both common idiosyncratic volatility and quantile risk are priced and do not convey the same pricing information.

As another related control, we use the tail risk (TR) factor of Kelly and Jiang (2014). As we can see, TR betas do not drive out the CIQ(τ) betas' effect, which remains significant, similarly to the univariate specification. Next, we control for related group of risk measures which consider the non-linear relationship between asset and market returns. By following the specifications of Harvey and Siddique (2000) and Ang et al. (2006), respectively, we control simultaneously for coskewness and cokurtosis. Once again, those measures do not drive out the significance of the CIQ(τ) betas. Coskewness possess the expected sign but it is not statistically significant. On the other hand, cokurtosis is borderline significant for $\tau \geq 0.5$ but with opposite sign than expected.

Another approach to capture non-linear dependence is via downside risk (DR) beta, which describes conditional covariance below some threshold level. We entertain the specification of Ang et al. (2006), which sets the threshold value equal to the average market return. As we can see, downside beta do not subsume the effect of the ΔCIQ(τ) factors, neither it is a significant predictor of future returns.

Another related left-tail risk measure is hybrid tail covariance risk (HTCR) measure proposed by Bali et al. (2014). Although, it is highly significant predictor of expected returns, it does not drive the effect of the CIQ(τ) risks out. Next, we include negative semibeta (β^-) of Bollerslev et al. (2021) in our bivariate regression. Similarly as in the previous cases, the exposure to the quantile factors yields a significant risk premium.

Then, to control for the effect of comovement asymmetry between left and right parts of the joint distribution of stock and market return, we include downside asymmetric comovement (DOWN ASY) measure of Jiang et al. (2018). This measure does not affect the relationship between expected returns and CIQ(τ) betas either.

To control for the effect of crashes in many risk factors, we control for multivariate crash risk (MCRASH) of Chabi-Yo et al. (2022).[19] MCRASH possess significant predictive power for the cross-section, which does not erase the effect of common idiosyncratic risk on the expected returns. Especially strong is the relationship between MCRASH and expected returns when controlling for CIQ(τ) risk in the left part of the joint distribution.

To control for the expectations of the coskewness, we also include stock-level predicted systematic skewness (COS PRED) of Langlois (2020) in the regressions. We can see that neither this variable drive out the effect of CIQ(τ) factors.

We also investigate whether the pricing information of the ΔCIQ(τ) factors

[19] We employ the baseline seven-factor version of their measure.

Table 3.9: *Fama-MacBeth regressions using CIQ(τ) factors and stock characteristics.*

	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
CIQ(τ)	0.68	0.72	0.83	1.14	1.91	0.63	0.11	0.67	0.03	-0.04	-0.08
	(2.20)	(2.13)	(2.28)	(2.47)	(2.42)	(0.25)	(0.05)	(1.19)	(0.07)	(-0.11)	(-0.20)
Size	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
	(-1.74)	(-1.75)	(-1.74)	(-1.76)	(-1.79)	(-1.89)	(-1.89)	(-1.73)	(-1.73)	(-1.76)	(-1.84)
Book-to-price	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12
	(1.76)	(1.75)	(1.71)	(1.71)	(1.79)	(1.90)	(1.92)	(1.94)	(1.94)	(1.93)	(1.90)
Net payout yield	0.95	0.88	0.91	0.93	1.05	1.23	1.26	1.27	1.26	1.17	1.03
	(1.19)	(1.12)	(1.13)	(1.07)	(1.11)	(1.21)	(1.24)	(1.26)	(1.35)	(1.33)	(1.33)
Turnover	-0.10	-0.10	-0.10	-0.10	-0.11	-0.11	-0.11	-0.11	-0.10	-0.10	-0.10
	(-2.05)	(-2.13)	(-2.16)	(-2.21)	(-2.23)	(-2.07)	(-2.08)	(-2.13)	(-2.09)	(-2.13)	(-2.10)
Illiquidity	1.86	1.86	1.86	1.85	1.85	1.95	1.95	1.97	1.97	1.96	2.00
	(1.08)	(1.09)	(1.09)	(1.08)	(1.10)	(1.18)	(1.17)	(1.14)	(1.13)	(1.13)	(1.13)
Profit	0.47	0.46	0.47	0.47	0.47	0.47	0.47	0.48	0.48	0.48	0.48
	(3.60)	(3.59)	(3.61)	(3.61)	(3.56)	(3.56)	(3.57)	(3.68)	(3.71)	(3.71)	(3.68)
Investment	-0.39	-0.39	-0.38	-0.38	-0.39	-0.39	-0.39	-0.39	-0.39	-0.39	-0.39
	(-7.13)	(-7.09)	(-7.07)	(-7.16)	(-7.08)	(-7.20)	(-7.18)	(-7.22)	(-7.24)	(-7.26)	(-7.28)

Note: The table contains estimated prices of risk and t-statistics from the Fama-MacBeth predictive regressions. Each segment contains prices of risk of ΔCIQ(τ) betas while controlling for various stock characteristics. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$. Note the coefficients are multiplied by 100 for clarity of presentation.

is not subsumed by stock characteristics based on accounting and trading information.[20] To that end, we provide the results of the multivariate cross-sectional regressions, in which we simultaneously control stock-level characteristics such as size, book-to-price, net payout yield, turnover, illiquidity, profit, and investment. We report the results in Table 3.9. We can see that the additional variables do not erase the pricing effect of the CIQ(τ) risks. The downside factors are significant determinants of the risk premium peaking at $\tau = 0.3$ with t-statistics of 2.47. On the other hand, exposure to the upside factors do not carry any significant pricing information.

Table 3.10 summarizes the results of controlling for the effect of past returns on the cross-section. Same as in the case of previous set of variables, we report estimation results from multivariate regression including variables maximum return, momentum, intermediate return, and lagged return. We observe that the additional variables slightly diminish the effect of the ΔCIQ(τ) factors for extreme left tail (τ between 0.1 and 0.2) but the effect for non-extreme downside risk remain strong. The effect of upside quantile factors remain insignificant even in this setting.

To summarize this subsection, we have shown that the CIQ(τ) results from the Fama-MacBeth regressions suggest that the exposure to the idiosyncratic downside common events is significantly priced in the cross-section of stock returns, and that none of the discussed risks drives out the significance of these

[20] We construct the variables in the same vein as in Langlois (2020).

Table 3.10: *Fama-MacBeth regressions using $\Delta CIQ(\tau)$ factors and momentum-type characteristics.*

	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
CIQ(τ)	0.49	0.52	0.63	0.88	1.79	0.83	0.13	0.72	0.13	0.10	0.08
	(1.66)	(1.69)	(1.90)	(2.11)	(2.49)	(0.37)	(0.06)	(1.31)	(0.34)	(0.29)	(0.25)
Maximum return	-11.17	-11.13	-11.12	-11.12	-11.12	-11.17	-11.22	-11.40	-11.49	-11.41	-11.24
	(-3.16)	(-3.15)	(-3.14)	(-3.05)	(-3.00)	(-2.99)	(-2.99)	(-3.09)	(-3.17)	(-3.21)	(-3.22)
Momentum	0.59	0.59	0.59	0.60	0.58	0.57	0.57	0.58	0.58	0.58	0.58
	(3.68)	(3.65)	(3.68)	(3.69)	(3.63)	(3.61)	(3.61)	(3.59)	(3.63)	(3.58)	(3.57)
Intermediate return	0.06	0.05	0.04	0.04	0.06	0.08	0.08	0.07	0.07	0.07	0.07
	(0.32)	(0.31)	(0.26)	(0.26)	(0.35)	(0.46)	(0.45)	(0.40)	(0.38)	(0.41)	(0.42)
Lagged return	-3.72	-3.73	-3.76	-3.75	-3.69	-3.64	-3.62	-3.60	-3.63	-3.65	-3.70
	(-7.05)	(-7.09)	(-7.12)	(-7.06)	(-6.89)	(-6.79)	(-6.73)	(-6.73)	(-6.84)	(-6.91)	(-7.07)

Note: The table contains estimated prices of risk and t-statistics from the Fama-MacBeth predictive regressions. Each segment contains prices of risk of CIQ(τ) betas while controlling for various momentum-type characteristics. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$. Note the coefficients are multiplied by 100 for clarity of presentation.

results. On the other hand, the exposure to the idiosyncratic upside potential captured by the quantile factors for $\tau \geq 0.5$ do not possess significant pricing implications for the cross-section of stock returns. This asymmetry further favors the hypothesis that the common volatility is not the reason behind the significant pricing consequences of the downside quantile factors.

3.4.2 Portfolio Sorts

Next, we asses performance of the ΔCIQ(τ) factors in terms of investment opportunities. To this end, we look at the returns of the portfolios sorted on the CIQ(τ) betas. Every month, we estimate CIQ(τ) betas for all stocks that possess 48 return observations during the last 60 months using data up to time t. We sort the stocks into ten portfolios based on their betas for every τ separately. We then record the portfolios' performances at time $t+1$ using either an equal-weighted or value-weighted scheme. Then we move one month ahead, re-estimate all the betas, and create new portfolios. We expect that, for $\tau < 0.5$, there will be an increasing pattern of returns from the low exposure to the high exposure portfolios, and vice versa for $\tau > 0.5$. The results for sorts based on ten portfolios summarizes Table 27. We observe an increasing return pattern for the portfolios with τ up to 0.4 for both equal-weighted and value-weighted schemes. This pattern practically disappears when we look at the portfolios formed on higher τ CIQ(τ) betas. This observation is in agreement with the results from the Fama-MacBeth regressions and suggests that only the exposure to the lower tail common movements is priced in the cross-section.

Moreover, to formally assess whether there is a compensation for bearing a

Table 3.11: *Portfolios sorted on the exposure to the $\Delta CIQ(\tau)$ factors.*

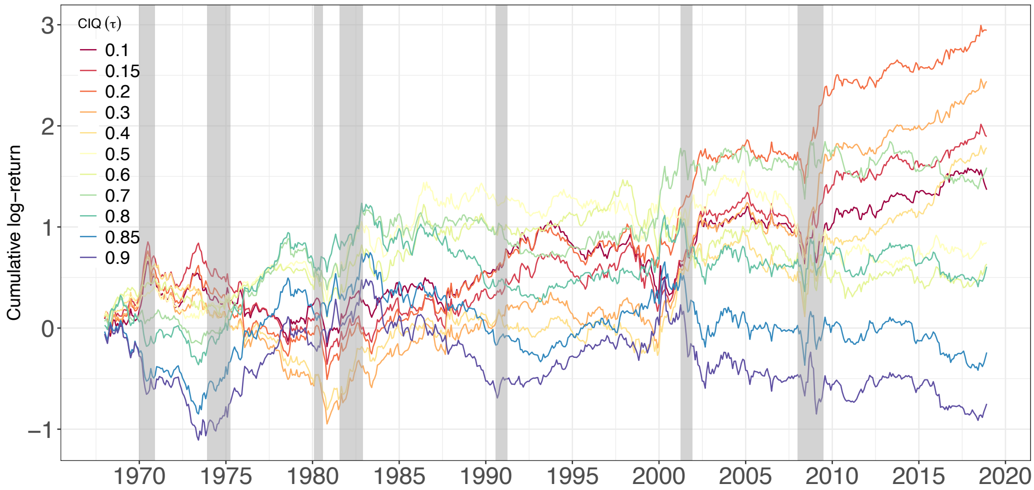
τ	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
							Equal-weighted							
0.10	4.88	7.89	8.20	9.19	8.75	9.47	9.99	10.65	10.71	9.63	4.74	2.76	4.80	2.55
0.15	4.41	7.78	8.88	8.95	9.59	9.39	10.11	10.36	10.10	9.79	5.38	3.07	5.60	3.04
0.20	4.37	7.43	8.82	9.33	9.10	10.14	10.11	10.06	9.86	10.12	5.74	3.18	6.36	3.28
0.30	4.41	7.54	8.49	9.15	9.87	9.71	10.15	10.28	10.16	9.59	5.19	3.10	5.76	3.37
0.40	4.65	8.11	8.82	9.39	9.05	9.64	10.22	10.35	9.90	9.22	4.57	2.95	4.79	3.01
0.50	6.77	9.01	9.95	9.38	9.29	9.48	9.61	9.73	9.11	7.02	0.25	0.14	-0.84	-0.41
0.60	6.35	9.25	9.77	9.16	9.66	9.75	9.84	9.20	8.96	7.42	1.07	0.63	-0.80	-0.43
0.70	6.28	8.84	9.86	9.11	9.19	9.01	9.54	9.40	9.57	8.55	2.27	1.61	0.15	0.09
0.80	8.05	9.34	9.43	9.02	8.84	9.39	9.23	8.91	8.78	8.36	0.32	0.20	-1.67	-0.96
0.85	8.19	9.13	9.54	8.97	9.02	9.40	9.61	8.88	8.57	8.03	-0.16	-0.10	-1.83	-0.99
0.90	8.14	9.69	9.40	8.89	9.11	9.58	9.32	8.89	8.87	7.45	-0.69	-0.38	-2.17	-1.13
							Value-weighted							
0.10	4.08	5.07	5.98	6.17	6.47	7.02	6.83	8.60	9.46	8.18	4.10	1.75	3.28	1.41
0.15	3.77	4.63	6.82	5.60	7.36	6.15	7.69	7.18	9.17	8.99	5.22	2.05	5.47	2.20
0.20	2.87	6.31	6.63	5.65	6.48	7.12	7.15	7.40	8.91	10.14	7.27	2.78	8.57	3.13
0.30	3.17	6.40	5.73	6.15	6.67	7.35	6.92	6.97	7.78	9.39	6.22	2.33	7.53	2.67
0.40	3.41	6.43	5.44	6.78	6.47	7.24	6.76	6.74	7.28	8.27	4.86	2.03	7.17	3.02
0.50	3.89	5.44	5.37	5.45	6.36	7.28	7.65	6.36	4.89	7.08	3.19	1.42	3.72	1.42
0.60	3.32	6.45	5.28	4.68	7.43	6.09	8.63	6.79	6.14	6.09	2.77	1.21	1.47	0.61
0.70	3.90	5.65	7.58	7.48	6.94	6.47	6.29	6.20	5.94	8.40	4.51	1.92	3.50	1.34
0.80	4.29	7.17	6.46	5.84	6.88	6.77	6.39	6.18	5.17	6.96	2.68	1.09	2.31	0.93
0.85	5.09	6.80	6.19	6.61	6.54	6.77	6.75	6.14	5.54	6.33	1.24	0.50	1.41	0.57
0.90	4.62	6.71	6.47	5.90	6.42	7.27	6.19	5.69	6.07	5.05	0.43	0.16	0.49	0.18

Note: The table contains annualized out-of-sample excess returns of ten portfolios sorted on the exposure to the ΔCIQ(τ) factors. We use all the CRSP stocks that have at least 48 monthly observations in each 60-month window. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We also report t-statistics for these alphas. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

risk of high exposure to the common movements in various parts of distributions of idiosyncratic returns, we present returns of high minus low portfolios. We obtain these returns as a difference between returns of portfolios with the highest CIQ(τ) betas and portfolios with the lowest CIQ(τ) betas. These portfolios are zero-cost portfolios and capture the risk premium associated with specific τ joint movements of idiosyncratic returns. As expected, we observe a significant positive premium for the difference portfolios only for τ being less or equal to 0.4. These premiums are both economically and statistically significant. In the case of the equal-weighted portfolios, the premium for CIQ(0.2) factors is 5.74% on the annual basis with a t-statistic of 3.18. The premiums are very similar in the case of the value-weighted portfolios – e.g., for $\tau = 0.2$ the premium is 7.27 with t-statistic of 2.78. This slightly lower significance in the case of the value-weighted portfolios may be partially caused by the fact that the value-weighted portfolios possess a higher concentration, which leads to more volatile returns.

To make sure that the estimated premiums cannot be explained by exposure to other risks previously proposed in the literature, we regress the returns of

Figure 3.2: *Performance of the CIQ(τ) portfolios.*



Note: The figure depicts cumulative log-return of high minus low portfolios obtained from sorting the stocks into decile portfolios based on their exposure to the CIQ(τ) factors and buying the portfolio with high exposure and selling the portfolio with low exposure. Returns of the portfolios are value-weighted.

the high minus low portfolios on four factors of Carhart (1997) and CIV shocks of Herskovic et al. (2016) and BAB factor of Frazzini and Pedersen (2014) and report corresponding annualized 6-factor alphas. From the results, we can see that the proposed factors do not capture the positive premium associated with the zero-cost portfolios. For the equal-weighted portfolio with $\tau = 0.2$, the estimated annualized alpha is 6.36% with t-statistic of 3.28, for value-weighted portfolios it is 8.57% premium with t-statistics being equal to 3.13.

To visually inspect the performance of the value-weighted CIQ(τ) portfolios, we present in Figure 3.2 cumulative log-return of the value-weighted high minus low portfolios for every τ. Consistent with the numerical results, only the portfolios based on CIQ factors for $\tau \leq 0.4$ provide strong performance during the sample period.

Next, in Table 3.12, we look at the performance of the CIQ(τ) sorted portfolios captured by the following twelve-month value-weighted returns. Each month, we construct portfolios as in the previous case. Instead of saving the next one-month return of the sorted portfolios, we record a twelve-month return, which follows after the formation period. Due to the passive approach for the following 12-month period, we focus on the value-weighted performance of the portfolios. We observe returns consistent with the results obtained using one-month returns. The high minus low portfolios with $\tau = 0.2$ yield 6.62% ($t = 2.43$). The other risk factors cannot explain these premiums as the 6-factor

Table 3.12: *Portfolio results with 1-year holding period.*

τ	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
0.10	2.83	4.37	6.01	6.25	5.91	5.77	6.74	9.18	9.42	9.17	6.35	2.64	5.49	1.82
0.15	2.78	4.72	6.09	5.81	6.54	5.92	7.87	8.70	9.10	8.95	6.17	2.05	5.49	1.43
0.20	2.60	5.23	6.04	7.08	6.29	5.56	7.58	8.73	8.88	9.22	6.62	2.43	6.44	2.29
0.30	2.93	5.00	5.90	6.95	6.48	6.22	7.10	7.18	8.73	8.30	5.37	1.87	7.35	2.43
0.40	3.34	4.53	6.37	6.43	6.03	6.69	6.43	7.86	7.46	6.33	2.99	0.87	5.31	1.60
0.50	4.82	4.61	5.06	5.46	6.18	8.64	7.42	6.47	5.58	6.72	1.90	0.78	0.75	0.25
0.60	4.68	5.12	5.22	5.96	5.85	7.05	8.00	6.65	6.26	6.11	1.43	0.73	-0.79	-0.29
0.70	2.88	6.04	5.82	6.56	7.25	6.80	6.95	6.48	6.56	6.48	3.60	2.31	3.67	1.16
0.80	4.02	6.53	5.12	6.71	7.15	6.93	7.26	6.59	5.89	4.22	0.20	0.09	-0.36	-0.09
0.85	4.27	5.78	5.46	6.63	7.58	7.08	6.99	6.91	5.65	4.92	0.65	0.24	-0.96	-0.24
0.90	5.06	6.13	5.10	6.33	6.99	7.12	6.60	6.74	5.78	4.63	-0.43	-0.18	-2.78	-0.74

Note: The table summarizes annualized out-of-sample returns of the CIQ(τ) portfolios which are held for one year after their formation. The returns are value-weighted.

alphas stay economically and statistically significant.

Due to the fact that only the exposures to the lower tail common movements are priced, the previous results suggest that the CIQ(τ) risks are not driven by the effect of the common volatility. If it were the case that the volatility is the main driver of the obtained results, we would observe that both exposures to the lower and upper parts of the joint movements are priced, which is not the case. But to explicitly control for the effect of the common idiosyncratic volatility, we perform dependent bivariate sorts by double sorting on betas for PCA-SQ factor and betas for the ΔCIQ(τ) factors. Every month, we first sort the stocks into ten portfolios based on their PCA-SQ betas. Then, within each of the PCA-SQ-sorted portfolios, we sort the stocks into ten portfolios based on their CIQ(τ) betas. Finally, for each CIQ(τ) portfolio, we collapse all the corresponding CIV portfolios into one CIQ(τ) portfolio. This procedure yields single-sorted portfolios which vary in their CIQ(τ) betas but possess approximately equal PCA-SQ betas. The obtained results summarizes Table 3.13. For the equal-weighted portfolios, we see that the risk premium captured by the returns of the high minus low portfolios for $\tau \leq 0.4$ remains significant with an annualized return of 4.48% ($t = 3.14$) for $\tau = 0.2$. In case of the value-weighted portfolios, the return remain close to the equal-weighted case with return of 4.51% for $\tau = 0.2$ ($t = 2.21$.). These observations suggest that the CIQ(τ) risk premium captures risk that is not explained to the common volatility as described by the PCA-SQ model.

The portfolio results show that holding risk associated with the common idiosyncratic downside risk is rewarded by a significant premium. On the other hand, exposure to the common idiosyncratic upside potential is not related to robust pricing consequences. In Appendix 3.A in Tables 26, 19, and 18, we

Table 3.13: *Dependent bivariate sorts on CIQ(τ) and PCA-SQ exposures.*

τ	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
							Equal-weighted							
0.10	6.09	7.97	7.92	9.28	9.06	9.71	9.37	9.51	10.35	10.13	4.05	3.01	4.04	2.71
0.15	6.09	7.76	8.73	8.87	9.03	8.93	10.16	9.51	10.28	10.02	3.92	2.75	3.85	2.52
0.20	5.63	8.49	8.14	9.01	9.46	9.27	9.53	9.70	10.08	10.11	4.48	3.14	4.60	2.83
0.30	5.43	8.40	8.30	8.95	9.18	9.92	9.26	9.76	10.30	9.94	4.51	3.19	4.34	2.83
0.40	5.57	8.76	8.64	8.94	9.10	9.40	9.22	10.10	10.09	9.60	4.03	2.70	3.60	2.26
0.50	6.77	9.69	9.26	9.66	9.37	9.56	9.20	9.31	9.09	7.48	0.71	0.44	-0.10	-0.05
0.60	6.16	10.00	9.56	9.35	9.63	9.11	9.93	8.70	9.32	7.66	1.49	0.96	0.14	0.09
0.70	6.52	8.93	9.40	9.65	8.81	9.07	9.34	9.37	9.19	9.13	2.61	2.05	1.44	1.06
0.80	7.74	9.21	9.36	9.34	8.94	8.93	8.47	9.11	9.41	8.87	1.14	0.89	-0.20	-0.14
0.85	7.68	9.10	9.06	8.86	9.16	9.03	8.99	9.61	9.02	8.86	1.18	0.87	0.11	0.07
0.90	7.89	9.45	8.89	9.14	9.10	9.15	9.25	8.81	9.17	8.54	0.65	0.45	-0.67	-0.42
							Value-weighted							
0.10	5.29	5.82	5.55	6.07	5.99	5.53	6.95	8.12	8.23	9.80	4.52	2.19	4.13	2.05
0.15	4.67	6.12	6.37	5.26	5.85	6.43	7.30	7.18	8.11	9.39	4.72	2.25	4.68	2.34
0.20	5.07	7.21	4.98	6.92	5.44	6.08	6.68	7.12	8.12	9.58	4.51	2.21	4.86	2.24
0.30	5.02	7.02	5.90	6.50	5.69	6.38	6.73	5.82	8.31	9.39	4.37	2.07	4.59	2.07
0.40	4.30	7.02	5.77	7.04	5.49	6.64	6.44	6.80	8.11	8.12	3.82	1.79	4.36	2.14
0.50	5.68	4.80	5.62	6.10	6.48	5.73	7.78	6.83	7.39	5.35	-0.33	-0.15	-0.85	-0.36
0.60	4.58	5.94	5.69	5.02	6.87	5.48	7.78	7.26	7.93	6.09	1.51	0.72	-0.18	-0.09
0.70	6.06	6.11	6.97	6.68	5.82	7.08	5.77	5.94	7.03	7.59	1.52	0.72	1.24	0.59
0.80	4.64	7.20	6.13	5.54	6.63	7.16	5.28	5.47	6.90	7.14	2.50	1.21	1.56	0.71
0.85	4.62	6.71	6.46	6.13	6.58	5.48	6.22	6.78	6.82	6.36	1.74	0.89	0.56	0.27
0.90	3.65	8.45	6.12	5.74	5.33	6.92	6.01	7.41	5.90	7.13	3.48	1.59	1.64	0.74

Note: The table contains annualized out-of-sample excess returns of ten portfolios sorted on the exposure to the ΔCIQ(τ) and PCA-SQ factor. Exposure to the PCA-SQ factor are approximately same across the portfolios. We use all the CRSP stocks that have at least 48 monthly observations in each 60-month window. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We also report t-statistics for these alphas. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

provide results of the same analysis using five portfolios instead of ten. The results are qualitatively very similar to the results from the above, confirming the robustness of our claim that the exposure to the common left tail events is priced in the cross-section of returns.

Finally, we also repeat the exercise on the simulated universe of stocks. We simulate stocks from location-scale model in order to see that risk factors will not be quantile dependent, and will all be coming from the volatility. At the same time the exercise will show that choice of small sample in the moving window does not bias the results. Detailed discussion in Appendix 3.B shows that the premium associated with exposures to the different quantile levels on simulated data are the same to the exposures on the PCA-SQ factors in magnitude. The risk premiums have identical significance, and is constant (with opposite sign for downside and upside) over the quantiles. Hence if the returns were generated from the location-scale model, then quantile risk would be equivalent across quantiles, and it would be captured by the volatility risk.

Table 3.14: *Portfolios sorted on relative CIQ(τ) betas.*

τ	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
							Equal-weighted							
0.10	5.00	7.68	8.41	8.97	8.83	9.55	9.80	10.73	10.56	9.81	4.81	2.71	4.66	2.37
0.15	4.32	7.89	8.76	9.13	9.55	9.19	10.33	10.08	10.05	10.04	5.73	3.16	5.75	2.93
0.20	4.37	7.54	8.73	9.19	8.93	9.85	10.63	10.09	9.71	10.31	5.94	3.22	6.45	3.17
0.30	4.51	7.01	8.43	9.31	10.06	9.66	10.19	10.11	10.00	10.08	5.57	3.17	5.96	3.28
0.40	4.57	8.03	8.52	9.22	9.75	9.85	9.33	9.96	10.43	9.69	5.11	3.22	5.14	3.20
0.50	6.77	9.01	9.95	9.38	9.29	9.48	9.61	9.73	9.11	7.02	0.25	0.14	-0.84	-0.41
0.60	7.52	8.96	8.37	9.53	9.18	10.06	8.89	9.76	9.45	7.64	0.13	0.09	-1.27	-0.84
0.70	6.55	9.44	9.47	8.58	9.33	8.87	9.52	9.19	9.76	8.63	2.09	1.56	0.18	0.12
0.80	8.25	9.09	9.40	8.99	8.68	9.41	9.39	8.89	9.09	8.15	-0.10	-0.07	-1.94	-1.17
0.85	8.30	9.37	9.20	9.32	9.10	8.81	9.46	9.30	8.65	7.82	-0.48	-0.29	-1.88	-1.03
0.90	8.38	9.19	9.54	9.13	9.18	9.53	9.10	9.07	9.00	7.23	-1.15	-0.63	-2.45	-1.31
							Value-weighted							
0.10	3.83	4.53	6.11	6.53	6.49	6.33	6.95	8.51	9.70	8.48	4.66	1.97	2.96	1.29
0.15	4.00	4.72	6.40	6.60	7.20	6.30	7.03	7.39	9.33	9.12	5.11	2.04	4.75	1.94
0.20	2.79	6.25	6.36	6.38	6.61	7.11	7.12	7.23	8.59	9.68	6.89	2.72	7.12	2.74
0.30	3.47	6.24	5.64	6.02	7.85	6.96	6.89	6.53	8.18	9.66	6.19	2.33	6.83	2.50
0.40	3.69	6.37	6.24	6.53	7.16	6.83	6.09	6.27	7.96	8.99	5.30	2.06	7.35	2.87
0.50	3.89	5.44	5.37	5.45	6.36	7.28	7.65	6.36	4.89	7.08	3.19	1.42	3.72	1.42
0.60	5.93	5.56	5.97	5.30	5.70	5.80	6.53	7.13	7.77	6.82	0.89	0.39	0.23	0.08
0.70	4.40	6.63	5.92	7.42	6.85	7.24	5.54	5.88	6.68	7.34	2.94	1.26	2.02	0.77
0.80	5.38	7.48	5.50	6.67	6.78	6.74	6.51	6.12	5.01	6.45	1.07	0.43	0.65	0.25
0.85	5.07	7.05	6.70	6.34	6.51	6.51	6.75	6.29	5.63	5.54	0.48	0.18	0.75	0.29
0.90	4.96	6.84	6.30	6.58	6.26	6.76	6.87	5.37	5.72	5.57	0.62	0.22	1.53	0.55

Note: The table contains annualized out-of-sample excess returns of ten portfolios sorted on relative CIQ(τ) betas. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We also report t-statistics for these alphas. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

3.4.3 Beyond CIQ(τ) Betas

To specifically capture additional information beyond median dependence from the lower and upper parts of the distribution, respectively, we define *relative* CIQ betas as

$$\beta_i^{rel}(\tau) := \beta_i(\tau) - \beta_i(0.5).$$

The results of the portfolio sorts based on relative betas are summarized in Table 3.14. These results are in the spirit of the CIQ betas' results presented above. The high minus low portfolio sorted on $\beta^{rel}(0.2)$ yields annual 5.94% excess return ($t = 3.22$) with 6-factor $\alpha = 6.45$ ($t = 3.17$) for the equal-weighted portfolio. In case of the value-weighted portfolios, we obtain annual return of 6.89% ($t = 2.72$) and $\alpha = 7.12$ ($t = 2.74$).

Because there is a little theory on which τ to choose when investing based on the exposure to the CIQ(τ) factors, we aim to aggregate the information from downside and upside factors into compressed measures. To summarize the dependence in the whole lower or upper part of the factor structure, we define *downside* and *upside* CIQ betas as

$$\beta_i^{down} := \sum_{\tau \in \tau_{down}} F\Big(\beta_i(\tau)\Big)$$
$$\beta_i^{up} := \sum_{\tau \in \tau_{up}} F\Big(\beta_i(\tau)\Big)$$

where $F\Big(\beta_i(\tau)\Big) = \frac{Rank(\beta_i(\tau))}{N_t+1}$. We obtain the downside and upside CIQ betas as an average cross-sectional rank of the CIQ betas for downside and upside τs, respectively. Results of the portfolio sorts based on those betas are summarized in Table 3.15. We can see that the long-short portfolios sorted on downside CIQ betas provide significant excess returns of 5.19% ($t = 3.02$) and 6.44% ($t = 2.48$) annual returns using equal- and value-weighted schemes, respectively. On the other hand, an investment strategy based on upside beta does not yield significant abnormal returns using either weighting approach.

To summarize the relative betas through the whole downside or upside parts of the joint structure, we introduce *downside* and *upside relative* betas

$$\beta_i^{down,rel} := \sum_{\tau \in \tau_{down}} F\Big(\beta_i^{rel}(\tau)\Big),$$
$$\beta_i^{up,rel} := \sum_{\tau \in \tau_{up}} F\Big(\beta_i^{rel}(\tau)\Big),$$

which are obtained as a mean cross-sectional rank of the relative betas associated with the exposure to the downside or upside CIQ(τ) factors, respectively. The associated returns are also summarized in Table 3.15. Similarly as in the case of the relative betas, downside relative betas provide investment strategy with significant abnormal returns of 6.02% ($t = 3.25$) and 7.40% ($t = 2.90$) on an annual basis using equal- or value-weighted returns, respectively. The returns of the portfolios based on relative upside betas remain insignificant.

3.5 Conclusion

We investigate the pricing implications of the exposures to the common idiosyncratic quantile factors. These factors capture non-linear common movements in various parts of the distributions across a large panel of stocks. Similarly, as the quantile regression extends the classical linear regression, our quantile factor model of asset returns extends the approximate factor models used in empirical asset pricing literature. We show that the downside quantile factors

Table 3.15: *Ten univariate sorted portfolios on combination CIQ betas.*

Weighting	Variable	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
Equal	β^{down}	4.71	7.20	8.54	9.23	9.48	10.20	9.26	10.49	10.34	9.90	5.19	3.02	5.66	3.19
	β^{up}	7.73	9.45	9.54	8.73	9.23	9.08	9.43	9.30	8.56	8.30	0.57	0.36	-1.43	-0.79
	$\beta^{down,rel}$	4.33	7.68	8.47	8.97	9.71	9.78	10.10	9.90	10.07	10.34	6.02	3.25	6.43	3.26
	$\beta^{up,rel}$	8.58	9.04	8.89	8.87	9.26	9.05	9.07	9.03	9.24	8.31	-0.27	-0.18	-2.00	-1.23
Value	β^{down}	3.08	6.41	5.90	5.85	5.72	8.06	6.96	7.59	8.31	9.52	6.44	2.48	7.15	2.88
	β^{up}	4.72	6.57	5.02	6.59	7.11	7.21	6.53	5.57	5.50	7.51	2.79	1.17	2.38	0.96
	$\beta^{down,rel}$	2.97	6.35	5.79	6.47	6.85	6.73	7.53	6.61	8.38	10.37	7.40	2.90	7.52	2.91
	$\beta^{up,rel}$	5.60	7.08	6.71	5.39	7.38	6.51	6.58	6.10	5.36	6.64	1.04	0.43	0.32	0.13

Note: The table contains annualized out-of-sample excess returns of ten portfolios sorted on downside (upside) and relative downside (upside) CIQ betas. We use all the CRSP stocks that have at least 48 monthly observations in each 60-month window. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We also report t-statistics for these alphas. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

can robustly predict the market return out-of-sample. We also provide evidence that the expected returns are associated with the exposures to the downside common movements in contrast to the upside movements. Importantly, the quantile dependent factors provides richer information to investors in comparison to other downside risk or volatility factors. We perform various robustness checks to show that these results are not attributable to other previously proposed risk factors. Most notably, we aim to prove that the common volatility does not drive the results.

Future research may focus on better interpretability of the quantile factor models using the characteristics-based quantile factor model proposed by Chen et al. (2023). This investigation may identify which stock characteristics are related to exposure to common extreme events. From a theoretical perspective, future endeavors could explore the link between theoretical quantile asset pricing models, such as the model of Ramos et al. (2020), and quantile factor models. Furthermore, an important direction may extend the arbitrage pricing theory into the quantile domain in the spirit of Renault et al. (2022).

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Appendix

3.A Additional Results

Table 16: *Correlations between CIQ(τ) and other factors.*

variable / τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
Panel A: *Levels of factors*											
PCA-SQ	-0.68	-0.66	-0.59	-0.47	-0.23	0.06	0.12	0.44	0.61	0.65	0.70
CIV	-0.29	-0.26	-0.24	-0.16	-0.03	0.04	0.07	0.19	0.28	0.28	0.30
TR	0.07	0.07	0.06	0.02	-0.04	-0.02	-0.04	-0.19	-0.19	-0.19	-0.17
VRP	0.04	0.05	0.06	0.07	0.10	-0.08	-0.03	0.03	0.03	0.00	0.01
VIX	-0.17	-0.15	-0.11	-0.01	0.16	0.08	0.13	0.30	0.33	0.30	0.28
Panel B: *Differences of factors*											
PCA-SQ	-0.54	-0.50	-0.44	-0.32	-0.11	0.17	0.17	0.35	0.51	0.55	0.60
CIV	-0.20	-0.17	-0.17	-0.12	-0.06	0.06	0.07	0.11	0.15	0.15	0.13
TR	0.11	0.09	0.09	0.04	-0.03	-0.03	-0.03	-0.24	-0.26	-0.27	-0.25
VRP	0.14	0.12	0.10	0.07	0.02	-0.05	-0.03	-0.06	-0.07	-0.11	-0.10
VIX	0.20	0.23	0.23	0.22	0.22	0.07	0.10	0.10	0.05	0.01	-0.06

Note: The table reports correlations between CIQ(τ) factors and factors related to the asymmetric and variance risk. Data contain the period between January 1963 and December 2018.

Table 17: *Portfolios sorted on the exposure to the $\Delta CIQ(\tau)$ factors.*

τ	Low	2	3	4	High	H - L	t-stat	α	t-stat
					Equal-weighted				
0.10	6.38	8.69	9.11	10.32	10.17	3.78	2.59	4.36	2.79
0.15	6.10	8.91	9.49	10.23	9.94	3.85	2.58	4.65	3.05
0.20	5.90	9.08	9.62	10.09	9.99	4.08	2.78	5.20	3.47
0.30	5.97	8.82	9.79	10.21	9.88	3.90	2.81	4.89	3.56
0.40	6.38	9.11	9.34	10.29	9.56	3.19	2.55	4.00	3.22
0.50	7.89	9.67	9.38	9.67	8.07	0.18	0.14	-0.04	-0.03
0.60	7.80	9.46	9.70	9.52	8.19	0.39	0.30	-0.55	-0.42
0.70	7.56	9.49	9.10	9.47	9.06	1.50	1.31	0.29	0.22
0.80	8.69	9.23	9.11	9.07	8.57	-0.12	-0.10	-1.37	-0.98
0.85	8.66	9.26	9.21	9.24	8.30	-0.36	-0.26	-1.59	-1.07
0.90	8.91	9.14	9.35	9.11	8.16	-0.75	-0.48	-1.83	-1.22
					Value-weighted				
0.10	4.74	6.10	6.63	7.58	9.16	4.42	2.20	4.26	2.24
0.15	4.36	6.13	6.71	7.38	9.08	4.72	2.39	5.40	2.90
0.20	4.98	6.07	6.75	7.15	9.07	4.09	2.09	5.39	3.05
0.30	5.06	5.87	7.00	6.82	8.03	2.97	1.57	4.43	2.59
0.40	5.14	5.96	6.83	6.71	7.57	2.42	1.36	4.40	2.53
0.50	4.67	5.44	6.80	7.07	5.43	0.77	0.46	0.90	0.45
0.60	5.24	4.85	6.51	7.64	6.11	0.87	0.50	-0.16	-0.09
0.70	4.96	7.37	6.52	6.08	6.75	1.79	1.06	1.48	0.84
0.80	6.11	6.13	6.69	6.28	5.99	-0.11	-0.06	-0.49	-0.28
0.85	6.12	6.31	6.58	6.50	5.92	-0.20	-0.11	-0.39	-0.22
0.90	5.92	6.18	6.91	5.90	5.86	-0.06	-0.03	-0.20	-0.11

Note: The table contains annualized out-of-sample excess returns of five portfolios sorted on the exposure to the ΔCIQ(τ) factors. We use all the CRSP stocks that have at least 48 monthly observations in each 60-month window. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We also report t-statistics for these alphas. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

Table 18: *Dependent bivariate sorts on CIQ(τ) and PCA-SQ exposures.*

τ	Low	2	3	4	High	H - L	t-stat	α	t-stat
					Equal-weighted				
0.10	6.91	8.58	9.39	9.77	10.02	3.11	2.78	3.22	2.57
0.15	7.03	8.64	9.39	9.68	9.93	2.90	2.57	2.97	2.44
0.20	6.84	8.65	9.61	9.46	10.12	3.28	2.91	3.63	2.91
0.30	6.84	8.55	9.81	9.43	10.04	3.20	2.86	3.42	2.84
0.40	7.03	8.98	9.19	9.85	9.63	2.60	2.29	2.69	2.22
0.50	8.07	9.61	9.35	9.37	8.27	0.20	0.16	-0.33	-0.25
0.60	8.07	9.59	9.35	9.30	8.36	0.30	0.24	-0.75	-0.60
0.70	7.59	9.68	8.90	9.31	9.19	1.59	1.59	0.93	0.86
0.80	8.28	9.58	8.81	8.95	9.06	0.78	0.80	-0.03	-0.03
0.85	8.36	9.05	8.97	9.63	8.67	0.30	0.31	-0.50	-0.44
0.90	8.49	9.10	9.27	9.23	8.59	0.11	0.09	-0.85	-0.70
					Value-weighted				
0.10	5.43	5.79	6.05	6.96	8.58	3.15	2.07	2.20	1.68
0.15	5.58	6.15	5.90	7.25	7.77	2.19	1.43	1.76	1.24
0.20	6.08	5.88	5.97	6.80	7.91	1.83	1.30	1.95	1.42
0.30	6.21	6.09	6.25	6.22	8.04	1.83	1.23	2.11	1.50
0.40	5.56	6.56	6.15	6.75	7.08	1.51	0.96	1.99	1.30
0.50	5.15	5.58	6.25	7.02	6.42	1.27	0.80	0.28	0.16
0.60	5.55	5.27	6.08	7.41	7.22	1.67	0.99	0.19	0.11
0.70	6.01	6.41	6.58	5.68	7.56	1.55	0.98	0.55	0.39
0.80	5.77	6.12	6.28	6.15	6.65	0.88	0.59	-0.56	-0.37
0.85	5.81	6.12	6.53	6.29	6.48	0.67	0.49	-0.59	-0.40
0.90	5.15	6.63	6.36	6.49	6.71	1.56	0.97	0.05	0.03

Note: The table contains annualized out-of-sample excess returns of five portfolios sorted on the exposure to the ΔCIQ(τ) and PCA-SQ factor. Exposure to the PCA-SQ factor are approximately same across the portfolios. We use all the CRSP stocks that have at least 48 monthly observations in each 60-month window. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 6-factor alphas with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). We also report t-statistics for these alphas. Data contain period between January 1963 and December 2018. In each window, we use all the CRSP stocks that have at least 48 monthly observations, and we exclude penny stocks with prices less than 1$.

Table 19: *Portfolio results with 1-year holding period.*

τ	Low	2	3	4	High	H - L	t-stat	α	t-stat
0.10	3.71	6.13	5.82	8.00	9.41	5.70	2.56	6.49	1.70
0.15	4.01	5.79	6.15	8.21	9.12	5.11	2.28	6.69	1.88
0.20	4.15	6.48	5.80	8.10	8.98	4.83	2.25	7.00	1.93
0.30	4.14	6.40	6.34	7.25	8.40	4.26	2.06	5.14	1.33
0.40	3.97	6.44	6.18	7.10	7.11	3.15	1.73	2.81	0.91
0.50	4.43	5.19	7.40	6.90	5.94	1.51	1.13	-1.32	-0.75
0.60	4.80	5.33	6.37	7.21	6.26	1.46	1.12	-2.38	-1.47
0.70	5.17	6.11	6.94	6.57	6.37	1.20	0.91	-0.81	-0.60
0.80	5.82	5.86	6.85	6.92	5.21	-0.61	-0.38	-2.45	-1.92
0.85	5.45	6.14	7.11	6.83	5.21	-0.25	-0.13	-2.49	-1.84
0.90	5.82	5.94	6.99	6.46	5.32	-0.50	-0.27	-2.36	-1.52

Note: The table summarizes returns of the CIQ(τ) portfolios which are held for one year after their formation. The returns are value-weighted.

3.B Simulation Study

We present a simulation exercise to illustrate how the CIQ(τ) premiums would look like if the driving force behind them were simply common volatility. We simulate the returns from the following model

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \gamma_i (V_t - \bar{V}) - \gamma_i \lambda^V + e_{i,t} \tag{18}$$

where V_t is the common variance factor, and the variance of the idiosyncratic error follows the factor structure proposed by Ding et al. (2022)

$$\begin{aligned} e_{i,t} &= \sqrt{V_{i,t}} z_{i,t}, \\ V_{i,t} &= V_t \exp(\mu_i + \sigma_i u_{i,t}) = V_t \tilde{V}_{i,t}, \\ z_{i,t},\ u_{i,t} &\sim i.i.d.\, N(0,1). \end{aligned} \tag{19}$$

Time-series variation of the returns drive two common factors – market factor, $r_{m,t}$, and variance factor V_t. The expected return of a stock is then equal to

$$\mathbb{E}[r_i] = \alpha_i + \beta_i \mathbb{E}[r_m] + \gamma_i \lambda^V. \tag{20}$$

We assume that the market factor follows a simple GARCH(1,1) process of Bollerslev (1986), which we fit on the market return from the empirical analysis. We assume that the log of the variance factor follows a modified HAR model of Corsi (2009)

$$\begin{aligned} \log V_{t+1} &= \theta_0 + \theta_m x_t^m + \theta_y x_t^y + v_{t+1} \\ v_{t+1} &\sim i.i.d.\, N(0, \sigma_v^2) \end{aligned} \tag{21}$$

where x_t^m and x_t^y are the previous month's log-variance and average log-variance over the last 12-month period, respectively. The common variance process is approximated by the cross-sectional average of the squared residuals from the time series regression of stock returns on the market factor. We fit the model from equation 21 on this time series. When simulating this time series, we initialize the process by randomly selecting 12 consequent observations of the common variance process estimated from the data and using those observations for iterating forward.

We calibrate the simulation setting to match the CRSP data sample we employ in the empirical investigation. We estimate stock-level market beta, β_i, using time-series regression of stock return on the market return. Exposure to the common variance, γ_i, is estimated by regressing the stock return on the estimate of the common variance process. Price of risk associated with the variance exposure, λ^V is chosen to be equal to 3×10^{-3}.[21] We estimate stock-level parameters of the idiosyncratic error variance–μ_i, σ_i–as the sample mean and standard deviation of $\log \tilde{V}_{i,t}$. To approximate the $\tilde{V}_{i,t}$, we use squared residuals from the time-series regression of the stock return on the market return. Then, to simulate these parameters, we approximate their distribution by normal distribution, with the mean equal to the estimates' cross-sectional average and the variance equal to the cross-sectional variance of the estimates.

We simulate the panel of 2,500 stocks with 120 observations. We repeat the simulation 1,000 times. Each time, we simulate stock returns by randomly choosing parameters for the stock-level process from the normal distribution with mean and variance corresponding to their sample counterparts. We remove the common time variation in stock returns by first forming the common linear factor

$$f_t = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}, \quad t = 1, \ldots, T \tag{22}$$

and then regressing the returns on this factor

$$r_{i,t} = \alpha_i + \hat{\beta}_i f_t + \hat{e}_{i,t}, \tag{23}$$

which yields the residuals $\hat{e}_{i,t}$. Those residuals are then used to form the common volatility and quantile factors. We construct the volatility factor as the first principal component of those squared residuals. ΔCIQ(τ) factors are estimated as discussed in Section 3.2. Exposures to those factors are then estimated using univariate time-series regressions of stock returns on the increments of the volatility or quantile factors, respectively.

Similarly, as in the empirical investigation, we sort stocks into decile portfolios based on their estimated exposure to the factors to infer the associated risk premiums. We proxy the premiums by computing high minus low returns of the portfolios. Table 20 reports the average premiums for all the CIQ(τ) factors. We observe that the premium is positive for the downside values of τ, negative for the upside ones and insignificant for the median. The magnitude

[21] This value corresponds to approximately 6% annual high minus low premium obtained from ten portfolios portfolios sorted on the exposure to the common variance. The choice of this value is not essential for the results that we present here.

Table 20: *Simulated risk premiums.*

τ	Premium	t-stat	Rejections
0.10	9.34	2.62	0.96
0.15	9.37	2.56	0.96
0.20	9.38	2.51	0.96
0.30	9.50	2.54	0.97
0.40	9.37	2.26	0.96
0.50	0.35	0.03	0.96
0.60	-9.61	-2.70	0.96
0.70	-9.60	-2.72	0.96
0.80	-9.52	-2.67	0.96
0.85	-9.42	-2.58	0.96
0.90	-9.35	-2.56	0.96

Note: The table contains average risk premiums computed from high minus low returns of decile portfolios sorted on exposure to the CIQ(τ) risks. We simulate the returns using common variance factor model proposed by Ding et al. (2022). We simulate panel of 2,500 stocks with 120 monthly observations. We perform the simulation 1,000 times. t-statistics are obtained by dividing the average premium by its standard deviation. We also report proportion of rejections of non-significance of CIQ(τ) betas from multivariate cross-sectional regressions of average returns on those betas and market betas.

of the premiums is comparable across all τ and, on average, in absolute value equal to 9.44%. The premium associated with the exposure to the PCA-SQ factor is -6.09%. We also compute associated t-statistics as a ratio between average premium and its standard deviation across all the simulations. All the premiums except for the median value are significant, with values around 2.6 in absolute value. The t-value associated with the PCA-SQ factor is -2.33. Next, we present the proportion of rejections of non-significance of CIQ(τ) betas at a 5% significance level from multivariate cross-sectional regressions of average returns on those betas and market betas. We can see that the proportions are virtually identical for both upside and downside betas of around 96%. The ratio for the PCA-SQ betas is 90%.

As we can see from the results, if there was a common volatility element present in the return, which is compensated in the cross-section, the CIQ(τ) risk premium would be symmetrical around the median. Moreover, the exposure to the PCA-SQ factor would be priced in this case. Overall, the evidence from the simulation exercise suggests that the CIQ(τ) risk premiums we observe in the data are not attributable to the common volatility compensation.

Chapter 4

Asymmetric Risks: Alphas or Betas?[1]

I show that systematic asymmetric risk measures, such as coskewness or tail risk beta, can complement each other when implementing an investment strategy based on them. I propose a simple approach to combining these measures and obtaining anomalous returns above the premiums associated with each measure separately. I show that various multivariate regression setups that combine the asymmetric risk measures perform poorly. Instead, I use instrumented principal component analysis and construct portfolios that are neutral with respect to the common sources of risk associated with these measures. The resulting portfolios enjoy abnormal returns that no other factor model can fully explain, although there is a clear relation between asymmetric risk measures and the momentum factor. I also show that some measures can contribute significantly to the performance of a model with a linear factor structure.

4.1 Introduction

The nonlinear systematic behavior of stock returns has been a fruitful area of research in the empirical asset pricing literature. Many statistical measures that capture the essence of these features have been proposed as significant cross-sectional predictors. They all attempt to capture the natural human aversion to extreme adverse events, especially in bad times. However, the definition of these extreme events and bad times tends to differ across specifications. There

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is no theoretical answer as to which specification is the right one. I propose an approach that combines these measures into a portfolio that efficiently exploits the associated premiums.

Many of the studied systematic asymmetric risk measures produce differing significance levels for their risk premium. This variability is contingent upon the research environment in which they are assessed. I intend to enhance their performance by merging these measures and averaging out the associated noise. Unfortunately, regression models based on Lewellen (2015) exhibit poor performance, even when I utilize regularization techniques like lasso or ridge regression.

Instead, I propose to use the instrumented principal component analysis (IPCA) by Kelly et al. (2019). Using the unrestricted version of their model, I am able to differentiate between risk compensation for bearing the risk related to the common factors and the risk associated with the non-linear features of the measures. I construct a portfolio that is conditionally neutral with respect to the exposures to the associated latent factors. Nevertheless, it yields an annualized Sharpe ratio of up to 0.97. This result shows that the employed asymmetric measures can be successfully used to yield significant alphas.

Furthermore, the abnormal returns cannot be explained by any other factor model, including IPCA factors estimated using the original dataset of 32 characteristics. However, the returns of this arbitrage portfolios are generally exposed to the momentum factor. Assuming a constant relationship between asymmetric risk measures and arbitrage portfolio formation, accounting for this exposure only partially diminishes the abnormal returns. When I allow for time variation in the relationship, the decline in efficiency causes momentum to fully capture the abnormal returns.

I also examine the alignment of asymmetric risk measures with exposures to common linear factors. A six-factor model using asymmetric risk measures as proxies for exposures to these latent factors is required to capture the anomaly returns associated with eleven measures. This result suggests that these variables have little redundancy for asset prices. In addition, a portfolio that is mean-variance efficient and has asymmetric risks explaining the factor loadings can result in a Sharpe ratio of approximately 1.15.

When evaluating the asymmetric risk measures in a controlled environment of 32 characteristics from Kelly et al. (2019), three measures significantly impact the fit of the latent factor model: downside beta, hybrid tail covariance risk, and negative semibeta. Additionally, when evaluated together, asymmetric risk measures generate mildly significant p-values of approximately 7% in this setting. These results show that some measures are related to the betas with respect to common factors.

The present analysis is related to several strands of the literature. The first deals with the emergence of the so-called factor zoo–many factors that are supposed to price the cross-section of stock returns. However, there is no clear consensus on what researchers should think about this claim. Some results suggest that a substantial fraction of the factors is a proxy for underlying common risks, and by including them, we can average out the noise associated with each factor and identify the driving force behind the formation of expected returns (Kozak et al. 2020).

Another ongoing discussion in empirical asset pricing regards characteristics vs. covariances. A risk-based explanation of expected returns claims that only exposures to common movements should constitute price determinants for the cross-section of asset returns. If a characteristic predicts future returns, it should be because this characteristic is a good proxy of systematic risk exposure. Similarly, as in the factor zoo discussion, there is still no obvious conclusion. Some results claim that we can form an arbitrage portfolio that enjoys abnormal returns without exposure to systematic risk (Kim et al. 2020; Lopez-Lira and Roussanov 2020), while others suggest that exposures capture all the essential pricing information (Kelly et al. 2019; 2023). Moreover, those exposures to the common fluctuations should be fully described by the betas, which are based on a simple covariance measure of dependence.

Much of the progress in recent years has been made in both strands of the literature, separately and simultaneously. Unfortunately, these research efforts tend to focus only on accounting variables and simple market friction characteristics, neglecting various measures of nonlinear systematic dependence between stocks and common factors. I relate to these studies by investigating a number of systematic asymmetric risk measures in a multivariate setting in the factor context.

Related studies have tended to shy away from this type of risk, probably due to the relatively greater difficulty in estimating them compared to conventional accounting variables. Nonetheless, investigating these risks is compelling in terms of revealing the factor structure of asset returns since they hold a distinct position among characteristics. In particular, they represent the joint behavior of stock returns and a general measure of risk that cannot be captured by the standard covariances with tradable factors. Due to their relationship to conventional measures based on covariance, it is challenging to determine the portion of the risk premium connected to the non-linear dependence versus the overall linear dependence for the asymmetric risk measures.

In response, I create arbitrage portfolios that are neutral with respect to the factors associated with these measures. I use asymmetric risk measures as proxies for exposure to common linear factors. I construct portfolios that exploit the premiums associated with their non-systematic components. My findings show significant efficiency and performance of the resulting portfolios using this method. Furthermore, I assess the added value of these measures for explaining the exposures to the common factors, controlling for conventional characteristics traditionally utilized in related studies. I show that some measures are suitable proxies for the exposures to the common factors. So, are the asymmetric measures of risk alphas or betas? I show that they can act as both.

4.1.1 Theoretical Motivation

The empirical research, centered around the expected utility assumption, focuses on the implementation of the equation

$$\mathbb{E}_t[m_{t+1} r_{i,t+1}] = 0, \tag{4.1}$$

which can be interpreted in terms of (co)variances as

$$\mathbb{E}_t[r_{i,t+1}] = \underbrace{\frac{\mathbb{C}ov_t(m_{t+1}, r_{i,t+1})}{\mathbb{V}ar_t(m_{t+1})}}_{\beta^m_{i,t}} \underbrace{\left(-\frac{\mathbb{V}ar_t(m_{t+1})}{\mathbb{E}_t[m_{t+1}]}\right)}_{\lambda_t}. \tag{4.2}$$

This statement implies that the priced exposure to the risk is adequately measured by the regression coefficient, $\beta^m_{i,t}$, obtained from regressing excess stock return on the stochastic discount factor, m_{t+1}. Further, if we assume linearity of the discount factor in some set of factors f, which proxy for the growth of marginal substitution, i.e., $m_{t+1} = \delta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}$, this leads to

$$\mathbb{E}_t[r_{i,t+1}] = \alpha_{i,t} + \lambda' \beta_{i,t} \tag{4.3}$$

$$r_{i,t+1} = \alpha_{i,t} + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{4.4}$$

where $\beta_{i,t}$ are the multiple regression coefficients of $r_{i,t}$ on f_t, and λ is vector of risk prices associated with factors f. In the case of tradable factors, λ is equal to the expected value of f. This line of reasoning constitutes a base for the empirical factor literature such as the arbitrage pricing theory of Ross (1976), the three-factor model of Fama and French (1993), etc. One of the main implications of the theory is that the non-systematic part of the risk, $\alpha_{i,t}$, should be equal to zero. Statistical tests such as Gibbons et al. (1989) provide inference on goodness of fit by testing this restriction.

On the other hand, there are models that deviate from the expected utility framework and/or linearity assumption of the stochastic discount factor. Examples of the former are models that introduce some form of behavioral bias, such as the disappointment aversion utility of Gul (1991). Based on that framework, Ang et al. (2006) introduced a cross-sectional relation between expected returns and downside beta, dependence between market and stock return conditional on the market being below its mean. A pioneer of the later violation is the work of Harvey and Siddique (2000), which assumes that the stochastic discount factor is *quadratic* in the market return, which introduces conditional systematic skewness as a priced risk characteristic. More recently, based on the recursive utility with disappointment aversion of Routledge and Zin (2010), Farago and Tédongap (2018) argue that betas with various asymmetric specifications of market return and volatility should be significantly priced in the cross-section.

Based on those arguments, risk exposure cannot be sufficiently captured by the simple betas with tradable factors. The cross-sectional relation between stock returns and risk changes to

$$\mathbb{E}_t[r_{i,t+1}] = \delta' g(r_{i,t+1}, f^*_{t+1}) + \lambda' \beta_{i,t} \tag{4.5}$$

$$r_{i,t+1} = \delta' g(r_{i,t+1}, f^*_{t+1}) + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{4.6}$$

where g is a function of asset return and some factor–*asymmetric risk measure* (ARM) where δ is a vector of related prices of risk. We can see that this specification leads to the rejection of the non-significant alpha assumption from above.

The uniqueness of an ARM can lie either in the choice of the dependence function g or in the choice of the factor f^*. In this study, I utilize two types of asymmetric risk measures. The first one captures systematic exposure using an asymmetric non-linear type of dependence with some conventional factor, such as the market return. These measures are typically related to the theoretical deviation from the expected utility theory. An example of this type of measure is the aforementioned downside beta of Ang et al. (2006) that measures covariance between market and stock return conditional on the market performing poorly.

The second one is defined by utilizing an asymmetric non-linear type of aggregate risk factor. This type is usually related to the violation of the linearity assumption regarding the stochastic discount factor. An example of such a factor would be the common time-varying component of return tails in the case of tail risk beta of Kelly and Jiang (2014).

In recent years, researchers have proposed many asymmetric risk measures to possess the ability to explain and predict stock returns. However, their ability to complement each other when implementing an investment strategy has yet to be researched. Related to that, there has yet to be an effort to investigate whether there is some small number of latent factors that would explain the abnormal returns related to these measures. Studies usually control for some pre-specified set of factors and conclude that abnormal returns cannot be explained by exposure to those factors. Because the choice of the factors will always be somewhat arbitrary, I will entertain the question of whether there is any set of factors that can eliminate significant alphas related to asymmetric risk measures. I investigate these questions using a representative set of eleven asymmetric risk measures in their multivariate setting.

The rest of the paper is structured as follows. Section 4.2 introduces data and asymmetric risk measures that I use in the further analysis. Section 4.3 investigates the arbitrage returns related to the asymmetric risk measures. Section 4.4 discusses the factor structure that the IPCA model yields. Section 4.5 verifies the results regarding the asymmetric risk measures and arbitrage returns. Section 4.6 entertains the possibility that the compensation for bearing asymmetric risk is time-varying. Section 4.7 inspects relation between the arbitrage returns and the momentum factor and characteristic. And finally, Section 4.8 concludes the whole investigation.

4.2 Asymmetric Risk Measures

In this section, I provide a first look at the asymmetric risk measures that are employed in the main analysis. I show they possess a sizable variation of the significance of the related anomaly premiums based on the research setting in which I estimate them. This observation supports the intention to evaluate the asymmetric risk measures jointly to extract the important component for the asset prices.

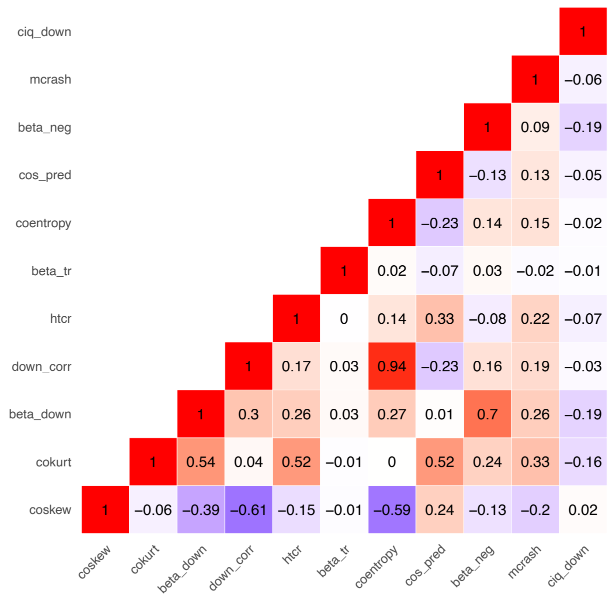
4.2.1 Data

In the empirical investigation, I employ a representative set of eleven asymmetric risk measures. Those measures are coskewness (`coskew`) of Harvey and Siddique (2000), cokurtosis (`cokurt`) of Dittmar (2002), downside beta (`beta_down`) of Ang et al. (2006), downside correlation (`down_corr`) based on Hong et al. (2006) and Jiang et al. (2018), hybrid tail covariance risk (`htcr`) of Bali et al. (2014), tail risk beta (`beta_tr`) of Kelly and Jiang (2014), exceedance coentropy measure (`coentropy`) based on Backus et al. (2018) and Jiang et al. (2018), predicted systematic coskewness (`cos_pred`) of Langlois (2020), negative semibeta (`beta_neg`) of Bollerslev et al. (2021), multivariate crash risk (`mcrash`) of Chabi-Yo et al. (2022), and downside common idiosyncratic quantile risk (CIQ) beta (`ciq_down`) of Barunik and Nevrla (2022). The choice of the variables corresponds to the fact that they capture different aspects of the return dependence in terms of non-linearity and asymmetry. I provide an overview of how the measures are estimated in Appendix 4.A. I estimate those measures using either daily or monthly return data from the CRSP database that starts in January 1963 and ends in December 2018.

In the further analysis, I also use a set of 32 characteristics from Freyberger et al. (2020), which is an intersection of data used by Freyberger et al. (2020) and Kelly et al. (2019). These characteristics are employed to estimate the baseline specification of the model of Kelly et al. (2019). I merge the dataset of ARMs with the characteristics dataset and include only observations that possess information about all the characteristics. Therefore, I work with a stock universe that is fully transparent for investors and eligible for trading based on a wide variety of strategies. The full merged dataset contains 1,519,754 stock-month observations of 12,505 unique stocks. To show the variability of the risk premiums significance related to the ARMs, I also employ a dataset that strips down penny stocks, which I define as stocks with a price less than \$5 or capitalization below 10% quantile of the NYSE-traded stocks each month. The dataset that excludes penny stocks yields 947,897 stock-month observations of 8,477 unique stocks.

I use an initial window of 5 years to estimate the ARMs; because of that, the first prediction period constitutes January 1968 in the case of in-sample

Figure 4.1: *Correlation structure across ARMs.*



Note: The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures. Data include the period between January 1968 and December 2018.

analysis. When performing out-of-sample exercises, I set the initial estimation period to be 60 months, so the out-of-sample prediction starts in January 1973.

4.2.2 Correlation Structure

First, to gain some intuition regarding the common variation of the ARMs, I investigate their correlation structure. Figure 4.1 contains correlations between ARMs themselves. Correlations are obtained as time-series averages of the cross-sectional correlations. We can see that the highest absolute values of correlations are between coentropy and downside correlation with a value of 0.94, downside beta and negative semibeta with a value of 0.70, and coskewness and downside correlation with a value of -0.61. The rest of the correlations vary quite a lot, with some being close to zero and some relatively high. Figure 6 shows that these correlations are relatively stable during distinct time periods and during non-recession and recession periods as defined by NBER.

The first column of Table 4.1 summarizes how each measure is generally related to the others by reporting average absolute correlations across all measures. We observe that the downside beta possesses the highest level of similarity with other measures, with the average absolute correlation equal to 0.29. On

Table 4.1: *Average correlations of ARMs.*

	Variables		*Managed portfolios*	
	with ARMs	with others	with ARMs	with others
`coskew`	0.24	0.02	0.32	0.16
`cokurt`	0.24	0.11	0.30	0.35
`beta_down`	0.29	0.08	0.40	0.36
`down_corr`	0.27	0.02	0.39	0.22
`htcr`	0.19	0.11	0.29	0.48
`beta_tr`	0.02	0.02	0.07	0.08
`coentropy`	0.25	0.02	0.39	0.24
`cos_pred`	0.20	0.12	0.39	0.43
`beta_neg`	0.19	0.13	0.36	0.47
`mcrash`	0.16	0.05	0.32	0.25
`ciq_down`	0.08	0.04	0.21	0.18

Note: Panel A of the table reports time-series averages of cross-sectional correlations for each ARM averaged across all other ARMs or 32 characteristics employed in Kelly et al. (2019). Panel B reports average correlations between managed portfolios. The average correlation for each ARM is obtained by averaging correlations across all other ARM portfolios or 32 characteristic managed portfolios. Data cover the period between January 1968 and December 2018.

the other hand, the least correlated measure is tail risk beta, with an average value of only 0.02.

The findings reveal potential variables associated with the common variation seen in ARMs. Conversely, some variables remain independent. In general, higher average correlations indicate ARMs that rely on non-linear measures of dependence with the market factor, like downside beta or downside correlation. The measures that capture non-linear factors unrelated to the market factor, specifically tail risk beta or downside CIQ beta, display lower correlations with the other measures and thus are expected to offer more pricing information when accounting for exposure to common factors.

4.2.3 Fama-MacBeth Regressions

Next, I present the first results on how ARMs align with the cross-section of asset returns. To do that, I run Fama and MacBeth (1973) cross-sectional regressions and report the results in Table 4.2 in Panel A. I report both univariate estimates and estimates obtained by controlling for four characteristics widely employed in the literature: market beta, size, book-to-market, and momentum. Below the estimated coefficients, I include t-statistics based on the Newey-West robust standard errors using the procedure of Newey and West (1994) to select the number of lags.

From the univariate results, it is evident that the cross-sectional pricing implications of ARMs vary considerably in their significance. Looking at the all-stock results, the highest significance possesses the downside CIQ beta with

Table 4.2: *Fama-MacBeth regressions.*

	Panel A: All stocks						*Panel B: No penny stocks*					
		multivariate						multivariate				
	ARM	ARM	β	Size	BM	MOM	ARM	ARM	β	Size	BM	MOM
coskew	-0.57	-0.39	-0.13	-0.15	0.22	0.49	-0.62	-0.36	-0.26	-0.12	0.12	0.53
	(-2.17)	(-1.62)	(-0.75)	(-1.72)	(3.23)	(3.23)	(-2.21)	(-1.56)	(-1.41)	(-1.63)	(1.41)	(3.41)
cokurt	-0.21	-0.12	-0.10	-0.14	0.21	0.51	-0.08	0.04	-0.27	-0.18	0.13	0.53
	(-3.15)	(-1.28)	(-0.47)	(-1.95)	(3.39)	(3.51)	(-1.24)	(0.60)	(-1.39)	(-2.58)	(1.45)	(3.35)
beta_down	-0.12	-0.14	-0.02	-0.15	0.21	0.50	-0.07	-0.05	-0.20	-0.12	0.12	0.53
	(-1.29)	(-2.43)	(-0.14)	(-1.58)	(3.08)	(3.26)	(-0.52)	(-0.63)	(-1.24)	(-1.55)	(1.36)	(3.43)
down_corr	0.18	-0.03	-0.13	-0.16	0.22	0.50	0.35	0.08	-0.25	-0.12	0.13	0.52
	(1.47)	(-0.32)	(-0.76)	(-1.66)	(3.20)	(3.19)	(2.38)	(0.83)	(-1.40)	(-1.57)	(1.51)	(3.31)
htcr	34.30	-1.55	-0.13	-0.16	0.19	0.53	201.36	140.20	-0.24	-0.17	0.12	0.51
	(0.76)	(-0.05)	(-0.75)	(-1.91)	(3.00)	(3.84)	(4.57)	(4.28)	(-1.35)	(-2.37)	(1.40)	(3.29)
beta_tr	0.16	0.15	-0.13	-0.14	0.21	0.51	0.28	0.25	-0.24	-0.11	0.12	0.51
	(1.89)	(2.10)	(-0.78)	(-1.52)	(3.06)	(3.31)	(2.77)	(3.52)	(-1.37)	(-1.52)	(1.38)	(3.29)
coentropy	0.13	-0.08	-0.13	-0.16	0.22	0.50	0.35	0.03	-0.25	-0.12	0.13	0.53
	(0.82)	(-0.64)	(-0.75)	(-1.72)	(3.21)	(3.22)	(1.76)	(0.21)	(-1.39)	(-1.64)	(1.52)	(3.36)
cos_pred	-3.05	-0.20	-0.15	-0.18	0.21	0.49	-1.97	1.29	-0.29	-0.19	0.14	0.56
	(-1.78)	(-0.11)	(-0.88)	(-2.69)	(3.33)	(3.17)	(-1.16)	(0.91)	(-1.68)	(-3.00)	(1.64)	(3.52)
beta_neg	-0.12	0.30	-0.26	-0.14	0.20	0.51	-0.53	-0.45	-0.06	-0.13	0.11	0.54
	(-0.29)	(0.78)	(-2.12)	(-1.65)	(2.92)	(3.48)	(-1.33)	(-1.39)	(-0.42)	(-1.81)	(1.25)	(3.51)
mcrash	0.24	0.29	-0.14	-0.17	0.23	0.49	1.55	1.19	-0.26	-0.14	0.13	0.52
	(0.29)	(0.50)	(-0.80)	(-1.78)	(3.34)	(3.18)	(1.85)	(2.04)	(-1.45)	(-1.78)	(1.54)	(3.30)
ciq_down	0.09	0.05	-0.12	-0.15	0.21	0.49	0.09	0.04	-0.25	-0.12	0.13	0.52
	(2.69)	(2.05)	(-0.72)	(-1.64)	(3.17)	(3.14)	(2.24)	(1.58)	(-1.43)	(-1.62)	(1.50)	(3.31)

Note: The table reports the risk premiums of the ARMs estimated using Fama-MacBeth regressions. Below the coefficients, I include their HAC t-statistics based on Newey and West (1987) using lag auto-selection of Newey and West (1994). I report results from univariate regressions and multivariate regressions while controlling for four characteristics from Carhart (1997). Panel A reports results using all stocks, Panel B excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

t-statistics of 2.69. Cokurtosis yields t-statistics of -3.15. Unfortunately, the sign of the coefficient is counterintuitive. Coskewness is, on the other side, significant with an expected sign. Tail risk beta is borderline significant with a t-stat of 1.89. The rest of the variables are deemed insignificant in the presented setting. When we move to the controlled setting, most variables become slightly less significant with few exceptions, such as tail risk beta, which becomes significant (t-stat=2.10), or downside beta, which becomes also significant, but with a negative sign.

Panel B of Table 4.2 reports the results using the dataset that excludes penny stocks. Generally, coefficients become more significant (or less significant if they possess a counterintuitive sign in the all-stock sample). For example, hybrid tail covariance risk (t-stat=4.57) or downside correlation (t-stat=2.38) become highly significant. Some variables become even more significant when controlling for other risk measures, such as multivariate crash risk (t-stat=2.04) or tail risk beta (t-stat=3.52).

4.2.4 Portfolio Sorts

Next, to briefly inspect the tradability of the ARMs, I perform simple univariate portfolio sorts. I focus here on a portfolio formation based on the following scheme

$$x_{t+1} = \frac{Z_t' r_{t+1}}{N_{t+1}} \tag{4.7}$$

where Z_t is a vector of an ARM observed at time t, r_{t+1} represents a vector of excess returns of the stocks in the next period, and N_{t+1} denotes the number of stock observations in a given month. I will refer to this type of portfolio as a *managed portfolio* with a corresponding return x_{t+1}. The managed portfolio's return is derived as a weighted average of stock returns, using the values of the ARM as weights, and normalized by the number of stock observations.

To calculate the weights for a given ARM, every month, I cross-sectionally rank their values, divide them by the number of observations in the month, and subtract 0.5. This procedure transforms the ARM into the interval $[-0.5, 0.5]$. By doing so, I eliminate the effect of outliers and the resulting return can be interpreted as a zero-cost portfolio return associated with the ARM.

Table 4.3 summarizes the annualized returns of these managed portfolios. In the case of all stocks, the highest absolute Sharpe ratio possesses the downside CIQ beta with a value of 0.42. In the case of non-penny stocks, the highest Sharpe ratio attains hybrid tail covariance risk with the same value of 0.42. As hinted from the Fama-MacBeth regressions, some variables possess a counterintuitive negative premium, e.g., cokurtosis yields a significantly negative risk premium in the universe of all stocks. Another notable example is downside beta, which attains negative risk premiums in both samples, but the associated average returns are not significantly different from zero.

Table 4.3 also reports annualized 6-factor alphas and their t-statistics with respect to six commonly used risk factors. As a general benchmark of risk, I employ four factors of Carhart (1997): market, size, value, and momentum. To control for the effect of the common volatility, which may be a driver of many tail events, I use the common idiosyncratic volatility (CIV) shocks of Herskovic et al. (2016). The betting-against-beta (BAB) factor of Frazzini and Pedersen (2014) aims at controlling the effect of the well-known beta mispricing anomaly. When I control for the exposures to those six factors, the significance of some of the ARM premiums deteriorates, such as in the case of tail risk beta in both samples. On the other hand, some of the premiums do not suffer any decrease in significance if we control for the exposure to these common factors. For example, controlled risk premiums associated with the downside CIQ betas deliver significant t-stats of 3.58 and 3.24 in the all-stock and no-penny datasets,

Table 4.3: *Managed portfolio returns.*

	Panel A: All stocks					*Panel B: No penny stocks*				
	Mean	t-stat	SR	α	t-stat	Mean	t-stat	SR	α	t-stat
`coskew`	-0.30	-2.51	-0.32	-0.23	-1.52	-0.28	-2.34	-0.30	-0.10	-0.66
`cokurt`	-0.39	-2.29	-0.29	-0.10	-0.57	-0.07	-0.48	-0.06	0.23	1.73
`beta_down`	-0.27	-1.27	-0.16	0.09	0.63	-0.13	-0.53	-0.07	0.11	0.84
`down_corr`	0.15	1.80	0.22	0.09	0.84	0.24	2.64	0.33	0.04	0.39
`htcr`	0.00	0.01	0.00	-0.14	-0.66	0.37	2.86	0.42	0.32	2.63
`beta_tr`	0.32	2.28	0.32	0.31	1.44	0.35	2.56	0.36	0.18	1.12
`coentropy`	0.11	1.37	0.16	0.07	0.61	0.18	2.03	0.25	-0.01	-0.08
`cos_pred`	-0.46	-1.76	-0.26	-0.50	-1.85	-0.22	-0.97	-0.14	-0.11	-0.56
`beta_neg`	-0.13	-0.38	-0.05	0.34	1.83	-0.35	-1.18	-0.16	-0.03	-0.26
`mcrash`	0.03	0.36	0.05	0.06	0.63	0.16	1.74	0.25	0.14	1.52
`ciq_down`	0.41	2.83	0.42	0.52	3.58	0.36	2.29	0.34	0.44	3.24

Note: The table contains annualized out-of-sample returns of the managed portfolios sorted on various asymmetric risk measures. It reports corresponding t-statistics, Sharpe ratio (SR), and annualized 6-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). I use the HAC t-statistics of Newey and West (1987) with six lags. Panel A reports results using all stocks. Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

respectively.

In Appendix 4.B, I employ a more conventional approach to the portfolio sorts. Tables 26 and 27 summarize portfolio returns from sorting the stocks into five and ten portfolios, respectively, with monthly rebalancing. Tables contain results using equal- and value-weighted schemes for both data samples. In the case of all stocks, the highest risk premium carries predicted coskewness using both equal- and value-weighted returns and sorting into either quintile or decile portfolios, although with varying significance levels.

These results show that there is a sizable variation in the magnitude and significance of the risk premiums associated with the ARMs. Those variations can be caused by selecting the weighting scheme, universe of stocks, number of portfolios, research design, or their combinations. Surprisingly, most of the ARMs perform better in the no-penny dataset. This may be caused by the fact that the penny stocks are less liquid and thus the ARMs are measured with higher noise. Consequently, the less precise measurement of the ARMs may obscure the true impact that they have on the asset prices. This effect is especially evident in the cases of downside correlation, hybrid tail covariance risk, and multivariate crash risk.

Moreover, common factors can explain some of the premiums related to the ARMs. Therefore, an effort to combine the ARMs to extract the important information for the expected returns makes sense. In addition, the resulting portfolio should aim at minimizing the exposure to the common factors to yield significant risk-adjusted returns.

Figure 7 in Appendix 4.B depicts the time-series correlations between managed portfolios sorted on ARMs. Similarly as in the case of values of ARMs, these correlations are quite stable across times and recession and non-recession periods as reported in Figure 8. This result supports the value of combining the asymmetric risk measures into an investment strategy, as correlations of these strategies do not peak during the bad times. Moreover, Table 4.1 also contains averages of those correlations for each ARM. Correlations are noticeably higher than in the case of the values of the characteristics, which we might expect. The most correlated with other ARMs is downside beta, closely followed by downside correlation, coentropy, and predicted coskewness. There is clearly some common structure, but the question remains whether the exposures to that structure represent priced determinants of risk. In addition, there are also ARMs that capture unrelated residual risk.

4.2.5 Naive Combination Approach

In this section, I investigate whether combining information from all ARMs using an approach based on multivariate regression can produce a portfolio that outperforms those arranged individually for each ARM. I form the portfolios based on the multivariate Fama-MacBeth regressions in the spirit of Lewellen (2015). Using the set of 11 ARMs, I estimate expanding- and moving-window regressions where on the left-hand side are stock returns at time $t + 1$ and on the right-hand side are the ARMs at time t. I use an out-of-sample setting with a 60-month initial or moving period.[2] I estimate the model up to time T and use the model to predict the return at time $T + 1$. I use the predicted values of the out-of-sample return to construct the portfolio and observe its realized return. Then, I expand the estimation window and repeat the procedure until the sample is exhausted. I use either the managed portfolio approach or the difference between high and low portfolios based on quintile or decile sorts. I weight the difference portfolios using an equal- or value-weighted scheme. These portfolios are referred to as *regression portfolios* in the text.

Table 4.4 summarizes the results. I use three approaches to estimate the Fama-MacBeth multivariate regressions. I utilize OLS estimation as the simplest benchmark and report the results in Panel A. To deal with potential problems related to the OLS estimator, such as overfitting in presence of corre-

[2] The results are not qualitatively altered by changing the size of the window. Extending its length only slightly improves the results.

Table 4.4: *Regression portfolio returns.*

Window	Sorting	Weighting	Mean	t-stat	SR	α	t-stat	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
Panel A: OLS estimation												
Expanding	Managed		0.44	2.02	0.31	0.33	1.35	-0.59	12.87	66.28	-39.35	44.30
	Quintile	Equal	4.18	1.93	0.30	3.20	1.31	-0.61	12.35	66.88	-38.81	44.01
		Value	4.65	1.77	0.27	3.41	1.09	-0.81	11.37	65.68	-38.56	41.61
	Decile	Equal	2.66	1.12	0.17	1.02	0.41	0.16	7.45	68.59	-28.74	41.83
		Value	4.54	1.65	0.25	1.91	0.64	0.20	6.37	58.47	-25.81	42.59
Moving	Managed		0.51	1.88	0.28	0.16	0.59	-0.38	9.35	66.11	-31.16	42.61
	Quintile	Equal	4.87	1.78	0.27	1.31	0.48	-0.48	9.01	66.57	-33.13	41.36
		Value	4.70	1.41	0.22	-0.12	-0.03	-0.60	8.28	66.20	-32.49	39.58
	Decile	Equal	4.63	1.62	0.23	1.78	0.65	0.49	10.84	74.02	-24.74	49.35
		Value	5.61	1.61	0.24	1.89	0.55	0.09	6.99	77.69	-26.95	43.38
Panel B: Ridge estimation												
Expanding	Managed		0.43	1.98	0.31	0.32	1.29	-0.56	12.41	65.28	-38.82	44.09
	Quintile	Equal	4.13	1.91	0.30	3.04	1.24	-0.58	11.71	65.91	-38.04	43.67
		Value	4.38	1.66	0.25	3.03	0.97	-0.84	11.37	67.02	-39.56	41.39
	Decile	Equal	2.77	1.19	0.18	0.96	0.39	0.25	7.43	67.95	-28.72	42.40
		Value	4.42	1.56	0.24	1.79	0.59	0.18	6.74	57.64	-26.87	42.85
Moving	Managed		0.51	1.84	0.27	0.15	0.54	-0.33	8.95	66.25	-30.64	42.26
	Quintile	Equal	4.60	1.67	0.25	0.91	0.33	-0.38	8.77	69.00	-32.51	41.73
		Value	5.08	1.51	0.23	0.26	0.08	-0.58	7.53	66.33	-31.44	38.30
	Decile	Equal	3.75	1.28	0.19	0.76	0.27	0.45	10.27	76.75	-24.67	48.55
		Value	7.29	2.09	0.31	3.86	1.06	0.21	6.62	71.25	-25.03	42.95
Panel C: Lasso estimation												
Expanding	Managed		0.43	1.98	0.31	0.32	1.30	-0.55	12.44	65.53	-38.31	44.25
	Quintile	Equal	4.02	1.85	0.29	3.04	1.25	-0.55	11.84	66.89	-37.48	43.82
		Value	4.45	1.69	0.26	3.13	1.00	-0.79	11.30	65.21	-37.86	41.94
	Decile	Equal	3.08	1.32	0.20	1.47	0.60	0.24	7.58	68.34	-29.14	42.67
		Value	4.87	1.72	0.26	2.39	0.78	0.22	6.65	59.41	-27.30	42.71
Moving	Managed		0.50	1.77	0.27	0.14	0.49	-0.34	8.85	66.08	-30.71	42.10
	Quintile	Equal	4.48	1.59	0.25	0.78	0.28	-0.45	8.42	68.73	-33.09	40.58
		Value	4.68	1.36	0.21	-0.10	-0.03	-0.57	7.91	67.36	-31.77	39.23
	Decile	Equal	3.96	1.37	0.20	1.23	0.43	0.47	9.98	78.57	-24.95	48.54
		Value	6.86	1.89	0.29	3.25	0.90	0.17	6.25	67.60	-25.18	41.99

Note: The table contains out-of-sample results for the regression portfolios estimated using Fama-MacBeth regressions and various weighting schemes. Predicted returns are estimated using either OLS, Ridge or Lasso regression. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with 6 lags, Sharpe ratio (SR), alpha and its t-statistic with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding (moving) window estimation with a 60-month initial (moving) period. Data cover the period between January 1968 and December 2018.

lated variables, I also estimate the models with ridge and lasso regressions and report the results in Panel B and C, respectively.[3] Notably, we observe that the returns of these portfolios are only somewhat significant. The above observation is further confirmed by the insignificant t-statistics with respect to the six-factor model previously applied in single-sorted portfolios. Additionally, returns exhibit leptokurtic behavior with slightly negative skewness in most instances. The last three columns employ rescaled returns so that the unconditional yearly volatility is 20%, and report maximum drawdown and worst- and best-month returns.

These results show that the simple portfolio formation based on multivariate regression cannot efficiently combine the information from the ARMs to yield

[3] I set the tuning parameters based on the best fit obtained from the three-fold cross-validation using the data up to time T. Extending the number of folds does not have any significant effect on the results.

abnormal returns beyond premiums associated with single sorts. In addition, the regression portfolios are highly exposed to the common factors and thus do not yield any significant risk-adjusted premium. High correlations between some ARMs may cause high estimation errors, which may be more attenuated in the out-of-sample setting with shorter estimation periods. The fact that the moving-window estimation approach yields lower significance of the results further supports this claim.

4.3 Combining Asymmetric Risk Measures

In this section, I present an approach to portfolio construction that enjoys the abnormal returns associated with the ARMs without being exposed to common sources of risk. I estimate a latent factor model that utilizes the ARMs to account for the maximal possible explanation of the factor loadings to the common factors. Then, I form a portfolio that is factor neutral and show that it still possess a significant risk premium not explained by any other factor model.

4.3.1 IPCA Model

To exploit the risk premium associated with the ARMs, I use the instrumented principal component analysis (IPCA) model of Kelly et al. (2019; 2020), which can be written as

$$\begin{aligned} r_{i,t+1} &= \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}, \\ \alpha_{i,t} &= z'_{i,t}\Gamma_\alpha + \nu_{\alpha,i,t},\ \beta_{i,t} = z'_{i,t}\Gamma_\beta + \nu_{\beta,i,t} \end{aligned} \tag{4.8}$$

where $r_{i,t+1}$ is an excess return and $\beta_{i,t}$ contains dynamic loadings on $(K \times 1)$ vector of latent factors f_{t+1}. The vector of factor loadings may depend on the instrument $(L \times 1)$ vector $z_{i,t}$ of observable asset characteristics (which includes a constant) through the matrix Γ_β. I use the set of eleven ARMs as the characteristics that may proxy for the exposure to the common factors (hence ARM-IPCA).[4] Mapping between characteristics and factor loadings serves two purposes. First, it enables the exploitation of other information than just simply return data for the estimation of latent factor loadings and thus makes

[4] Same as in the case of managed portfolios, I standardize the variables to have zero mean and range between -0.5 and 0.5.

the estimation more efficient. Second, it naturally makes the loadings time-varying as they are a function of the characteristics and thus makes it valuable tool for estimation of conditional risk premium. Moreover, the model admits the possibility that the characteristics align with the returns in addition to their relation to systematic risk through the $(L \times 1)$ vector of coefficients Γ_α that maps the characteristics into their anomaly intercepts.

This feature can be used to investigate how well the ARMs proxy for the exposure to the systematic risk and to test whether they contain some important information beyond that and yield some anomaly (mispricing) returns. To do that, I can examine features of the Γ_α estimate and test the null hypothesis that the ARMs do not proxy for the anomaly alpha. Throughout the text, I use the fit of two specifications of the Model 4.8. First, the *restricted* model is estimated by setting the Γ_α vector to zero. Second, the *unrestricted* model is obtained by allowing expected returns to align with the ARMs beyond their relation with the systematic risk exposure, and thus Γ_α is estimated freely.

To construct the portfolio that combines the information from all ARMs and exploits their abnormal returns, I use the estimates of the unrestricted model. I estimate the unrestricted model and form corresponding arbitrage portfolio with weights set equal to

$$w_{t-1} = Z_{t-1}(Z'_{t-1}Z_{t-1})^{-1}\Gamma_\alpha, \tag{4.9}$$

which yields conditional factor neutrality. This portfolio efficiently combines assets in proportion to their conditional expected returns beyond the exposure to the common factors. I denote this portfolio as *pure-alpha* portfolio.

The proposed approach is particularly suitable for combining ARMs for various reasons. First, by using ARMs to approximate the exposures to common factors, I can extract the risk premium associated solely with the non-linear features related to the measures. Moreover, the algorithm minimizes the risk that other risk factors will span the resulting abnormal returns. This is especially critical for the market factor. From the previous literature, see, e.g., Hou et al. (2018), it is a well-documented fact that the exposure to the market factor is negatively priced across stock returns, even though it represents a counter-intuitive observation. It is reasonable to expect that the linear relation with the overall market will dilute some asymmetric risk measures. As the market return usually explains the most time-series variation of stock returns, IPCA considers this fact, and the effect of this puzzle is mitigated for the pure-alpha portfolios.

Second, this procedure also alleviates potential issues of multicollinearity among the ARMs. If some variables proxy for the exposure to the common factors, IPCA controls these associations when setting the weights for the pure-alpha portfolio by letting them to explain the systematic risk.

The performance of the pure-alpha portfolio provides a straightforward test for abnormal returns connected to ARMs beyond exposure to common factors. The pure-alpha portfolio offers investors a chance to avoid systematic risk associated with common linear factors and enjoy the premium related to the ARMs. A factor model that captures risk compensation appropriately should not offer such an opportunity. Naturally, the performance of the pure-alpha portfolio provides an alternate approach for merging information from ARMs, resulting in abnormal returns beyond single-variable sorts.

Following Kelly et al. (2019), estimation of the restricted model with $\Gamma_\alpha = 0$ is performed using *alternating least squares* and iterating between the first-order conditions for Γ_β and f_{t+1}

$$f_{t+1} = \left(\hat{\Gamma}'_\beta Z'_t Z_t \hat{\Gamma}_\beta\right)^{-1} \hat{\Gamma}'_\beta Z'_t r_{t+1}, \quad \forall t \tag{4.10}$$

and

$$\mathrm{vec}(\hat{\Gamma}'_\beta) = \left(\sum_{t=1}^{T-1} Z'_t Z_t \otimes \hat{f}_{t+1}\hat{f}'_{t+1}\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \hat{f}'_{t+1}\right]' r_{t+1}\right) \tag{4.11}$$

where r_{t+1} is the $N \times 1$ vector of stock returns and Z_t is the $N \times L$ matrix of stock characteristics. The identifying restrictions are that $\hat{\Gamma}'_\beta \hat{\Gamma}_\beta = \mathbb{I}_K$, the unconditional second moment matrix of f_t is diagonal with descending diagonal entries, and the mean of f_t is non-negative.[5] In the case of the unrestricted version of the model with $\Gamma_\alpha \neq 0$, the estimation proceeds similarly, the only difference is that we augment the vector of factors to include a constant.[6]

4.3.2 Pure-Alpha Portfolios

To combine the ARMs while hedging exposure to common factors, I form the pure-alpha portfolios and investigate their out-of-sample performance. The models are estimated using an expanding window. First, I estimate the ARM-IPCA model with the first 60 observations of the sample and use the estimate

[5]Those restrictions do not possess any economic implications for the model.

[6]I thank Seth Pruitt for making the code for the IPCA estimation publicly available.

Table 4.5: *Pure-alpha portfolio returns.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	14.36	4.73	0.72	0.09	3.59	41.40	-31.14	25.53
2	19.36	6.27	0.97	0.15	2.87	31.17	-25.47	27.23
3	16.78	5.35	0.84	-0.00	6.59	43.45	-39.64	25.67
4	8.20	3.04	0.41	-0.12	5.41	45.88	-40.07	24.14
5	8.06	2.86	0.40	0.34	3.69	38.36	-32.42	27.70
6	5.97	2.05	0.30	0.79	3.20	51.45	-17.98	27.29
7	1.07	0.34	0.05	0.48	2.25	73.12	-20.19	26.80
8	-2.53	-0.81	-0.13	-0.40	2.76	89.97	-31.47	21.61

Note: The table contains out-of-sample results for the pure-alpha portfolios estimated using the ARM-IPCA model ranging between one and eight latent factors. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with a 60-month initial period. Data cover the period between January 1968 and December 2018.

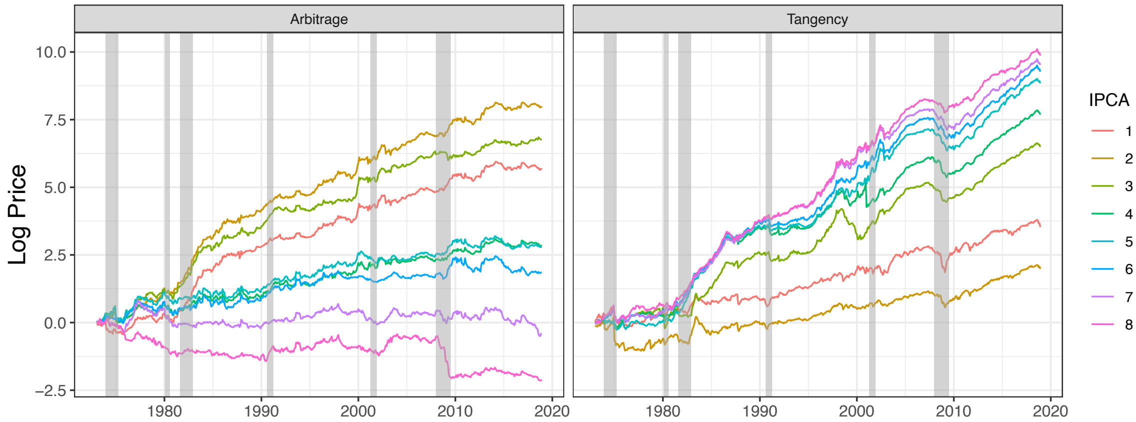
of $\hat{\Gamma}_\alpha$ to form the pure-alpha portfolio and record the out-of-sample return in the next period. Then, I expand the estimation period by one observation and predict the next. I repeat the procedure until the dataset is exhausted. The first out-of-sample prediction period corresponds to January 1973. For comparability reasons, I scale the portfolio returns to have an unconditional standard deviation of 20% p.a. over the whole sample, which does not affect the significance of the results. Later in the text, I also report results of a volatility-targeted weights, which yield the same qualitative and quantitative conclusions.

Table 4.5 summarizes the basic features of the pure-alpha portfolios for the ARM-IPCA model with one to eight common latent factors. Results show that portfolios estimated using one to five factors yield highly significant returns with the HAC t-statistic of Newey and West (1987) with 6 lags of up to 6.27, corresponding to the ARM-IPCA(2) specification. Sharpe ratio achieves a value of up to 0.97. I also report skewness and kurtosis of the pure-alpha portfolios. These values do not indicate any extreme behavior of the portfolios as the return distributions are close to symmetric and without signs of heavy tails.

In comparison to the results obtained using the regression portfolios based on Fama-MacBeth regression, returns of the pure-alpha portfolios exhibit features much closer to the normal distribution. Moreover, I include the maximum drawdowns that every portfolio yielded and their best and worst months. In Appendix 4.C in Table 28, I include summary results for the pure-alpha portfolios estimated separately in two disjoint sub-intervals. I show that the implications hold similarly over these periods.

To further assess the performance of the pure-alpha portfolios, the left panel

Figure 4.2: *Performance of the ARM-IPCA portfolios.*



Note: The figure shows out-of-sample performance results of the pure-alpha and tangency portfolios estimated using IPCA models with the ARMs as instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted ARM-IPCA model, and pure-alpha portfolios on the unrestricted model. Data cover the period between January 1973 and December 2018.

of Figure 4.2 captures the cumulative log return of those portfolios. We see that the pure-alpha portfolios based on up to five latent factors grow constantly over the whole period without a noticeable sign of slowing down. These results suggest that it is possible to strip the ARMs down from their exposures to the common linear factors and combine them into a highly profitable strategy. This strategy provides a Sharpe ratio more than twice as big as the best strategy based on a single-variable sort. Moreover, the features of the pure-alpha portfolios suggest that the resulting returns do not exhibit extreme behavior that may be expected due to the nature of the ARMs.

4.3.3 Risk-Adjusted Returns

Next, I investigate whether the arbitrage returns associated with the pure-alpha portfolios are not driven by exposures to other known factors. I regress returns of the pure-alpha portfolios on various sets of factors that were proven successful in capturing the risk premium. I report the annualized alphas and their HAC t-statistics of Newey and West (1987) with six lags. Table 4.6 reports risk-adjusted returns when controlling for the exposures to the three- and five-factor models of Fama and French (1993) and Fama and French (2015), while also using the specification of Carhart (1997) and combining it with the CIV shocks of Herskovic et al. (2016), and the BAB factor of Frazzini and Pedersen (2014). We can see that the returns of the pure-alpha portfolios are

Table 4.6: *Fama-French risk-adjusted returns of the pure-alpha portfolios.*

K factors	CAPM	FF3	FF3+MOM	FF3+MOM +CIV	FF3+MOM +CIV+BAB	FF5	FF5+MOM	FF5+MOM +CIV	FF5+MOM +CIV+BAB
1	14.31	13.58	9.12	9.16	6.27	12.38	8.77	8.79	6.87
	(4.75)	(4.54)	(2.71)	(2.74)	(1.85)	(3.69)	(2.53)	(2.54)	(2.00)
2	19.65	18.70	13.28	13.31	10.63	17.39	13.00	13.02	11.18
	(6.54)	(6.19)	(3.95)	(3.98)	(3.15)	(5.02)	(3.73)	(3.73)	(3.23)
3	17.04	16.88	11.49	11.50	10.03	15.97	11.59	11.60	10.34
	(5.68)	(5.41)	(4.23)	(4.22)	(3.46)	(4.65)	(4.06)	(4.03)	(3.51)
4	8.44	6.68	5.87	5.90	5.22	6.67	6.02	6.03	5.37
	(3.27)	(2.55)	(2.27)	(2.28)	(1.88)	(2.67)	(2.32)	(2.32)	(1.91)
5	7.89	6.57	5.59	5.60	5.61	7.41	6.54	6.55	6.14
	(2.94)	(2.32)	(1.94)	(1.94)	(1.94)	(2.76)	(2.33)	(2.33)	(2.13)
6	6.07	4.14	4.51	4.52	4.57	5.90	6.07	6.07	5.61
	(2.13)	(1.47)	(1.48)	(1.48)	(1.52)	(2.13)	(2.01)	(2.01)	(1.88)

Note: The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with six lags obtained by regressing the pure-alpha portfolio returns on various factor models and their combinations: Fama and French (1993), Carhart (1997), Fama and French (2015), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

Table 4.7: *Exposures of the ARM-IPCA pure-alpha portfolios.*

K	α	Mkt	SMB	HML	RMW	CMA	MOM	CIV	BAB
1	6.87	0.05	0.06	0.06	-0.21	-0.06	0.33	-0.02	0.48
	(2.00)	(0.69)	(0.38)	(0.39)	(-0.97)	(-0.30)	(2.73)	(-0.47)	(3.63)
2	11.18	0.02	0.08	0.10	-0.27	0.02	0.42	-0.02	0.46
	(3.23)	(0.26)	(0.53)	(0.57)	(-1.39)	(0.12)	(3.35)	(-0.62)	(3.88)
3	10.34	0.04	-0.03	-0.05	-0.42	0.27	0.44	0.01	0.31
	(3.51)	(0.57)	(-0.21)	(-0.24)	(-2.19)	(1.12)	(4.50)	(0.33)	(2.76)
4	5.37	0.04	-0.21	0.27	-0.27	0.16	0.06	-0.04	0.17
	(1.91)	(0.59)	(-1.12)	(1.13)	(-1.61)	(0.60)	(0.67)	(-1.14)	(1.37)
5	6.14	0.09	-0.23	0.26	-0.40	0.11	0.09	-0.01	0.10
	(2.13)	(1.24)	(-1.38)	(1.16)	(-2.75)	(0.48)	(0.88)	(-0.37)	(0.98)
6	5.61	-0.01	-0.05	0.35	-0.46	-0.07	-0.04	0.00	0.11
	(1.88)	(-0.17)	(-0.55)	(2.50)	(-3.20)	(-0.37)	(-0.45)	(-0.10)	(1.09)

Note: The table reports estimated coefficients and their t-statistics from regressing returns of the pure-alpha ARM-IPCA(K) portfolios on five factors of Fama and French (2015), augmented by momentum factor of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

not subsumed by those other specifications. However, it is evident that the momentum factor and betting-against-beta factor capture a non-trivial part of the returns of the pure-alpha portfolios.

Table 4.7 summarizes the exposures of the pure-alpha arbitrage portfolios to eight factors based on the five-factor model of Fama and French (2015), augmented by the momentum factor of Carhart (1997), CIV shocks of Herskovic et al. (2016), and the BAB factor of Frazzini and Pedersen (2014). The pure-alpha portfolios of the ARM-IPCA models possess significant exposures to the momentum and betting-against-beta factors. Although these exposures diminish the abnormal returns, the remaining risk premium remains significant.

Next, I control for the exposure to the q-factor models of Hou et al. (2014) and Hou et al. (2020), augmented by the momentum factor, CIV shocks, and

Table 4.8: *Q-model risk-adjusted returns of the pure-alpha portfolios.*

K factors	Q4	Q5	Q5+MOM	Q5+MOM +CIV	Q5+MOM +CIV+BAB
1	8.03	7.40	7.81	7.64	6.29
	(2.23)	(2.15)	(2.42)	(2.37)	(1.95)
2	12.22	11.05	11.58	11.41	10.16
	(3.27)	(3.13)	(3.60)	(3.51)	(3.17)
3	11.39	8.69	9.29	9.24	8.54
	(2.98)	(2.57)	(3.06)	(3.00)	(2.74)
4	5.98	6.00	6.04	5.91	5.37
	(2.01)	(1.93)	(1.96)	(1.89)	(1.69)
5	6.11	6.50	6.63	6.59	6.39
	(1.97)	(2.03)	(2.11)	(2.08)	(2.04)
6	6.33	6.81	6.83	6.81	6.40
	(2.16)	(2.16)	(2.18)	(2.18)	(2.08)

Note: The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with six lags obtained by regressing the pure-alpha portfolio returns on factor models of Hou et al. (2014) and Hou et al. (2020), augmented by momentum factor, CIV shocks, and BAB factor. Data cover the period between January 1973 and December 2018.

BAB factor.Table 4.8 summarizes the results. The abnormal returns of the pure-alpha portfolios cannot be erased by those combinations, either. Especially strong remain the abnormal returns for portfolios constructed from two- or three-factor specifications of the ARM-IPCA model.

Finally, I put the anomaly returns of the pure-alpha portfolios against their closest competitor. I investigate whether the out-of-sample IPCA factors estimated using the original set of 32 characteristics from Kelly et al. (2019) can explain the abnormal returns related to the pure-alpha portfolios from the ARM-IPCA model. The results of this analysis are in Table 4.9. We observe that returns of the pure-alpha portfolios of the ARM-IPCA models with one to five latent factors cannot be explained by the original IPCA factors. Using even five- or six-factor versions of the original IPCA model cannot span the highly significant performance of the pure-alpha portfolios.

4.3.4 Variable Importance

This section investigates which ARMs contribute the most to the performance of the pure-alpha portfolios. Table 4.10 reports estimates of the Γ_α vector from the out-of-sample procedure in the last prediction period. Because the ARM variables are standardized, their magnitudes are comparable. We can observe that the coefficients of some variables change considerably across the range of common latent factors. This fact is caused by using more ARMs as proxies for exposures to common factors as the number of latent factors goes up, and

Table 4.9: *IPCA risk-adjusted returns of the pure-alpha portfolios.*

K factors	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
1	14.12	14.60	12.95	8.60	10.62	12.07
	(4.77)	(4.99)	(2.71)	(2.01)	(2.34)	(2.69)
2	18.85	19.50	18.83	12.15	13.65	18.33
	(6.30)	(6.89)	(3.57)	(2.54)	(2.74)	(3.62)
3	16.40	16.95	21.09	16.29	16.42	18.87
	(5.26)	(6.07)	(4.12)	(3.21)	(3.03)	(3.25)
4	7.12	7.47	3.29	3.04	7.94	10.61
	(2.62)	(2.94)	(0.88)	(0.81)	(2.03)	(2.16)
5	7.25	7.40	4.44	3.82	7.96	11.83
	(2.58)	(2.76)	(1.24)	(1.05)	(2.12)	(2.58)
6	5.47	4.52	1.90	3.12	2.12	4.50
	(1.92)	(1.65)	(0.65)	(0.98)	(0.64)	(1.22)

Note: The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with six lags obtained by regressing the pure-alpha portfolio returns on out-of-sample IPCA factors with one to six latent factors and 32 characteristics from Kelly et al. (2019) as instruments. Data cover the period between January 1973 and December 2018.

Table 4.10: *Estimated coefficients of* Γ_α *vector.*

	K factors					
	1	2	3	4	5	6
`coskew`	-5.50	-4.11	0.08	-0.78	-0.54	-1.57
`cokurt`	2.61	1.72	2.70	1.88	2.03	2.55
`beta_down`	-9.10	-7.71	-1.82	-2.76	-3.32	-3.00
`down_corr`	2.62	1.84	0.18	1.04	0.42	-0.13
`htcr`	2.92	3.10	5.87	4.89	4.56	2.73
`beta_tr`	2.43	1.71	0.71	2.71	-1.19	0.81
`coentropy`	-4.18	-3.12	-1.72	-2.40	-1.66	-0.71
`cos_pred`	-4.23	-4.85	-4.92	-1.45	-0.36	-0.40
`beta_neg`	-2.53	-2.32	0.11	0.92	1.14	1.11
`mcrash`	0.95	0.95	1.52	1.34	1.77	1.14
`ciq_down`	3.82	3.30	2.98	3.52	1.21	-1.32

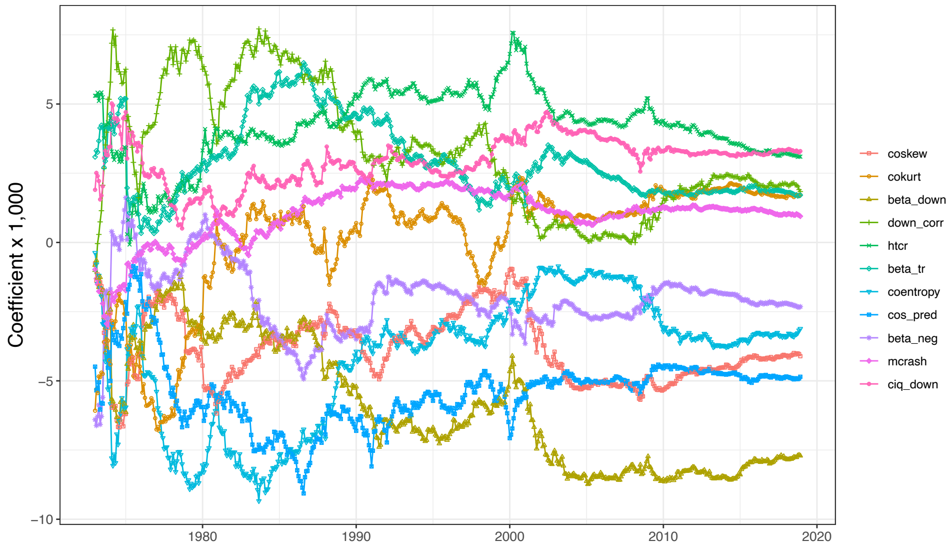
Note: The table summarizes the estimated coefficients of Γ_α vector of the ARM-IPCA model. This vector is used for the construction of the pure-alpha portfolios. Reported are coefficients estimated using the last prediction window before exhausting the entire dataset. Coefficients are multiplied by 1,000 for better readability. Data cover the period between January 1973 and December 2018.

potentially losing some predictive ability for anomaly returns of the pure-alpha portfolio.

Moreover, in Figure 4.3, I capture the estimates of Γ_α from the expanding window estimation of the ARM-IPCA(2) model. We can see that the coefficients are relatively stable across time, and the variables possess the same sign during most of the period.

Next, I assess the variable importance for the out-of-sample results based on setting the effect of a variable on the formation of the pure-alpha portfolio to zero. More specifically, I estimate the unrestricted IPCA model using all ARMs for a given number of latent factors. Then, when forming the arbitrage portfolio, I set the element of Γ_α corresponding to the investigated ARM to zero and record the out-of-sample return next period. I exhaust the entire dataset

Figure 4.3: Γ_α *estimates from the out-of-sample estimation.*



Note: The figure shows estimates of the Γ_α vector from the unrestricted ARM-IPCA(2) model using expanding window estimation and a 60-month initial period. Data cover the period between January 1973 and December 2018.

and compute the realized out-of-sample Sharpe ratio. I repeat this procedure for each ARM and range between one and six factors.[7]

Table 4.11 reports these effects. We can see three variables with highly negative omission impact across all six specifications of the pure-alpha portfolios: downside correlation, coentropy, and downside CIQ beta. These variables noticeably improve the performance of the pure-alpha portfolios. Hybrid tail covariance risk contributes positively to the first three specifications of the pure-alpha portfolios, which possess the highest Sharpe ratios among the specifications.

4.4 ARM Latent Factors

Although we can exploit arbitrage returns related to the ARMs, I also investigate how the ARMs can be used as an approximation for the exposures to the

[7]I avoid the analysis based on entirely leaving a variable out from the whole estimation procedure of an unrestricted model because, in this case, the effect on the Sharpe ratio combines two forces. First, there is less information that can be used for the formation of the arbitrage portfolio. This effect should generally lead to a decrease in the out-of-sample Sharpe ratio. Second, leaving one variable out restricts the information that can be used for the exploitation of the common factor structure of the returns. Consequently, this effect saves more potential pricing information for the construction of the arbitrage portfolio, which should generally lead to an increase in the Sharpe ratio.

Table 4.11: *Variable Importance of the ARMs for the pure-alpha portfolios.*

	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6
Sharpe ratio	0.72	0.97	0.84	0.41	0.40	0.30
	Decrease of Sharpe ratio in %					
`coskew`	15.17	9.19	7.21	26.12	14.21	34.82
`cokurt`	11.24	-1.56	-14.72	4.65	-6.52	26.40
`beta_down`	5.96	0.48	-10.53	-40.93	-47.86	-71.74
`down_corr`	-15.21	-4.98	-13.34	-26.57	-29.53	-57.84
`htcr`	-5.44	-6.91	-22.21	3.71	15.90	21.43
`beta_tr`	0.09	2.27	3.97	-3.61	-66.96	-142.62
`coentropy`	-39.70	-32.00	-25.15	-2.26	-16.14	-41.76
`cos_pred`	1.35	-21.29	-7.45	25.48	11.38	32.18
`beta_neg`	-1.99	0.64	7.24	-12.51	-61.93	-51.72
`mcrash`	1.27	-1.63	-1.61	-0.07	0.03	-10.14
`ciq_down`	-31.39	-25.61	-15.85	-22.89	-15.85	-74.26

Note: The table reports decreases of the out-of-sample Sharpe ratios in the pure-alpha portfolios from the leave-one-out procedure. For each ARM, I report the difference (in % points) between the Sharpe ratio obtained without the ARM and the Sharpe ratio obtained from the model with all ARMs. Data cover the period between January 1968 and December 2018.

common factors. In this section, I dissect the IPCA model fit using mainly the restricted specification of the ARM-IPCA model. I investigate which variables proxy for the exposures to the common factors and how they relate to the original IPCA model results using the set of 32 variables.

4.4.1 Model Fit and Tests

I evaluate the performance of the IPCA models in terms of two metrics. The first one, *total* R^2, describes how the model is able to capture time variation of the realized returns using conditional loadings and factor realizations

$$\text{Total}\, R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)^2}{\sum_{i,t} r^2_{i,t+1}}. \tag{4.12}$$

The total R^2 aims to quantify the model's success at capturing the riskiness of the assets. Total R^2 is related to the estimation procedure. Similarly, as in the case of principal component analysis, the estimation targets to maximize the model's explanatory power of the time variation of returns. In the case of the out-of-sample fits, the model parameters are estimated using the information up to time t, the same as the factors that are formed using the information up to time t, and the out-of-sample realized factor returns are then recorded.

The second metric, *predictive* R^2, captures how the model is capable of explaining the conditional expected returns

$$\text{Predictive } R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta\hat{\lambda})\right)^2}{\sum_{i,t} r^2_{i,t+1}} \tag{4.13}$$

where $\hat{\lambda}$ is a vector of factor means. In the case of out-of-sample analysis, $\hat{\lambda}$ is estimated up to time t. The predictive R^2 captures how much the model is able to describe the risk-return trade-off of the assets. We can use the restriction of $\Gamma_\alpha = 0$ to compare the performance with the unrestricted model. When we impose the restriction, the predictive R^2 tells us how much the risk compensation can be explained by the systematic risk with the exposures approximated by the ARMs. When we do not impose this restriction, the predictive R^2 summarizes how much of the variation of the expected returns can be explained through the characteristics via their relation to either systematic risk exposure or anomaly intercepts.

Moreover, the IPCA model has a natural interpretation in terms of managed portfolios. Using managed portfolio interpretation is important for estimation (e.g., for initial guess of the numerical optimization), its relation to the classical PCA estimator, and for various bootstrap testing procedures. More importantly for the presented analysis, I will use both single stock and managed portfolio returns to evaluate the performance of the IPCA models. Asset pricing literature frequently prefers to use portfolios because of their lower levels of unrelated idiosyncratic risk. The corresponding metrics are defined as

$$\text{Total } R^2 = 1 - \frac{\sum_t \left(x_{t+1} - Z'_tZ_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta\hat{f}_{t+1})\right)'\left(x_{t+1} - Z'_tZ_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta\hat{f}_{t+1})\right)}{\sum_t x'_{t+1}x_{t+1}} \tag{4.14}$$

and

$$\text{Predictive } R^2 = 1 - \frac{\sum_t \left(x_{t+1} - Z'_tZ_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta\hat{\lambda})\right)'\left(x_{t+1} - Z'_tZ_t(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta\hat{\lambda})\right)}{\sum_t x'_{t+1}x_{t+1}}. \tag{4.15}$$

To formally decide between restricted or unrestricted model specification for given number of latent factors *in-sample*, I follow Kelly et al. (2019). Using Model 4.8, I test a null hypothesis of $H_0 : \Gamma_\alpha = 0_{L\times 1}$ against an alternative hypothesis $H_1 : \Gamma_\alpha \neq 0_{t\times 1}$. Under the null hypothesis, the characteristics do not yield significant alphas after controlling for their explanatory power regarding the loadings on latent factors. The procedure follows three steps.

First, the unrestricted IPCA model is estimated and the parameters and the residuals are saved. I compute a Wald-type test statistic that measures the distance between the restricted and unrestricted model, $W_\alpha = \hat{\Gamma}'_\alpha \hat{\Gamma}_\alpha$. Second, the inference regarding the test statistic is performed using residual bootstrap. In each bootstrap replication, I generate a sample of new managed portfolio returns using the estimated residuals, estimate $\hat{\Gamma}_\beta$ (both from the original unrestricted model) and the restricted model's specification (setting $\Gamma_\alpha = 0$). Then, the generated sample is used to estimate the unrestricted model and the simulated test statistic is saved. Third, the resulting inference is obtained from the simulated distribution of bootstrapped test statistics. A resulting p-value of the test is calculated as a proportion of bootstrapped test statistics that exceed the value of the test statistic from the actual data.

The choice of using the bootstrap to draw an inference regarding the IPCA fits throughout the investigation is mainly driven by its robustness features. I exploit the fact that bootstrap enjoys favorable statistical properties in finite samples. Furthermore, I can perform statistical testing without making strong distributional assumptions regarding the model residuals.

To assess the fit of the restricted and unrestricted model *out-of-sample*, I investigate the performances of two portfolios. Beside the pure-alpha portfolio studied in Section 4.3, I use the restricted model to form a factor tangency portfolio. Each time t, I estimate the restricted model and set weights of the factor portfolios proportional to $\Sigma_t^{-1}\mu_t$, where Σ_t and μ_t are a covariance matrix and vector of average returns of the IPCA factors, respectively, both estimated using information up to time t. The portfolio weights are re-scaled to target 1% monthly volatility based on the historical estimate. The performance of this portfolio indicates how well the ARMs align with the exposures to the common factors and whether those exposures are priced.

4.4.2 IPCA Estimation Results

Panel A of Table 4.12 summarizes the in-sample results of both restricted and unrestricted versions of the IPCA models with varying numbers of latent factors. The models are estimated over the whole sample. The first segment of each panel captures the results using individual stocks. The second segment

Table 4.12: *ARM-IPCA results.*

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: In-sample results									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	15.95	17.30	17.99	18.46	18.70	18.83	18.94	19.02
	$\Gamma_\alpha \neq 0$	16.02	17.36	18.00	18.47	18.71	18.83	18.94	19.02
Predictive R^2	$\Gamma_\alpha = 0$	0.29	0.31	0.35	0.35	0.36	0.36	0.35	0.36
	$\Gamma_\alpha \neq 0$	0.37	0.37	0.36	0.36	0.36	0.36	0.36	0.36
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.28	98.35	99.45	99.66	99.79	99.85	99.90	99.94
	$\Gamma_\alpha \neq 0$	96.35	98.41	99.46	99.67	99.79	99.85	99.90	99.94
Predictive R^2	$\Gamma_\alpha = 0$	1.85	1.88	1.95	1.94	1.95	1.95	1.94	1.95
	$\Gamma_\alpha \neq 0$	1.97	1.96	1.96	1.96	1.96	1.96	1.96	1.95
Asset pricing test									
W_α p-value		0.00	0.00	4.70	0.80	2.50	16.40	7.60	77.60
Panel B: Out-of-sample results									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	15.49	16.81	17.47	17.99	18.25	18.38	18.49	18.57
	$\Gamma_\alpha \neq 0$	15.47	16.80	17.37	17.98	18.24	18.36	18.48	18.57
Predictive R^2	$\Gamma_\alpha = 0$	0.23	0.23	0.26	0.26	0.27	0.28	0.28	0.28
	$\Gamma_\alpha \neq 0$	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	96.30	98.35	99.28	99.63	99.77	99.83	99.89	99.93
	$\Gamma_\alpha \neq 0$	95.91	98.04	99.08	99.56	99.74	99.81	99.88	99.92
Predictive R^2	$\Gamma_\alpha = 0$	1.55	1.56	1.64	1.67	1.69	1.69	1.69	1.69
	$\Gamma_\alpha \neq 0$	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.70
Tangency portfolios									
Mean		9.74	6.64	16.36	19.11	21.37	22.37	22.93	23.68
t-stat		3.10	2.27	4.45	6.00	6.06	6.66	7.10	7.43
Sharpe		0.49	0.33	0.82	0.96	1.07	1.12	1.15	1.18

Note: The table reports in-sample and out-of-sample results of the ARM-IPCA models with varying numbers of latent factors. The asset pricing test reports p-values of the null hypothesis that $\Gamma_\alpha = 0$. Data cover the period between January 1968 and December 2018.

describes the results using the managed portfolios. The third segment then reports the test results regarding the zero alpha assumption.

The test rejects the null hypothesis of non-significant alphas for the first five IPCA specifications. The predictive R^2s suggest little difference between the restricted and non-restricted models for the IPCA(3) models. However, it is difficult to assess the importance of those differences as only a tiny increase of R^2 may lead to large investment gains. They may play even more significant role if we look at the out-of-sample results, which the results of the pure-alpha portfolios confirm.

Generally, the results are similar to the results obtained by Kelly et al. (2019) or Kelly et al. (2023) in the sense that only a few instrumented latent factors are needed to explain the asset returns. These results suggest that if we let the ARMs explain the exposures into latent factors, their residual abnormal alpha returns vanish. The main difference between the results here and the results obtained by Kelly et al. (2019) is that their dataset contains 36 characteristics and needs six latent factors to not reject the null hypothesis of $\Gamma_\alpha = 0$. In the present case, I use only 11 characteristics and need the same number of factors to not reject the hypothesis.

The out-of-sample estimation proceeds the same as in the case of the formation of the arbitrage portfolios. The models are estimated using an expanding window with the 60-month initial period. Results regarding total and predictive R^2 hold similarly as in the case of the in-sample analysis. The results of the pure-alpha portfolios from Section 4.3 show that we have to include around six factors to eliminate statistically significant arbitrage returns. Those observations enable us to understand better the small differences between the predictive R^2s for the restricted and unrestricted models. Predictive R^2s for the restricted and unrestricted IPCA(5) models are 0.27 and 0.28, respectively, but the pure-alpha portfolio of the unrestricted model still delivers abnormal returns of 8.06% p.a. with significant t-statistics of 2.86. However, once we get to seven latent factors, those arbitrage opportunities vanish.

These out-of-sample results are similar to the results of the bootstrap tests obtained from the in-sample analysis. We see a need to include multiple latent factors to erase the significant effect of the ARM characteristics. This observation suggests there is less duplicity in the information regarding the expected returns among the ARMs than one might expect. The proportion of the number of factors needed to eliminate arbitrage opportunity and the number of ARMs is more than half.

Based on the performances of the tangency portfolios, the results also suggest that ARMs successfully proxy for the exposures to the common factors. Tangency portfolio yields up to around 1.15 Sharpe ratio. The right panel of Figure 4.2 captures the cumulative log return of those portfolios. We see tangency portfolios grow over the whole period without a noticeable sign of slowing down.

I also perform the out-of-sample analysis over two sub-intervals as a simple robustness check. Table 28 in Appendix 4.C summarizes the out-of-sample results of the ARM-IPCA models using all stocks estimated separately in two disjoint periods. The first period covers the range between January 1968 and December 1993, and the second spans time between January 1994 and December 2018. Results regarding the tangency and arbitrage portfolios agree with those obtained over the entire period. Generally, the results are stable over disjoint periods, as the number of latent factors needed to eliminate the arbitrage opportunities is around six.

Table 4.13: *Summary statistics of the ARM-IPCA factors.*

	In-sample			Out-of-sample		
Factor	Mean	Std. Dev.	Sharpe	Mean	Std. Dev.	Sharpe
1	4.10	33.72	0.12	2.02	34.24	0.06
2	6.94	18.35	0.38	3.69	18.04	0.20
3	3.84	13.80	0.28	5.76	13.53	0.43
4	0.37	9.72	0.04	5.70	10.91	0.52
5	8.92	8.58	1.04	8.35	9.32	0.90
6	3.56	7.06	0.50	-0.06	8.22	-0.01

Note: The table reports summary statistics of the instrumented principal components from the IPCA(6) model. The factors are standardized to have an unconditional standard deviation of 20% p.a.

4.4.3 Factors and Characteristic Importance

This section delves further into the features of the latent factors of the ARM-IPCA model. Table 4.13 summarizes the latent factors from the ARM-IPCA(6) model. The higher Sharpe ratios, both in-sample and out-of-sample, possess mainly higher-order (third and higher) factors. The first instrumented principal component, which explains the most time variation of the returns, leaves the predictive power to the other factors. This observation is similar to the result obtained by Lettau and Pelger (2020), which also reports high Sharpe ratios for higher-order factors.

Figure 10 from Appendix 4.C shows loadings of the ARMs on the latent factors from the restricted IPCA(6). The first two factors are clearly related to the negative semibeta and predicted coskewness, respectively. The fifth factor, which possesses the highest Sharpe ratio both in- and out-of-sample, noticeably loads on tail risk beta and downside CIQ betas.

To formally assess the importance of each variable for the performance of the restricted IPCA model, I perform a bootstrap test proposed by Kelly et al. (2019). For the given IPCA model with K latent factors, let the l^{th} row in the matrix $\Gamma_\beta = [\gamma_{\beta,1}, \ldots, \gamma_{\beta,L}]$ maps the l^{th} characteristic to the loadings on the K latent factors. The null hypothesis assumes that the l^{th} row is equal to zero, i.e., this characteristic does not proxy for the dynamics of the factor loadings. To test the hypothesis, I estimate the alternative model that admits the possibility of the contribution of the l^{th} characteristic and form a Wald-type characteristic of the form $W_{\beta,l} = \hat{\gamma}'_{\beta,l}\hat{\gamma}_{\beta,l}$. I save the estimated model parameters, factors, and managed portfolio residuals. Then, I simulate a new bootstrap sample under the null hypothesis of $\gamma_{\beta,l}$ being equal to zero by resampling the returns of the characteristic-managed portfolios using the wild bootstrap procedure and the estimated parameters. Using the new sample, I estimate the alternative model

Table 4.14: *Variable importance of the ARMs.*

	IPCA1	IPCA2	IPCA3	IPCA4	IPCA5	IPCA6	IPCA7	IPCA8
coskew	22.60	16.90	9.10	2.30	3.30	59.90	46.90	0.00
cokurt	17.70	18.30	9.80	4.80	7.10	55.60	1.90	0.50
beta_down	9.90	4.90	0.20	0.30	0.10	0.80	0.00	0.00
down_corr	0.00	3.00	18.40	7.30	9.20	13.80	33.30	64.90
htcr	0.00	4.20	0.10	0.80	0.40	0.00	0.30	0.00
beta_tr	97.80	8.60	18.80	21.70	0.00	0.00	0.00	0.00
coentropy	2.50	2.90	25.70	17.10	18.10	18.20	40.40	51.40
cos_pred	0.10	26.60	46.30	0.00	0.00	0.00	0.00	0.00
beta_neg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
mcrash	49.40	6.40	2.80	3.60	2.90	1.40	4.70	8.90
ciq_down	75.40	8.90	13.30	4.00	0.00	0.00	0.00	0.00

Note: The table reports p-values (in %) of the bootstrap tests that given ARM does not significantly contribute to the restricted ARM-IPCA model's fit in-sample. Data cover the period between January 1968 and December 2018.

and form test statistic $\tilde{W}^b_{\beta,l}$. The resulting p-value of the test is calculated as the proportion of $\tilde{W}^b_{\beta,l}$ that exceeds $W_{\beta,l}$.

Table 4.14 reports simulated p-values for each variable and each specification of the IPCA model. We know that around six latent factors are needed to eliminate the arbitrage opportunity, so I focus on the IPCA(6) specification here. In this case, seven variables are highly significant and drive the explanatory power of the model – downside beta, hybrid tail covariance risk, predicted coskewness, negative semibeta, MCRASH, and downside CIQ beta.

4.4.4 ARMs and other Characteristics

In this section, I investigate how the ARMs relate to other characteristics that have been proven to be significant proxies for factor exposures. To do that, I use data from Freyberger et al. (2020) and Kim et al. (2020) and select 32 variables that were employed in Kelly et al. (2019). Those variables are: market beta (`beta`), assets-to-market (`a2me`), total assets (`at`), sales-to-assets (`ato`), book-to-market (`beme`), cash-to-short-term-investment (`c`), capital turnover (`cto`), ratio of change in property, plants and equipment to the change in total assets (`dpi2a`), earnings-to-price (`e2p`), cash flow-to-book (`freecf`), idiosyncratic volatility with respect to the FF3 model (`idiovol`), investment (`invest`), market capitalization (`lme`), turnover (`lturnover`), net operating assets (`noa`), operating accruals (`oa`), operating leverage (`ol`), price-to-cost margin (`pcm`), profit margin (`pm`), gross profitability (`prof`), Tobin's Q (`q`), price relative to its 52-week high (`rel_to_high_price`), return on net operating assets (`rna`), return on assets (`roa`), return on equity (`roe`), momentum (`cum_return_12_2`), intermediate momentum (`cum_return_12_7`), short-term reversal (cum_return_1_0), long-term reversal (cum_return_36_13), sales-to-price (s2p), bid-ask spread (spread_mean), and unexplained volume (suv).[8]

Figure 4.4 contains correlations between ARMs and characteristics used in Kelly et al. (2019). The highest correlation is between market beta and negative semibeta with an average value of 0.75, and market beta and downside beta with a value of 0.58. Both these correlations are expected to be quite high as their definitions are closely related. Negative semibeta is also highly correlated with idiosyncratic volatility with an average correlation of 0.49. Table 4.1 summarizes the average absolute correlations between each ARM and all other characteristics. We observe that the average values are noticeably lower than in the case of correlations with other ARMs. The lowest correlated ARMs are coskewness, downside correlation, tail risk beta and coentropy with value around 0.02. The highest average correlation possesses negative semibeta with a value of 0.13.

Right panel of Table 4.1 reports average correlations between returns of the ARM-managed portfolios and managed portfolios sorted on other characteristics. Naturally, we observe higher correlations than in the case of the raw variables. The highest correlations possess hybrid tail covariance risk and negative semibeta, the lowest average correlations possess tail risk beta.

Table 4.15 reports correlations between out-of-sample latent factors estimated using the original dataset of 32 variables and latent factors estimated using 11 ARMs. Generally speaking, there is only a little commonality between those two sets of factors. Only the first IPCs from the all-stock dataset are noticeably correlated with a value of a 0.43. This observation suggests that the ARMs possess a specific common factor structure without a clear link to the structure obtained from the original dataset.

4.4.5 Model with All Characteristics

Next, I investigate whether the ARMs possess additional information for the factor exposures over the variables that were previously employed. To do so, I estimate the restricted and unrestricted IPCA models that utilize both the original set of 32 variables of Kelly et al. (2019) and 11 additional ARM variables, hence All-IPCA. Table 29 from Appendix 4.C reports the in-sample

[8] Due to availability in the updated sample, I have omitted four variables relative to the original IPCA specification from Kelly et al. (2019). Those variables are: capital intensity (d2a) fixed costs-to-sales (fc2y) leverage (lev), the ratio of sales and price (s2p). None of the variables was shown to be significant in the baseline IPCA(5) specification.

Figure 4.4: *Correlations between ARMs and other characteristics.*

	coskew	cokurt	beta_down	down_corr	htcr	beta_tr	coentropy	cos_pred	beta_neg	mcrash	ciq_down
suv	0	−0.02	−0.01	0	−0.02	0	0	−0.03	0	−0.01	0
spread_mean	0.08	−0.33	−0.06	−0.04	−0.42	0	−0.03	−0.24	0.26	−0.09	0.02
s2p	0.01	−0.17	−0.04	0	−0.15	0.02	0	−0.21	0.06	−0.06	0.06
roe	0	0.09	0.01	−0.01	0.09	0	−0.01	0.11	−0.07	0.04	−0.02
roa	−0.01	0.15	−0.03	−0.03	0.17	−0.04	−0.02	0.22	−0.19	0.06	−0.02
rna	0	0.01	0	0	0.01	−0.01	0	0.02	−0.01	0	−0.01
rel_to_high_price	−0.03	0.11	−0.17	0.05	0.35	0.02	0.02	0.02	−0.46	0.08	0.06
q	0.01	0.14	0.15	−0.01	0.04	−0.02	0	0.1	0.14	0.04	−0.08
prof	0	−0.02	0.02	0	−0.04	0.01	0	−0.05	0.05	0	−0.01
pm	0.01	0.06	−0.03	−0.02	0.08	−0.01	−0.02	0.11	−0.1	0.02	0.02
pcm	0.01	0.05	−0.01	−0.01	0.05	−0.02	−0.01	0.08	−0.04	0.02	−0.01
ol	−0.01	−0.11	−0.01	0	−0.11	0.01	0.01	−0.2	0.05	−0.05	0.03
oa	0	0.01	0	0	0	0	0	0	0	0	−0.01
noa	0	0.04	0.05	−0.01	0.01	−0.03	−0.01	0.03	0.05	0.01	−0.03
lturnover	−0.03	0.15	0.31	0.05	0	0.01	0.05	−0.04	0.34	0.04	−0.13
lme	0.01	0.26	0.02	−0.02	0.15	−0.02	−0.02	0.27	−0.07	0.1	0
investment	−0.01	0.08	0.09	0	0.03	0	0.01	0.06	0.08	0.03	−0.06
idio_vol	0.05	−0.3	0.1	−0.01	−0.48	0.02	0	−0.36	0.49	−0.15	−0.03
free_cf	−0.01	0.04	−0.04	−0.01	0.06	−0.02	−0.01	0.06	−0.11	0.02	0.01
e2p	−0.03	0.11	−0.05	0	0.19	0	0	0.15	−0.23	0.06	0.01
dpi2a	0	0.06	0.07	0	0.03	−0.03	0	0.06	0.06	0.02	−0.05
cum_return_36_13	−0.03	0.16	0.12	0.01	0.11	0.04	0.01	0.1	0.03	0.08	−0.07
cum_return_1_0	0	−0.01	−0.01	0.03	0.05	0.03	0.02	−0.17	−0.05	0	0
cum_return_12_7	−0.06	0.08	0.08	0.1	0.12	0.05	0.08	−0.12	−0.07	0.08	−0.01
cum_return_12_2	−0.07	0.06	0.06	0.13	0.18	0.07	0.1	−0.21	−0.11	0.08	−0.01
cto	−0.01	−0.05	0.02	0	−0.06	0.01	0	−0.12	0.04	−0.03	0
c	0	−0.05	0.08	0.02	−0.06	0.03	0.03	−0.13	0.14	−0.02	−0.05
beta	0	0.34	0.58	0.03	0.01	0.04	0.02	0.08	0.75	0.08	−0.28
beme	0.03	−0.24	−0.14	−0.01	−0.16	0.03	−0.01	−0.23	−0.03	−0.1	0.11
ato	−0.01	−0.01	0.01	0	−0.01	−0.01	0	−0.03	0.01	0	0
at	0	0.21	0.02	−0.02	0.12	0	−0.02	0.21	−0.04	0.07	0.02
a2me	0.01	−0.1	−0.08	−0.01	−0.07	0.05	−0.01	−0.07	−0.03	−0.04	0.06

Note: The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures and characteristics used in Kelly et al. (2019). Data include all available stocks and the period between January 1968 and December 2018.

IPCA results. Based on the p-values of a test that $\Gamma_\alpha = 0$, similarly as in the case ARM-IPCA, around six factors are needed to obtain an appropriate model that provides an adequate description of the behavior of stock returns.

Table 4.16 reports the p-values of the variable importance tests for each ARM. I focus on specifications with five and six latent factors due to their best fit. We can see that three ARM variables significantly contribute to the model performance: downside beta, hybrid tail covariance risk, and negative semi-beta. These non-linear systematic measures of risk can significantly improve the description of the stock exposures to the common linear factors.

To assess how the ARMs contribute to the fit of the model as a whole, I test whether ARMs jointly possess coefficients significantly different from zero. This is a generalization of the test discussed earlier, which inspects the

Table 4.15: *Correlations between original IPCA and ARM-IPCA factors.*

	ARM-IPC1	ARM-IPC2	ARM-IPC3	ARM-IPC4	ARM-IPC5	ARM-IPC6
IPC1	0.43	-0.38	-0.38	0.05	-0.12	-0.11
IPC2	0.30	0.15	-0.13	-0.13	0.04	0.05
IPC3	-0.08	0.32	-0.03	0.02	-0.12	-0.10
IPC4	-0.28	-0.32	0.16	0.03	0.00	-0.11
IPC5	-0.23	0.22	0.26	-0.05	0.41	0.08
IPC6	-0.03	0.07	0.18	0.01	0.21	0.10

Note: The table reports correlations between IPCA latent factors estimated using set of original 32 variables and IPCA latent factors estimated using 11 ARMs.

Table 4.16: *Variable importance results from the All-IPCA models.*

	coskew	cokurt	beta_down	down_corr	htcr	beta_tr	coentropy	cos_pred	beta_neg	mcrash	ciq_down	Joint test
All-IPCA(5)	6.8	28.7	0.6	28.6	1.8	8	22.5	16.9	2.2	58.6	26.1	6.7
All-IPCA(6)	24.2	37.3	2.5	23.9	2.4	11.7	26.2	8.2	1.2	94.9	17	6.8

Note: The table reports p-values (in %) of the significance tests regarding the importance of the ARMs in relation to the restricted All-IPCA model fit. It also contains results regarding the joint importance of the ARMs for the model fit. The All-IPCA model is estimated using set of original 32 variables from Kelly et al. (2019) and 11 ARMs.

importance of each variable separately. The testing procedure follows the same logic based on wild bootstrap. One difference is the definition of the Wald-type test statistic. In this case, we test whether a subset of J characteristics contributes significantly to the performance, so the statistic is $W_{\beta,l_1,\ldots,l_J} = \hat{\gamma}'_{\beta,l_1}\hat{\gamma}_{\beta,l_1} + \ldots + \hat{\gamma}'_{\beta,l_J}\hat{\gamma}_{\beta,l_J}$. In the resampling procedure, restricted model then sets contribution to all J tested characteristics to zero. The logic behind the rest of the test is the same.

The resulting tests for the All-IPCA models with five and six latent factors possess mildly significant p-values of 6.7% and 6.8%, respectively. This result suggests that the ARMs can contribute to the explanation of the stock returns based on a common factor structure.

4.5 Robustness Checks

In this section, I provide some robustness checks regarding the pure-alpha portfolio results. First, I show that the results are not significantly altered by focusing purely on universe of highly-liquid stocks. Second, I show that the results are not driven by time-varying volatility nature of the portfolios. Thirdly, I show that the results remain strong even if we rebalance the pure-alpha portfolios annually instead of monthly.

Table 4.17: *Pure-alpha portfolio returns without penny stocks.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	13.91	4.32	0.70	0.59	8.89	49.48	-33.90	42.04
2	15.95	4.58	0.80	0.59	5.57	42.30	-25.85	32.85
3	11.64	3.49	0.58	-0.06	8.66	57.62	-42.35	31.94
4	2.61	0.96	0.13	1.00	15.62	71.64	-37.15	52.39
5	0.68	0.25	0.03	1.58	21.83	76.26	-35.92	58.40
6	1.26	0.49	0.06	2.03	28.57	70.85	-35.95	63.04

Note: The table contains out-of-sample results for the pure-alpha portfolios estimated using the ARM-IPCA model ranging between one and six latent factors. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with a 60-month initial period. I exclude stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

4.5.1 Excluding Penny Stocks

Here, I provide a simple check regarding the universe of stocks that I exploit in the estimation of the IPCA model and formation process of the pure-alpha portfolios. I employ no-penny dataset discussed earlier. This dataset is characterized by exclusion of stocks with price less than \$5 or market capitalization below 10% quantile of NYSE stocks. I estimate the IPCA models and form the pure-alpha portfolios using this dataset in the same way as in the case of all stocks.

The results are summarized in Table 4.17. We see that the portfolio returns are highly significant with Sharpe ratios of up to 0.8. Statistical features of the portfolios are quite similar to the results obtained for all stocks. The main difference can be seen in the number of factors for which we can control to obtain significant abnormal results. In the case of all stocks, we obtain significant returns for up to five latent factors, in the case of no-penny dataset, this number reduces to three.

4.5.2 Volatility Targeting of the Pure-Alpha Portfolios

In this section, I provide results regarding the pure-alpha portfolios, which target in-sample volatility. More specifically, each time during the out-of-sample procedure, I scale the weights of the pure-alpha portfolio given by equation 4.9 so that the in-sample volatility of the portfolio is 20% p.a. This is a simple approach how one may proceed when setting up a portfolio.

The results for both all-stock and no-penny datasets are summarized in Table 4.18. We observe very similar results as in the case of the pure-alpha

Table 4.18: *Volatility-targeted pure-alpha portfolio returns.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
Panel A: All stocks								
1	15.21	4.80	0.72	0.13	2.62	36.25	-23.74	24.61
2	20.11	6.22	0.94	0.06	3.26	34.38	-27.49	26.32
3	17.36	5.49	0.85	0.46	3.62	32.09	-23.40	30.99
4	9.69	3.37	0.46	0.48	2.05	33.41	-24.09	26.54
5	9.38	3.08	0.45	0.67	1.67	45.53	-16.03	24.57
6	7.04	2.15	0.33	0.65	1.94	57.02	-15.24	28.09
Panel B: No penny stocks								
1	13.72	4.43	0.69	0.17	4.10	53.10	-25.69	27.86
2	15.41	4.52	0.77	0.11	3.79	56.86	-24.08	30.69
3	10.03	3.07	0.52	0.33	3.82	77.03	-23.57	30.69
4	2.63	0.84	0.13	-0.02	2.83	83.24	-26.12	28.75
5	0.24	0.08	0.01	0.16	3.43	87.09	-27.85	30.62
6	0.24	0.08	0.01	0.05	3.62	90.78	-29.78	29.16

Note: The table contains out-of-sample results for the pure-alpha portfolios estimated using the ARM-IPCA model ranging between one and six latent factors. Weights of the portfolios target in-sample volatility od 20% p.a. Table reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with a 60-month initial period. No-penny dataset excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

portfolios standardized over the whole period. We can conclude that the success of the pure-alpha portfolios is not driven by the time-varying volatility.

4.5.3 Annual Returns

I also investigate how the pure-alpha portfolios align with annual returns. Monthly rebalancing of the portfolios may be costly for investors and the annual frequency may mirror their investment horizon better. To inspect the relation, I take the weights of the pure-alpha portfolios employed in the previous sections and use them to weight average stock returns from month $t+1$ to $t+12$. I report results for both all-stock and no-penny universes in Table 4.19. The results are qualitatively very similar to the results using monthly rebalancing. Returns and their t-stats even increase for both datasets. We can see that the returns are not driven by short-lived features present among illiquid stocks.

4.6 Time-Varying Risk Premium

The IPCA framework may only fully capture the arbitrage opportunities if the compensation for bearing risk associated with the ARMs is stable across time. To investigate and potentially exploit the time-varying nature of the risk premium associated with the ARMs, I employ the projected principal

Table 4.19: *Pure-alpha portfolio annual returns.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Worst month	Best month
Panel A: All stocks							
1	30.69	5.06	1.53	0.37	0.11	-14.37	22.79
2	39.70	6.41	1.99	0.25	0.27	-14.85	25.42
3	38.55	6.34	1.93	0.62	0.59	-13.01	26.19
4	21.12	3.73	1.06	0.39	0.47	-17.39	21.51
5	14.10	2.41	0.71	0.68	1.29	-15.00	23.45
6	9.93	1.73	0.50	1.08	3.85	-16.62	32.38
Panel B: No penny stocks							
1	27.61	4.38	1.38	0.25	0.55	-15.40	19.78
2	35.45	5.65	1.77	0.28	0.39	-12.75	21.12
3	24.06	3.89	1.20	0.27	0.85	-21.53	22.39
4	9.40	1.69	0.47	0.15	1.63	-20.39	22.31
5	3.74	0.67	0.19	0.22	2.20	-20.77	23.23
6	5.91	1.05	0.30	0.26	3.08	-22.60	24.81

Note: The table contains out-of-sample results for the pure-alpha portfolios estimated using the ARM-IPCA model ranging between one and six latent factors. Portfolios are annually rebalanced. Table reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with a 60-month initial period. No-penny dataset excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

component analysis (PPCA) framework proposed by Fan et al. (2016) and extended by Kim et al. (2020). Compared to the IPCA framework, PPCA enables changes in cross-sectional relations between alphas/betas and characteristics. This variation may be potentially important if the relation between ARMs and risk/mispricing changes over time due to various reasons, such as being arbitraged away or beta-ARM relation changes.

An example of the former constitutes the results of Mclean and Pontiff (2016), which state that the relation changes due to the investors' usage of academic publications to learn about mispricing and forming their investment decisions based on that. An example of the latter represents Cho (2020), who argues that financial intermediaries turn alphas into betas through their arbitrage process and exposure to funding liquidity and arbitrageur wealth portfolio shocks.

The PPCA framework first assigns maximal explanatory power of the characteristics to the systematic risk exposures before relating the characteristics to their alphas. The resulting arbitrage portfolio thus aims to hedge sources of systematic risk related to the characteristics while enjoying the residual returns associated with the ARMs. Moreover, it enables the arbitrage portfolios to reflect the time variation in compensation for the ARMs by being consistently estimated over short samples. This feature comes at the cost of less efficiency if the relationship between characteristics and model parameters are constant because we use less data to estimate the model and form the arbitrage portfolio.

4.6.1 PPCA Model

Similarly, as in the case of the IPCA model, I assume that the excess return of stock follows the structure

$$r_{i,t} = \alpha_i + \beta_i f_t + \epsilon_{i,t} \tag{4.16}$$

where the main difference in comparison to IPCA is that now I assume that the return-generating process for individual stocks (characterized by α_i and β_i) is stable over short time periods (12 months in the empirical investigation) $t = 1, \dots, T$. In a matrix format for N assets over T periods, this can be rewritten as

$$\boldsymbol{R} = \boldsymbol{\alpha} \mathbf{1}'_T + \boldsymbol{B}\boldsymbol{F}' + \boldsymbol{E} \tag{4.17}$$

where $\boldsymbol{R}$ is the $(N \times T)$ matrix of returns, $\boldsymbol{\alpha}$ is the $(N \times 1)$ mispricing vector, $\boldsymbol{B}$ is the $(N \times K)$ matrix with i-th row corresponding to factor exposure β'_i, $\boldsymbol{F}$ is the $(T \times K)$ matrix of latent factors with t-th row being $f'_t = [f_{1,t}, \dots, f_{K,t}]$. This specification allows the systematic exposure matrix $\boldsymbol{B}$ and vector of mispricing being nonparametric functions of the asset-specific characteristics. I stack each of the L characteristics into the $(N \times L)$ matrix $\boldsymbol{Z}$ and impose the following structure

$$\boldsymbol{\alpha} = \boldsymbol{G}_\alpha(\boldsymbol{Z}) + \Gamma_\alpha \tag{4.18}$$

$$\boldsymbol{B} = \boldsymbol{G}_\beta(\boldsymbol{Z}) + \Gamma_\beta \tag{4.19}$$

where the mis-pricing function is defined as $\boldsymbol{G}_\alpha(\boldsymbol{Z}) : \mathbb{R}^{N \times L} \to \mathbb{R}^N$, and the factor loading function is $\boldsymbol{G}_\beta(\boldsymbol{Z}) : \mathbb{R}^{N \times L} \to \mathbb{R}^{N \times K}$, and the $(N \times 1)$ vector Γ_α and the $(N \times K)$ matrix Γ_β are cross-sectionally orthogonal to the characteristics $\boldsymbol{Z}$. To estimate this model, I follow the projected principal component analysis (PPCA) proposed by Fan et al. (2016) and generalized by Kim et al. (2020) to allow for the presence of the mispricing contained in $\boldsymbol{\alpha}$.

The formation of the arbitrage portfolio proceeds in three steps. First, I demean the returns and apply PCA to obtain an estimate of $\boldsymbol{G}_\beta(\boldsymbol{Z})$. Second, I cross-sectionally regress the average returns on the characteristics space which is orthogonal to the estimate $\boldsymbol{G}_\beta(\boldsymbol{Z})$ from the first step to obtain the estimate of $\boldsymbol{G}_\alpha(\boldsymbol{Z})$. Third, I use the estimate of $\boldsymbol{G}_\alpha(\boldsymbol{Z})$ to form the portfolio, which is held for the next period. I denote this portfolio as *arbitrage portfolio*.

The main advantage of this methodology over the IPCA framework is that it is suited for the estimation over short time periods and thus enables to exploit the dynamics of the compensation for the ARMs. The model is estimated on a rolling-window basis, setting T to a short time period. This freedom allows for a change in cross-sectional relation between ARMs and returns either in terms of systematic risk or mispricing. Moreover, the model does not require to have all relevant characteristics for risk and mispricing, as the missing information may be contained in Γ_α and Γ_β. The aim of this model is to exploit mispricing captured by $\boldsymbol{\alpha}$ while hedging the systematic risk characterized by the ARMs and captured by $\boldsymbol{B}$.[9]

This greater flexibility comes at a cost, however. The methodology does not exploit the time-variation of the characteristics during the estimation window. It employs only the values of characteristics at the first estimation period and assumes that these values proxy sufficiently for characteristics in the subsequent periods during the window. If the true relationship between characteristics and the model is constant, this will lead to a loss of estimation efficiency.

Following the original empirical PPCA implementation, I cross-sectionally demean the characteristics so that the resulting arbitrage portfolio costs zero dollars. Moreover, I target the in-sample volatility of the portfolio at 20% per year. I report the results for a range between one and ten latent factors. All the results are purely out-of-sample as the model is fitted using 12 months of data, the arbitrage portfolio is formed at the end of this period using the value of the characteristic at the beginning of the holding period, and then the return in the next month is recorded.

4.6.2 Arbitrage Portfolios

Table 4.20 summarizes the performances of the arbitrage portfolios that exploit the ARMs. We can see that when we use between two and ten factors in the model, we can obtain significant abnormal returns that are hedged against the exposure to common risks. The annual risk premium that we can obtain constitutes around 7.5% per year with a Sharpe ratio of around 0.45 and highly significant t-statistics of the average return of around three. Although the arbi-

[9] I thank Andreas Neuhierl for making the code for the extended PPCA estimation publicly available.

Table 4.20: *Summary of the arbitrage portfolio returns.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	3.29	1.30	0.20	0.79	8.13	60.81	-28.64	32.97
2	8.63	3.07	0.47	0.10	5.03	60.14	-31.22	30.56
3	7.73	2.99	0.49	0.40	6.92	55.43	-29.57	30.11
4	7.76	2.83	0.43	-0.00	4.61	56.98	-30.22	28.39
5	7.66	2.92	0.45	0.15	5.89	61.76	-28.80	30.67
6	7.84	2.96	0.46	-0.08	6.19	50.07	-31.86	27.03
7	7.71	3.01	0.46	-0.49	8.05	46.79	-34.48	26.16
8	8.90	3.35	0.53	-0.30	8.81	47.13	-33.95	30.00
9	6.32	2.35	0.39	-0.06	7.05	60.23	-29.87	28.11
10	5.68	2.26	0.35	-0.52	18.54	52.76	-44.09	32.44

Note: The table contains out-of-sample results for the arbitrage portfolios estimated using the extended PPCA framework of Kim et al. (2020) using a rolling window estimation of 12 months and various numbers of latent factors. It reports the annualized mean return, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Data cover the period between January 1968 and December 2018.

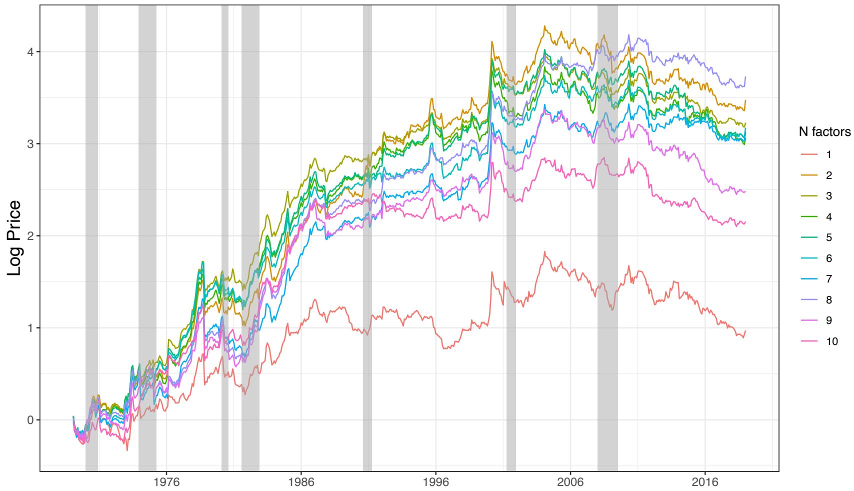
trage portfolios yield significant hedged returns, they do not result in noticeably better performance than single-sorted portfolios with the Sharpe ratio being at a maximum equal to 0.53 compared to the Sharpe ratio of downside CIQ beta with a value of 0.42 in the case of its managed portfolio.

Regarding the distributional features of the returns, we see that they are close to symmetrically distributed. On the other hand, the estimated kurtosis values suggest that the returns are more heavy-tailed than the pure-alpha portfolios estimated using the ARM-IPCA model. This fact also affects the maximum drawdowns of the portfolios, which are also higher in the case of the arbitrage portfolios than in the case of pure-alpha portfolios. Figure 4.5 plots the cumulative returns of the arbitrage portfolios. We see that the portfolios constantly grow up until around the financial crisis. Around that time, returns sizably deteriorate and have not recovered since then.

Table 4.21 summarizes the risk-adjusted returns of the arbitrage portfolios with respect to various factor models based on three-factor model of Fama and French (1993). While three- and five-factor models of Fama and French (1993) and Fama and French (2015) are not able to explain the associated anomaly returns, results that include momentum factor erase their significance.

To further investigate the relationship between arbitrage returns and other factors, I report in Table 4.22 exposures to the six-factor model based on four factors of Carhart (1997) augmented by CIV shocks of Herskovic et al. (2016), and the BAB factor of Frazzini and Pedersen (2014). We observe that the six-factor alpha significantly shrinks to around 4% per annum, and the corresponding t-statistic falls below two in all models. Similarly, as in the case of

Figure 4.5: *Cumulative return of the arbitrage portfolios.*



Note: The figure depicts the cumulative logarithm price of the arbitrage portfolios based on the PPCA framework of Kim et al. (2020), with the number of latent factors between one and ten. Arbitrage returns are purely out-of-sample. Data cover the period between January 1968 and December 2018.

pure-alpha portfolios, the arbitrage portfolio possess a significant exposure to the momentum factor. In this case, however, the momentum strategy, along with other factors, explains the whole significant part of the arbitrage returns. Well-documented momentum crashes may partially explain the leptokurtic features of the portfolio, similarly they may be related to the high drawdowns that the portfolios experienced.

We observe that the arbitrage returns of ARMs do not benefit from considering the time-varying nature of the model setting. The claim is apparent since the arbitrage portfolios do not produce abnormal returns beyond common factor exposures, particularly when factoring in relation to the momentum factor. These observations indicate that the loss of efficiency from short-window estimation outweighs any potential benefits from time-varying risk prices for ARMs. This conclusion was already suggested by the regression portfolios that performed better when estimated using the expanding window versus the moving window. Likewise, the alphas of those portfolios were much less impacted when estimated using the expanding window.

In comparison, the pure-alpha portfolio returns obtained from the IPCA procedure using up to five factors yield a significant premium after controlling for those six common factors. Moreover, the Sharpe ratios that attain the pure-alpha portfolios are considerably higher than those of the arbitrage portfolios based on the PPCA. All these results suggest that the relationship between

Table 4.21: *Fama-French risk-adjusted returns of the arbitrage portfolios.*

K factors	CAPM	FF3	FF3+MOM	FF3+MOM +CIV	FF3+MOM +CIV+BAB	FF5	FF5+MOM	FF5+MOM +CIV	FF5+MOM +CIV+BAB
1	2.38	1.85	1.36	1.36	3.44	2.81	2.40	2.40	3.38
	(0.90)	(0.70)	(0.46)	(0.46)	(1.06)	(0.91)	(0.71)	(0.71)	(0.99)
2	8.31	7.80	3.02	3.02	4.74	7.53	3.71	3.71	4.51
	(2.78)	(2.51)	(1.07)	(1.07)	(1.57)	(2.20)	(1.22)	(1.22)	(1.47)
3	7.04	6.55	2.73	2.73	4.21	6.64	3.57	3.57	4.20
	(2.61)	(2.43)	(1.09)	(1.09)	(1.53)	(2.18)	(1.27)	(1.27)	(1.47)
4	7.31	7.00	2.12	2.12	3.42	6.88	2.95	2.94	3.47
	(2.57)	(2.43)	(0.82)	(0.82)	(1.20)	(2.10)	(1.03)	(1.03)	(1.18)
5	7.04	6.71	2.49	2.48	3.72	6.68	3.28	3.28	3.77
	(2.57)	(2.42)	(0.95)	(0.96)	(1.29)	(2.11)	(1.14)	(1.15)	(1.27)
6	7.37	7.04	2.73	2.73	3.84	7.27	3.77	3.77	4.14
	(2.66)	(2.51)	(1.05)	(1.05)	(1.33)	(2.29)	(1.33)	(1.33)	(1.41)
7	7.24	7.11	2.72	2.72	3.83	7.30	3.73	3.73	4.09
	(2.68)	(2.59)	(1.05)	(1.05)	(1.33)	(2.31)	(1.30)	(1.30)	(1.38)
8	8.44	8.65	4.38	4.38	5.19	8.44	5.02	5.02	5.23
	(3.06)	(3.09)	(1.60)	(1.60)	(1.72)	(2.62)	(1.66)	(1.66)	(1.68)
9	6.02	5.78	1.92	1.92	2.58	5.40	2.33	2.33	2.50
	(2.17)	(2.09)	(0.74)	(0.74)	(0.94)	(1.74)	(0.84)	(0.84)	(0.90)
10	5.05	4.69	0.58	0.58	1.28	4.51	1.21	1.20	1.43
	(1.93)	(1.75)	(0.23)	(0.23)	(0.46)	(1.40)	(0.43)	(0.43)	(0.50)

Note: The table reports annualized alphas and their HAC t-statistics of Newey and West (1987) with six lags obtained by regressing the arbitrage portfolio returns on various factor models and their combinations: Fama and French (1993), Carhart (1997), Fama and French (2015), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Data cover the period between January 1973 and December 2018.

Table 4.22: *Exposures of the arbitrage portfolios.*

N factors	α	Mkt	SMB	HML	CIV	BAB	MOM
1	3.44	0.11	0.34	0.27	-0.05	-0.31	0.14
	(1.06)	(1.58)	(2.31)	(2.73)	(-1.72)	(-2.88)	(1.32)
2	4.74	0.11	0.30	0.40	-0.02	-0.26	0.53
	(1.57)	(1.47)	(2.19)	(3.23)	(-0.97)	(-2.68)	(5.84)
3	4.21	0.15	0.30	0.34	-0.03	-0.22	0.43
	(1.53)	(2.44)	(2.27)	(3.37)	(-1.50)	(-2.38)	(5.03)
4	3.42	0.13	0.26	0.33	-0.03	-0.19	0.53
	(1.20)	(1.94)	(1.88)	(2.81)	(-1.07)	(-2.07)	(5.95)
5	3.72	0.14	0.27	0.30	-0.03	-0.19	0.46
	(1.29)	(2.35)	(1.96)	(2.99)	(-1.19)	(-1.96)	(5.12)
6	3.84	0.12	0.28	0.30	-0.02	-0.17	0.46
	(1.33)	(1.86)	(2.12)	(2.92)	(-0.71)	(-1.82)	(5.49)
7	3.83	0.13	0.19	0.26	-0.02	-0.17	0.47
	(1.33)	(2.01)	(1.48)	(2.58)	(-0.72)	(-1.95)	(5.40)
8	5.19	0.11	0.21	0.16	-0.02	-0.12	0.45
	(1.72)	(1.59)	(1.60)	(1.61)	(-0.87)	(-1.27)	(4.79)
9	2.58	0.08	0.26	0.23	-0.01	-0.10	0.40
	(0.94)	(1.25)	(2.17)	(2.30)	(-0.32)	(-1.10)	(4.57)
10	1.28	0.13	0.35	0.27	-0.02	-0.11	0.43
	(0.46)	(2.07)	(2.82)	(3.46)	(-0.84)	(-1.20)	(5.05)

Note: The table reports estimated coefficients and their t-statistics from regressing returns of the arbitrage portfolios on four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and the BAB factor of Frazzini and Pedersen (2014). The formation of the arbitrage portfolios is based on the extended PPCA framework of Kim et al. (2020) using a rolling window estimation of 12 months. Arbitrage returns are purely out-of-sample. Data cover the period between January 1968 and December 2018.

ARMs and anomalous returns is quite stable over time.

4.7 Momentum Relation

The fact that the pure-alpha portfolios are exposed to the momentum risk relates to recent results in the literature. Much work has been done investigating the relationship between momentum returns and tail risk. More specifically, some studies investigate momentum crashes and propose methods to avoid them. Barroso and Santa-Clara (2015) propose a volatility-managed approach to solving the problem of extreme drawdowns and excess kurtosis related to the momentum strategy. Daniel and Moskowitz (2016) propose an alternative approach that maximizes the Sharpe ratio based on predicting both risk and return of the momentum strategy.

Min and Kim (2016) investigate the momentum strategy in relation to the economic states. They find that the strategy performs poorly when the marginal utility of wealth is the highest captured by the expectation of the market risk premium. They conclude that the momentum premium is substantially related to the downside risk. Atilgan et al. (2020) report the presence of left-tail momentum that is characterized by the continuation of extreme left-tail events of stocks that experienced such events in the past. Unlike their results, my pure-alpha portfolios that invest in stocks with high systematic left-tail risk report economically intuitive positive returns and positive exposure to the momentum factor.

Although the pure-alpha portfolios are significantly exposed to the momentum factor, they do not possess such extreme behavior. During the investigated period, momentum possesses a negative skewness of -1.35. On the other hand, the lowest value of skewness that a pure-alpha portfolio yields is -0.12, obtained from the IPCA model with four latent factors. On top of that, unlike the distribution of the momentum returns that exhibit highly leptokurtic features with a value of kurtosis equal to 10.92, the pure alpha portfolio attains a value of 6.59 at the highest.

Kelly et al. (2021) investigate momentum in relation to the IPCA model. They conclude that the momentum premium is explainable since it significantly proxies for the exposure to the common factors. Even though the original set of IPCA factors can erase the abnormal returns of the momentum factor, pure-alpha portfolios cannot be explainable by this set of factors.

I investigate consequences of including the momentum factor into the ARM-IPCA model. By doing this, I can infer whether the ARMs proxy for the exposure to the momentum factor. The answer to this question may help to

Table 4.23: *Coefficients from the model that includes momentum.*

	coskew	cokurt	beta_down	down_corr	htcr	beta_tr	coentropy	cos_pred	beta_neg	mcrash	ciq_down
Γ_α	-5.17	-1.83	-8.51	0.81	0.06	3.69	-2.80	-0.58	9.70	1.13	4.48
	(-8.11)	(-2.20)	(-6.78)	(0.61)	(0.09)	(6.99)	(-2.10)	(-0.86)	(9.44)	(2.22)	(8.77)
Γ_δ	-0.05	0.28	-0.19	0.15	0.38	-0.09	-0.14	-0.57	-0.67	0.04	-0.11
	(-2.18)	(9.04)	(-4.06)	(2.98)	(15.57)	(-4.58)	(-2.72)	(-23.94)	(-18.36)	(2.41)	(-6.04)

Note: The table reports coefficients estimated in-sample of the model in which ARMs explain anomaly alphas (Γ_α) and betas with respect to the momentum factor (Γ_δ). Below the coefficients, I include HAC t-statistics of Newey and West (1987) with six lags. Model is estimated using OLS. Data cover the period between January 1968 and December 2018.

better understand why the pure-alpha portfolios are partly diminished by the momentum factor. I also investigate pure-alpha portfolios that uses not only ARMs but also momentum characteristic to see its effect on the returns.

4.7.1 Momentum Factor

In this section, I augment the IPCA model to not only contain latent factors, but also include the momentum factor. The model changes to

$$\begin{aligned} r_{i,t+1} &= \alpha_{i,t} + \beta_{i,t} f_{t+1} + \delta_{i,t} g_{t+1} + \epsilon_{i,t+1}, \\ \delta_{i,t} &= z'_{i,t}\Gamma_\delta + \nu_{\delta,i,t} \end{aligned} \tag{4.20}$$

where g_{t+1} is the momentum factor, Γ_δ is the mapping from ARMs to loadings on the momentum factor, and the rest follows the same specification as model 4.8. I investigate how ARMs relate to the exposures to the momentum factor.

First, I present in-sample results of a model that does not include any latent factors. The model is in the form

$$r_{i,t+1} = z'_{i,t}\Gamma_\alpha + z'_{i,t}\Gamma_\delta g_{t+1} + \epsilon_{i,t+1} \tag{4.21}$$

Because the factor g_{t+1} is observable, I can estimate Γ_α and Γ_δ using OLS by setting the right-hand variables to $z_{i,t}$ and $g_{i,t} \otimes z_{i,t}$. By inspecting the estimate of Γ_δ, we can see how ARMs explain the exposures into the momentum factor. Table 4.23 summarizes the result. We can see that many of the variables proxy significantly not only for the abnormal returns but also for the exposures to the momentum factor. The highest significance of the explanatory power possesses the predicted coskewness.

Out-of-sample results with one to six latent factors of the Model 4.20 summarizes Table 4.24. We can see that when the ARMs are allowed to explain the exposures to the momentum factor, the corresponding pure-alpha portfo-

Table 4.24: *Pure-alpha portfolio returns with the momentum factor.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	7.20	2.26	0.36	0.31	3.67	74.30	-32.84	28.88
2	9.28	2.77	0.46	1.05	5.40	47.94	-21.50	39.17
3	4.58	1.64	0.23	0.33	4.71	49.12	-34.36	29.27
4	3.85	1.46	0.19	0.07	4.40	66.11	-35.66	28.05
5	8.41	3.09	0.42	-0.19	6.26	44.60	-40.07	28.08
6	4.72	1.69	0.24	0.77	4.12	52.72	-19.42	27.96

Note: The table contains out-of-sample results for the pure-alpha portfolios estimated using the ARM-IPCA model ranging between one and six latent factors. Model also includes momentum factor and ARMs are allowed to proxy for the corresponding exposure. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with a 60-month initial period. Data cover the period between January 1968 and December 2018.

Table 4.25: *Pure-alpha portfolio returns with momentum characteristic.*

K factors	Mean	t-stat	SR	Skewness	Kurtosis	Maximum drawdown	Worst month	Best month
1	11.51	4.25	0.58	-1.44	11.59	55.51	-48.58	22.36
2	14.88	4.83	0.74	-0.49	7.81	58.79	-42.96	25.29
3	9.98	3.04	0.50	0.55	4.38	64.70	-25.04	30.12
4	7.65	2.52	0.38	0.29	3.59	68.88	-23.70	27.50
5	5.01	1.61	0.25	0.25	4.18	80.58	-29.16	29.03
6	6.84	2.30	0.34	0.05	3.40	50.96	-30.18	24.13

Note: The table contains out-of-sample results for the pure-alpha portfolios estimated using the ARM-IPCA model ranging between one and six latent factors. Model also includes momentum characteristic for each stock as an instrument. It reports annualized mean, corresponding HAC t-statistics of Newey and West (1987) with six lags, Sharpe ratio (SR), skewness, kurtosis, the maximum drawdown, and best- and worst-month returns. Values are in percentages. I use expanding window estimation with a 60-month initial period. Data cover the period between January 1968 and December 2018.

lios are noticeably lower than what can be achieved without controlling for the exposure. This partly explains why the pure-alpha portfolios from the previous section are related to the momentum factor.

4.7.2 Momentum Characteristic

Next, I include the momentum characteristic into the ARM-IPCA model. I investigate whether the momentum characteristic can alter the latent factor structure of the model and diminish the significance of the pure-alpha portfolios. Results in Table 4.25 are very similar to the results obtained from the models that do not include momentum characteristic. This observation suggests that the pure-alpha portfolios are not noticeably affected by the inclusion of the momentum characteristic into the set of instrumental variables.

4.8 Conclusion

I investigate asymmetric risk measures that capture the non-linear systematic behavior of stock returns. I present an approach to combining their pricing information to form corresponding portfolios. This approach is based on the instrumented principal component analysis that enables to model anomaly intercepts and factor loadings as functions of stock characteristics. I use asymmetric risk measures as these characteristics and let the model decide whether they yield risk premium because they proxy for the exposure to the common factor structure or because they constitute anomalies in terms of latent linear pricing factors. I form so-called pure-alpha portfolios based on the estimated relation between asymmetric risk measures and anomaly returns.

I show that these portfolios enjoy abnormal returns without being subsumed by exposures to other common sources of risk. Various factor pricing models, such as the model of Fama and French (1993), Carhart (1997), Fama and French (2015), Hou et al. (2014), Hou et al. (2020), or Kelly et al. (2019), cannot explain the premium. These results hold strong even after accounting for exposure to CIV shocks of Herskovic et al. (2016) or the BAB factor of Frazzini and Pedersen (2014). Thus, I show that some of the asymmetric risk measures can be successfully exploited as alphas. I also investigate features of the pure-alpha portfolios concerning various robustness checks, such as split samples and the exclusion of penny stocks, among others. These results have significant implications for fund managers who try outperforming benchmarks and are evaluated based on exposures to various factor models. Investing that utilizes the pricing information of ARMs can significantly improve the manager's risk-adjusted returns.

On the other hand, I show that various alternative approaches to combing the pricing information of the asymmetric risk measures do not yield good results. I employ portfolio formations based on ordinary least square regression, ridge regression, and lasso regression predictions. These approaches yield insignificant results, regardless of the type of estimation window or sorting strategy. Furthermore, projected principal component analysis that allows for a time-varying nature of the relation between pricing and asymmetric risk measures cannot fully exploit the abnormal returns related to the asymmetric risk measures.

I also investigate how asymmetric risk measures relate to the joint factor structure. First, I report a strong relation between the asymmetric risk measures and the momentum factor. I show that the asymmetric risk measures significantly explain the exposure to this factor. Second, while controlling for previously researched characteristics, I demonstrate that some measures possess significant information explaining the stock return behavior concerning common latent sources of risk. This observation suggests that some of the researched measures can be employed to capture betas of the stocks better.

Further research may focus on generalizing the underlying factor model to allow for non-linearities. For example, by employing the autoencoder asset pricing model of Gu et al. (2021), we can relax the restriction of the linear relationship between asymmetric risk measures and factor loadings. Moreover, their framework also enables a non-linear approach to constructing the latent factor structure. In the presented results, I have shown that the linear factor structure in betas and factors cannot erase significant abnormal returns of the asymmetric risk measures with a reasonable number of latent factors. By allowing for non-linearities, we may better infer the dimensionality of pricing information regarding the asymmetric risk measures.

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Appendix

4.A Definitions of the ARMs

This Appendix provides a brief exposition of the estimation process of each of the asymmetric risk measures employed in the main text. For further details regarding the nuances of the related computations, consult the original papers.

I use two sources of data to compute the asymmetric risk measures. First, I use either daily or monthly data of stock returns from the CRSP database. Second, I use the value-weighted return of the CRSP stocks from Kenneth French's online library to approximate the overall market return.

Variables are estimated using moving windows of various lengths following the procedures proposed in their original papers. In the case of measures estimated from the daily stock returns, I use mostly a moving window of one year. I require at least 200 daily observations during the window to be included. I estimate measures based on monthly return data using a window of at least 60 months and demand at least 36 monthly observations.

The measures are estimated following the definition proposed in the literature. In some cases, I slightly change the requirements regarding the minimal history of stocks to be included in the analysis. This modification aims at the precision of the estimates as well as the broadest possible dataset.

Throughout the section, I use $r_{i,t}$ and $r_{i,t}^e$ to denote a raw and excess return of an asset i at time t, respectively. The raw and excess market return is denoted by f_t and f_t^e. Corresponding variables with a bar denote their time-series averages computed in a given window.

4.A.1 Coskewness

Coskewness (`coskew`) of Harvey and Siddique (2000) is estimated using daily excess returns and is defined as

$$CSK_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)^2}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)^2}\frac{1}{T}\sum_{t=1}^{T}(f_t^e - \bar{f}^e)^2}. \tag{22}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

4.A.2 Cokurtosis

Cokurtosis (`cokurt`) of Dittmar (2002) is estimated using daily data and is defined as

$$CKT_i = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)^3}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{i,t}^e - \bar{r}_i^e)^2}\frac{1}{T}\Big(\sum_{t=1}^{T}(f_t^e - \bar{f}^e)^2\Big)^{3/2}} \tag{23}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

4.A.3 Downside Beta

Downside (`beta_down`) beta of Ang et al. (2006) is estimated using daily data and is defined as

$$\beta_i^{DR} = \frac{\sum\limits_{f_t^e<\bar{f}^e}(r_{i,t}^e - \bar{r}_i^e)(f_t^e - \bar{f}^e)}{\sum\limits_{f_t^e<\bar{f}^e}(f_t^e - \bar{f}^e)^2}. \tag{24}$$

Estimation window is set to 1 year, at least 200 daily observations are required.

4.A.4 Downside Correlation

Downside correlation (`down_corr`) based on Hong et al. (2006) and Jiang et al. (2018) is estimated using daily data and is defined as

$$\mathbb{C}or_i^- = \mathbb{C}or(r_i, f|r_i < 0, f < 0) - \mathbb{C}or(r_i, f|r_i > 0, f > 0) \tag{25}$$

using empirical counterpart of the correlation. Minimum of 200 observations in the 1-year window is demanded.

4.A.5 Hybrid Tail Covariance Risk

Hybrid tail covariance risk (`htcr`) of Bali et al. (2014) is estimated using daily data using 6-month window with at least 80 daily observations as

$$HTCR_i = \sum_{r_{i,t}<h_i} (r_{i,t} - h_i)(f_t - h_f) \tag{26}$$

where h_i and h_f are the 10% empirical quantiles of stock and market return, respectively.

4.A.6 Tail Risk Beta

Tail risk beta (`beta_tr`) of Kelly and Jiang (2014) is estimated using monthly return data using 120-month window with requirement of at least 36 monthly observations. Beta is computed by means of least-square estimator from the predictive regression of the form

$$r_{i,t+1} = \mu_i + \beta_i^{TR}\lambda_t + \epsilon_{t+1,i} \tag{27}$$

where the tail risk factor is obtained as

$$\lambda_t = \frac{1}{K_t}\sum_{k=1}^{K_t} \ln\frac{e_{k,t}}{u_t} \tag{28}$$

where $e_{k,t}$ is the kth daily idiosyncratic return that falls below an extreme value threshold u_t during month t, and K_t is the total number of such exceedences within month t. Idiosyncratic return is computed relative to 3-factor model of Fama and French (1993), and the threshold value is taken to be 5% quantile from the monthly cross-section of daily returns.

4.A.7 Exceedance Coentropy

Exceedance coentropy (`coentropy`) measure based on Backus et al. (2018) and Jiang et al. (2018) using daily data and 1-year estimation window with at least 200 observations is based on

$$C^+(0, r_i, f) = \frac{L(r_i f) - L(r_i) - L(f)}{L(r_i) + L(f)}\bigg|(r_i > 0, y > 0) \tag{29}$$

$$C^-(0, r_i, f) = \frac{L(r_i f) - L(r_i) - L(f)}{L(r_i) + L(f)}\bigg|(r_i < 0, y < 0) \tag{30}$$

where $L(x) = \ln\mathbb{E}(x) - \mathbb{E}(\ln x)$. The measure is then defined as

$$Coentropy = C^{-}(0, r_i, f) - C^{+}(0, r_i, f). \tag{31}$$

4.A.8 Predicted Systematic Coskewness

Predicted systematic coskewness (`cos_pred`) of Langlois (2020) is based on

$$Cos_{i,t} = \mathbb{C}ov_{t-1}\Big(r_{i,t}, f_t^2\Big), \tag{32}$$

then, each month I run the panel regression using all available stocks and history of data

$$F\Big(Cos_{i,k-12\to k-1}\Big) = \kappa + F\Big(Y_{i,k-24\to k-13}\Big)\theta + F\Big(X_{i,k-13}\Big)\phi + \epsilon_{i,k-12\to k-1} \tag{33}$$

where $Cos_{i,k-12\to k-1}$ is the coskewness from Equation 32 computed using daily returns from month $k-12$ to month $k-1$, $Y_{i,k-24\to k-13}$ are risk measures (volatility, market beta, etc.) estimated using daily data from month $k-24$ to month $k-13$, and $X_{i,k-13}$ are characteristics (size, book-to-price, etc.) observed at the end of month $k-13$. The function $F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t+1}$ transforms the original variable into its normalized rank in the cross-section of variable x_t, which possess N_t observations.

The predicted systematic coskewness for each stock is then obtained using the estimated coefficients of $\hat{\kappa}, \hat{\theta}, \hat{\phi}$ as

$$F\Big(\widehat{Cos_{i,t\to t+11}}\Big) = \hat{\kappa} + F\Big(Y_{i,t-12\to t-1}\Big)\hat{\theta} + F\Big(X_{i,t-1}\Big)\hat{\phi}. \tag{34}$$

The choice of risk measures and characteristics employed in the prediction of systematic skewness follows closely Langlois (2020).

4.A.9 Semibeta

Negative semibeta (`beta_neg`) of Bollerslev et al. (2021) is estimated using daily data with 1-year moving window as

$$\beta_i^N = \frac{\sum\limits_{r_{i,t}<0, f_t<0} r_{i,t} f_{i,t}}{\sum\limits_t f_t^2} \tag{35}$$

with the requirement of at least 200 daily observations.

4.A.10 Multivariate Crash Risk

Multivariate crash risk (`mcrash`) of Chabi-Yo et al. (2022) is estimated using daily data with 1-year window and minimum of 200 observations in the following steps. First, for each stock separately, using stock and N factor returns, I estimate $N+1$ GARCH(1,1) models of Bollerslev (1986) to obtain a series of conditional distribution functions $F_{i,t}(h) = \mathbb{P}_{t-1}[r_{i,t} \leq h]$ and use it to compute probability integral transforms as $\hat{u}_{i,t} = F_{i,t}(r_{i,t})$. Second, I estimate MCRASH as

$$\text{MCRASH}_{i,t} = \frac{\sum_t \mathbb{I}(\{\hat{u}_{1,t} \leq p\}) \cdot \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})}{\sum_t \mathbb{I}(\cup_{j=2}^{N+1}\{\hat{u}_{j,t} \leq p\})} \tag{36}$$

where $\mathbb{I}$ denotes the indicator function and p is set to 0.05. I follow the baseline specification of Chabi-Yo et al. (2022) and use the five factors of Fama and French (2015), momentum factor of Carhart (1997) and betting-against-beta factor of Frazzini and Pedersen (2014).

4.A.11 Downside CIQ Beta

Downside common idiosyncratic quantile risk beta (`ciq_down`) of Barunik and Nevrla (2022) is estimated using monthly data with 60-month window and requirement of at least 48 observations as

$$\beta_i^{down} = \sum_{\tau \in \tau_{down}} F\Big(\beta_i(\tau)\Big) \tag{37}$$

which gives the average cross-sectional rank of the common idiosyncratic quantile (CIQ) betas for downside τ CIQ factors. CIQ betas are estimated from time-series regression of stock returns on the increments of CIQ factors. The CIQ factors are estimated using residuals from Fama and French (1993) factors and following the quantile factor model of Chen et al. (2021).

4.B Appendix B – ARM Portfolio Returns

Table 26: *Quintile portfolio sorts.*

Variable	Low	2	3	4	High	H-L	t-stat	α	t-stat
Panel A: All stocks									
Equal-weighted									
coskew	11.08	10.61	9.68	8.84	8.43	-2.65	-2.24	-1.69	-1.12
cokurt	11.69	10.44	9.84	8.90	7.78	-3.91	-2.40	-1.10	-0.63
beta_down	11.08	9.88	9.76	9.91	8.01	-3.07	-1.44	0.64	0.47
down_corr	8.73	9.51	10.14	9.96	10.31	1.57	1.98	0.99	0.97
htcr	9.99	9.00	9.98	9.93	9.75	-0.24	-0.12	-1.63	-0.74
beta_tr	8.20	9.07	9.44	10.48	11.47	3.27	2.36	3.10	1.47
coentropy	9.11	9.35	9.77	10.01	10.40	1.29	1.61	0.87	0.83
cos_pred	12.43	10.60	9.16	8.51	7.95	-4.48	-1.78	-4.69	-1.79
beta_neg	9.67	10.40	10.38	10.02	8.17	-1.50	-0.43	3.30	1.84
mcrash	10.03	9.84	9.47	10.00	9.91	-0.13	-0.13	0.18	0.19
ciq_down	7.16	9.51	10.22	10.22	11.54	4.38	2.98	5.51	3.67
Value-weighted									
coskew	6.93	7.57	7.66	6.18	4.47	-2.46	-1.60	1.39	0.71
cokurt	5.30	7.26	6.89	6.62	5.74	0.45	0.24	4.34	2.57
beta_down	5.92	7.05	6.69	6.34	5.18	-0.74	-0.27	1.51	0.70
down_corr	5.70	5.10	6.97	7.41	7.91	2.21	1.84	-1.35	-0.93
htcr	5.79	5.66	6.37	6.57	5.92	0.13	0.06	1.10	0.65
beta_tr	4.34	5.98	7.11	7.72	8.88	4.54	2.59	5.85	2.58
coentropy	4.73	6.05	6.62	7.25	7.71	2.98	2.18	-0.64	-0.42
cos_pred	11.66	8.56	8.10	6.43	5.57	-6.09	-2.31	-3.42	-1.44
beta_neg	7.06	6.56	6.60	5.94	2.85	-4.21	-1.17	-0.65	-0.31
mcrash	4.99	7.04	6.64	6.02	6.42	1.43	1.05	-0.07	-0.04
ciq_down	5.18	5.68	7.07	7.07	8.08	2.90	1.52	4.27	2.49
Pabel B: No penny stocks									
Equal-weighted									
coskew	9.30	9.06	8.83	7.98	6.59	-2.71	-2.24	-0.84	-0.56
cokurt	8.07	9.08	8.53	8.49	7.58	-0.48	-0.34	2.36	1.76
beta_down	8.26	8.64	9.09	9.14	6.62	-1.63	-0.66	0.96	0.69
down_corr	6.83	7.97	8.97	8.73	9.25	2.42	2.76	0.48	0.51
htcr	5.65	8.12	8.90	9.89	9.19	3.54	2.82	3.24	2.72
beta_tr	6.10	8.38	8.26	9.26	9.75	3.64	2.62	1.70	1.07
coentropy	7.12	8.07	8.94	8.77	8.85	1.73	1.98	0.05	0.05
cos_pred	9.68	8.58	8.16	7.75	7.59	-2.09	-0.94	-1.03	-0.56
beta_neg	8.82	9.38	9.39	9.03	5.15	-3.67	-1.21	-0.21	-0.16
mcrash	7.47	7.75	8.64	8.62	9.13	1.66	1.71	1.49	1.54
ciq_down	5.31	8.85	9.09	8.91	9.59	4.28	2.64	5.17	3.71
Value-weighted									
coskew	6.68	6.99	7.42	7.14	4.23	-2.44	-1.64	1.25	0.70
cokurt	5.93	6.73	5.97	7.16	5.53	-0.40	-0.25	3.51	2.31
beta_down	6.01	7.09	7.02	5.57	5.31	-0.71	-0.27	1.30	0.69
down_corr	5.49	5.31	6.69	7.47	7.72	2.23	1.92	-1.37	-0.96
htcr	4.92	6.42	6.82	6.11	6.00	1.09	0.71	1.89	1.20
beta_tr	4.86	6.24	6.48	7.48	8.21	3.36	2.10	3.61	1.77
coentropy	4.99	5.99	6.34	7.40	7.43	2.44	1.93	-1.13	-0.80
cos_pred	9.76	7.75	7.03	5.47	5.86	-3.90	-1.65	-0.66	-0.31
beta_neg	6.52	6.54	6.69	5.42	3.60	-2.92	-0.92	0.37	0.20
mcrash	5.98	6.20	6.32	5.81	6.34	0.35	0.28	-1.00	-0.63
ciq_down	4.74	5.66	6.45	6.96	7.67	2.93	1.54	3.58	2.34

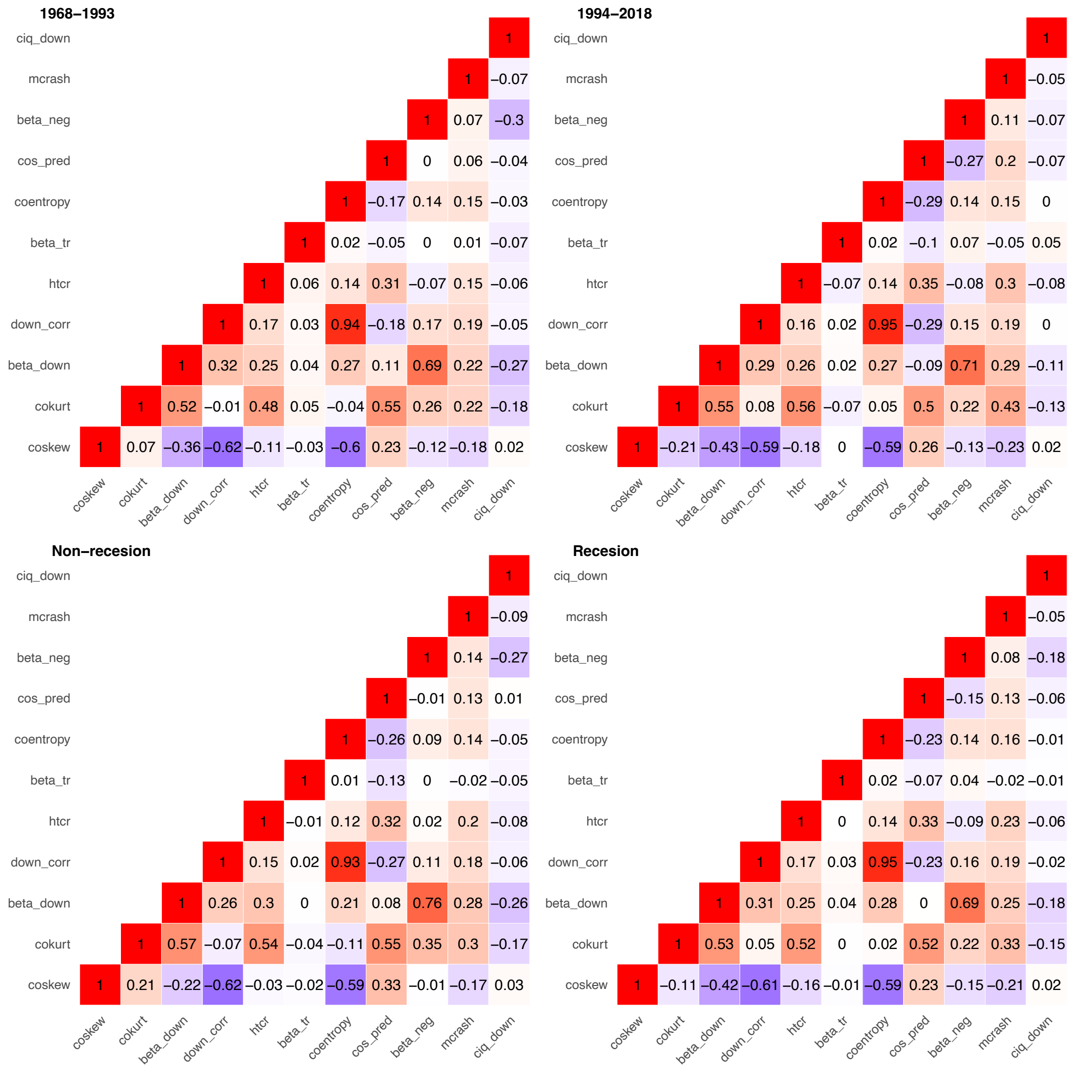
Note: The table contains annualized out-of-sample returns of five monthly rebalanced portfolios sorted on various asymmetric risk measures. It also reports returns of the high minus low (H - L) portfolios, HAC t-statistics of Newey and West (1987) with six lags, and annualized six-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Panel A reports results using all stocks. Panel B excludes stocks with a price less than $5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

Table 27: *Decile portfolio sorts.*

Variable	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	α	t-stat
Panel A: All stocks														
Equal-weighted														
coskew	11.46	10.70	10.58	10.64	10.08	9.29	9.21	8.47	9.40	7.46	-4.00	-2.86	-2.62	-1.44
cokurt	12.34	11.04	10.69	10.19	9.60	10.08	8.97	8.82	8.33	7.23	-5.11	-2.65	-2.14	-1.06
beta_down	11.82	10.35	9.85	9.91	10.08	9.44	9.78	10.04	8.79	7.24	-4.58	-1.79	0.17	0.11
down_corr	9.26	8.21	9.49	9.52	9.46	10.81	9.65	10.27	10.45	10.16	0.90	0.91	0.25	0.20
htcr	10.90	9.07	9.25	8.76	9.96	9.99	9.51	10.35	10.10	9.40	-1.51	-0.61	-3.06	-1.13
beta_tr	8.40	7.99	9.03	9.10	9.69	9.18	10.17	10.79	11.40	11.53	3.13	1.71	3.25	1.19
coentropy	9.56	8.66	9.40	9.29	10.00	9.54	10.46	9.56	10.92	9.89	0.33	0.35	-0.24	-0.19
cos_pred	13.10	11.75	11.17	10.04	8.95	9.37	8.22	8.80	8.31	7.59	-5.52	-1.77	-5.71	-1.80
beta_neg	9.18	10.15	10.27	10.53	10.03	10.74	10.54	9.51	8.90	7.44	-1.74	-0.41	4.30	1.84
mcrash	9.86	8.71	10.48	9.85	8.22	9.38	11.47	9.77	9.06	10.55	0.68	0.53	0.61	0.50
ciq_down	6.33	7.99	9.03	10.00	9.90	10.55	10.03	10.40	11.51	11.57	5.24	2.97	5.71	3.14
Value-weighted														
coskew	8.06	6.67	7.66	7.66	8.21	6.97	6.31	6.20	5.75	2.78	-5.28	-2.92	-0.93	-0.39
cokurt	6.94	4.52	8.07	7.03	6.33	7.36	6.32	6.80	7.04	5.35	-1.59	-0.72	3.44	1.65
beta_down	6.75	6.37	7.02	7.23	6.96	6.36	6.20	6.85	5.83	4.42	-2.33	-0.64	0.23	0.08
down_corr	5.01	6.02	5.42	4.86	6.05	8.34	7.16	7.83	8.74	6.73	1.72	1.13	-2.67	-1.80
htcr	5.94	5.47	6.20	5.35	6.39	6.39	6.18	6.89	7.20	5.08	-0.86	-0.31	0.35	0.15
beta_tr	5.01	3.94	5.50	6.56	7.10	7.32	8.03	7.55	8.66	8.75	3.75	1.63	4.43	1.54
coentropy	4.60	4.95	5.86	6.29	6.07	7.04	7.13	7.66	8.44	6.83	2.23	1.41	-2.33	-1.40
cos_pred	12.52	11.04	9.85	7.50	8.67	7.85	7.15	6.14	5.24	5.81	-6.70	-2.05	-4.53	-1.44
beta_neg	7.03	7.59	7.04	6.34	6.46	6.92	6.01	6.02	4.67	-0.62	-7.65	-1.77	-4.11	-1.58
mcrash	5.00	4.62	9.77	5.85	6.65	6.32	5.74	7.07	6.22	7.02	2.02	0.97	-0.28	-0.12
ciq_down	3.05	6.65	5.36	5.85	6.04	7.90	6.50	7.68	7.84	9.75	6.70	2.55	7.60	2.94
Panel B: No penny stocks														
Equal-weighted														
coskew	9.51	9.09	9.25	8.87	8.78	8.87	8.25	7.71	7.38	5.80	-3.72	-2.52	-1.32	-0.70
cokurt	8.07	8.07	8.65	9.51	8.71	8.34	8.66	8.33	8.27	6.90	-1.16	-0.67	2.11	1.32
beta_down	7.97	8.55	8.26	9.01	9.35	8.82	8.82	9.47	7.78	5.46	-2.50	-0.80	0.73	0.42
down_corr	6.64	7.02	7.38	8.55	8.15	9.78	8.64	8.83	9.29	9.22	2.57	2.28	0.27	0.23
htcr	4.74	6.57	7.98	8.25	9.12	8.68	9.10	10.68	9.46	8.93	4.18	2.73	3.98	2.65
beta_tr	4.73	7.48	8.05	8.72	8.32	8.19	9.22	9.30	9.39	10.10	5.37	3.04	3.52	1.71
coentropy	7.10	7.14	7.68	8.46	8.49	9.38	8.90	8.64	8.75	8.95	1.85	1.64	-0.16	-0.14
cos_pred	10.47	8.88	8.48	8.68	8.16	8.15	8.18	7.32	8.42	6.76	-3.71	-1.36	-2.61	-1.14
beta_neg	9.01	8.62	9.44	9.32	9.10	9.67	9.69	8.37	7.78	2.52	-6.50	-1.74	-2.14	-1.31
mcrash	7.35	7.06	9.28	8.12	7.68	12.10	7.38	12.21	8.37	9.30	1.95	1.62	1.60	1.36
ciq_down	4.24	6.38	8.86	8.85	8.84	9.34	9.42	8.40	9.40	9.77	5.53	2.77	5.91	3.27
Value-weighted														
coskew	6.85	6.99	6.99	7.34	6.59	8.00	7.29	7.00	4.36	4.36	-2.49	-1.37	2.47	1.15
cokurt	6.35	5.56	6.88	6.71	6.46	5.62	7.49	7.03	6.12	5.25	-1.10	-0.56	2.87	1.64
beta_down	5.41	6.88	6.38	7.54	6.66	7.40	6.24	5.52	6.13	4.29	-1.12	-0.32	1.51	0.62
down_corr	5.94	5.20	5.29	5.21	6.01	7.48	7.93	6.92	8.42	6.88	0.94	0.66	-3.22	-2.16
htcr	4.14	5.49	6.15	6.72	6.29	7.05	6.28	6.36	7.12	4.98	0.84	0.39	2.11	1.14
beta_tr	4.02	5.23	5.32	7.17	6.69	6.50	7.13	7.77	8.37	8.95	4.94	2.39	5.21	1.99
coentropy	5.43	4.89	5.65	6.26	5.59	7.03	7.81	7.47	8.10	6.69	1.25	0.79	-3.11	-1.84
cos_pred	11.30	9.00	6.74	8.29	7.43	6.62	6.30	4.86	5.93	5.88	-5.43	-1.92	-2.24	-0.79
beta_neg	7.40	6.11	6.86	6.47	6.97	6.52	6.35	4.97	4.34	1.93	-5.46	-1.31	-2.17	-0.90
mcrash	4.94	7.09	6.40	7.63	6.04	9.35	3.51	9.83	6.12	6.48	1.54	0.91	-0.50	-0.25
ciq_down	3.20	5.75	5.85	5.79	6.88	6.57	7.16	7.07	7.33	8.70	5.50	2.37	5.09	2.44

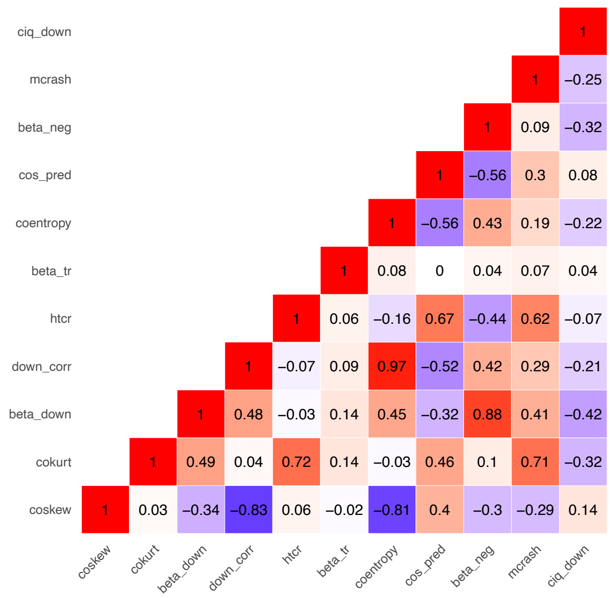
Note: The table contains annualized out-of-sample returns of ten monthly rebalanced portfolios sorted on various asymmetric risk measures. It also reports returns of the high minus low (H - L) portfolios, HAC t-statistics of Newey and West (1987) with six lags, and annualized six-factor alphas and their t-statistics with respect to the four factors of Carhart (1997), CIV shocks of Herskovic et al. (2016), and BAB factor of Frazzini and Pedersen (2014). Panel A reports results using all stocks. Panel B excludes stocks with a price less than \$5 or market cap below 10% quantile of NYSE stocks. Data cover the period between January 1968 and December 2018.

Figure 6: *Correlation structure across ARMs over different periods.*



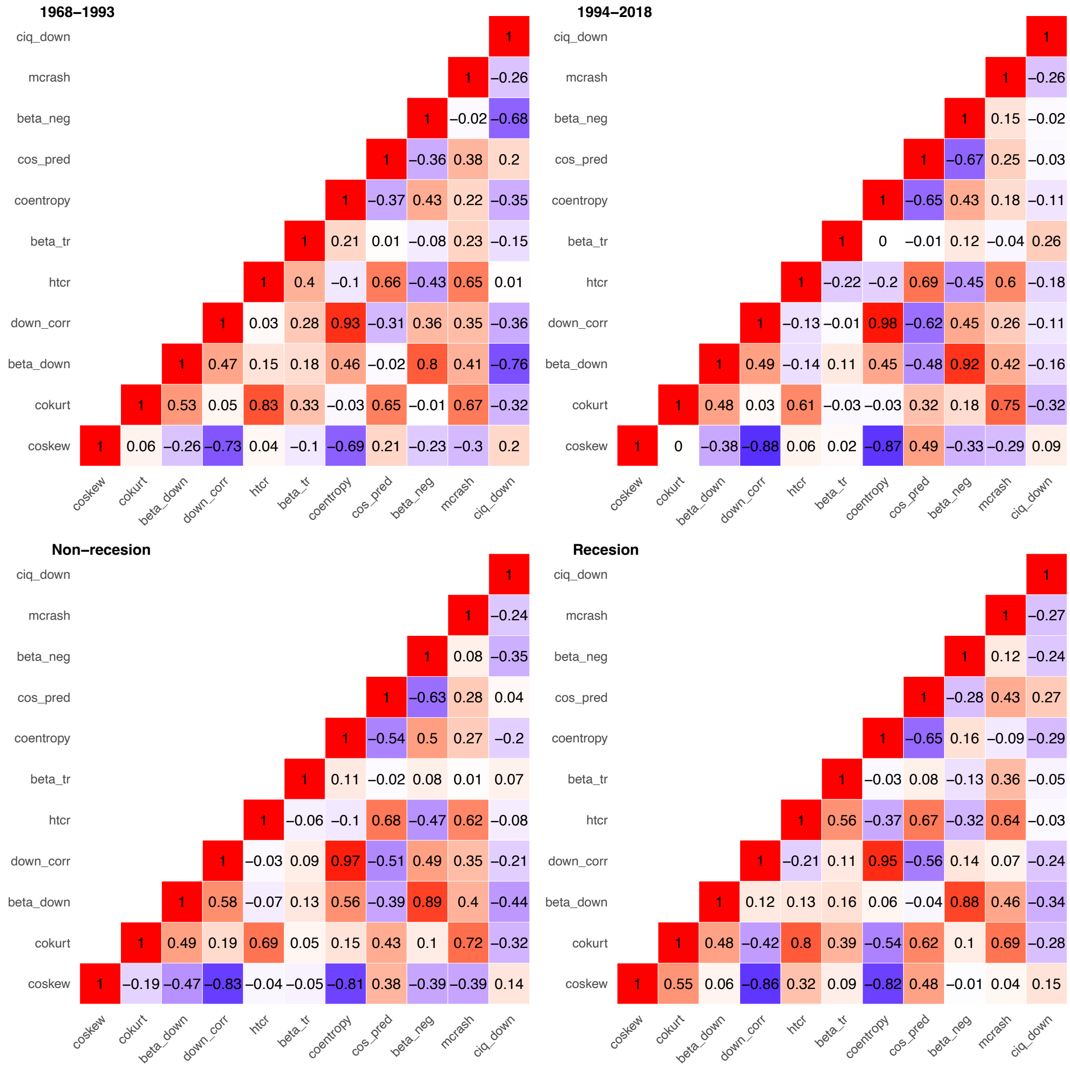
Note: The figure captures time-series averages of cross-sectional correlations between asymmetric risk measures during distinct time periods and recession and non-recession times as defined by NBER. Data include the period between January 1968 and December 2018.

Figure 7: *Correlation structure across ARM-managed portfolios.*



Note: The figure captures the time-series correlations between managed portfolios sorted on various asymmetric risk measures. Data cover the period between January 1968 and December 2018.

Figure 8: *Correlation structure across ARM managed portfolios over different periods.*



Note: The figure captures time-series averages of cross-sectional correlations between managed portfolio returns sorted on asymmetric risk measures. Data include the period between January 1968 and December 2018.

4.C Appendix: IPCA Estimation Results

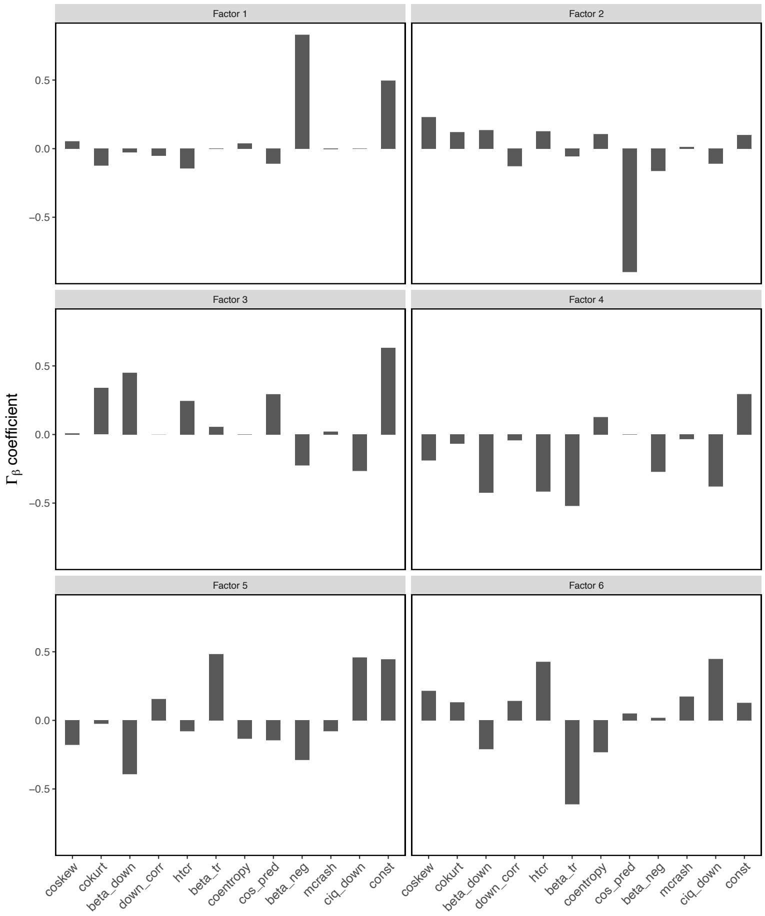
This Appendix provides some estimation results of the ARM-IPCA models.

Table 28: *Out-of-sample ARM-IPCA results using all stocks and split samples.*

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: Period 1/1968-12/1993									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	19.09	20.43	20.87	21.30	21.51	21.61	21.72	21.81
	$\Gamma_\alpha \neq 0$	18.99	20.42	20.87	21.29	21.51	21.60	21.71	21.80
Predictive R^2	$\Gamma_\alpha = 0$	0.11	0.10	0.11	0.15	0.16	0.16	0.16	0.16
	$\Gamma_\alpha \neq 0$	0.16	0.17	0.16	0.17	0.17	0.17	0.17	0.16
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	97.49	98.99	99.43	99.74	99.82	99.86	99.91	99.94
	$\Gamma_\alpha \neq 0$	96.87	98.76	99.39	99.68	99.79	99.84	99.90	99.93
Predictive R^2	$\Gamma_\alpha = 0$	0.59	0.57	0.60	0.68	0.69	0.70	0.70	0.69
	$\Gamma_\alpha \neq 0$	0.68	0.69	0.69	0.70	0.70	0.70	0.70	0.70
Tangency portfolios									
Mean		8.34	3.62	13.45	21.59	21.28	21.43	24.19	23.68
t-stat		1.85	0.83	2.92	4.15	3.90	4.37	5.31	5.07
Sharpe		0.42	0.18	0.67	1.08	1.06	1.07	1.21	1.18
Pure-alpha portfolios									
Mean		16.89	25.17	20.85	8.98	9.99	9.12	3.63	-1.54
t-stat		3.87	5.83	4.72	2.31	2.54	2.26	0.78	-0.38
Sharpe		0.84	1.26	1.04	0.45	0.50	0.46	0.18	-0.08
Panel B: Period 1/1994-12/2018									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	14.71	16.06	16.81	17.49	17.73	17.86	17.98	18.08
	$\Gamma_\alpha \neq 0$	14.72	15.94	16.62	17.48	17.72	17.86	17.98	18.08
Predictive R^2	$\Gamma_\alpha = 0$	0.19	0.19	0.19	0.25	0.24	0.25	0.25	0.25
	$\Gamma_\alpha \neq 0$	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.24
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	94.47	97.52	98.79	99.54	99.69	99.79	99.86	99.90
	$\Gamma_\alpha \neq 0$	94.24	96.29	97.27	99.51	99.66	99.76	99.85	99.89
Predictive R^2	$\Gamma_\alpha = 0$	2.25	2.21	2.22	2.24	2.24	2.25	2.24	2.24
	$\Gamma_\alpha \neq 0$	2.26	2.25	2.24	2.24	2.23	2.24	2.24	2.24
Tangency portfolios									
Mean		9.72	9.65	13.06	21.65	22.42	23.47	21.82	23.28
t-stat		2.00	2.17	2.27	3.64	3.70	3.69	3.45	3.79
Sharpe		0.49	0.48	0.65	1.08	1.12	1.17	1.09	1.16
Pure-alpha portfolios									
Mean		13.38	15.61	17.70	11.71	10.72	9.30	2.56	0.40
t-stat		2.67	3.01	3.74	2.49	2.23	1.93	0.59	0.09
Sharpe		0.67	0.78	0.88	0.59	0.54	0.47	0.13	0.02

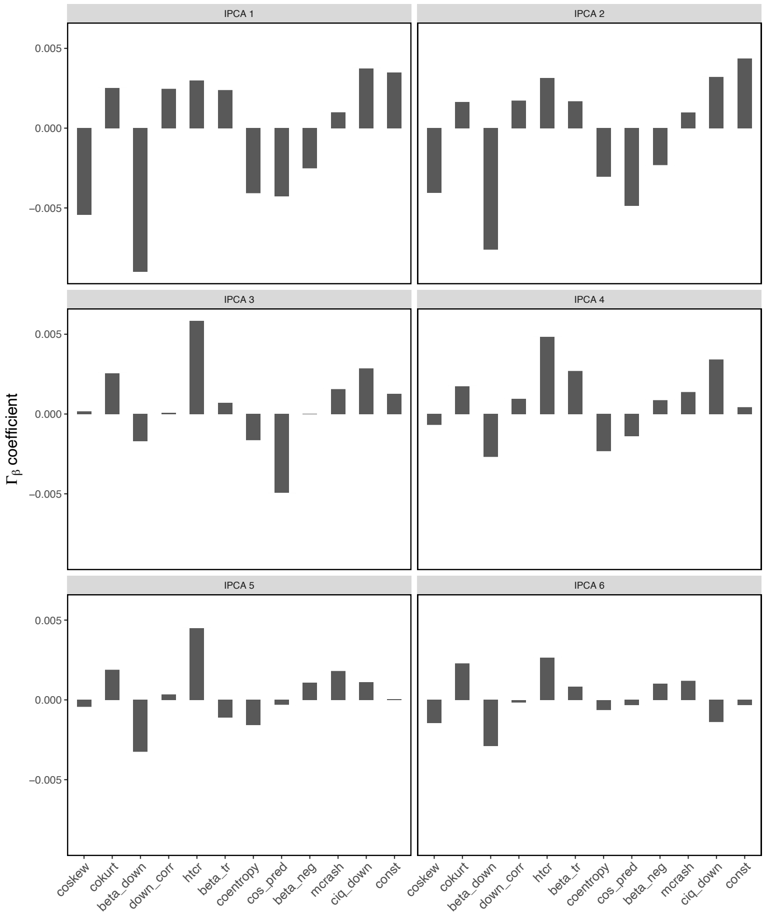
Note: The table reports out-of-sample results of the IPCA models with varying numbers of latent factors and using ARMs as the instruments. Models are estimated with an expanding window and a 60-month initial period. Tangency portfolios are based on the restricted IPCA model, the pure-alpha portfolios are based on the unrestricted model. I include all available stocks. The first period covers the interval between January 1968 and December 1993, and the second spans January 1994 and December 2018.

Figure 9: *Factor loadings of the restricted ARM-IPCA(6) model.*



Note: The figure reports columns of the estimated Γ_β IPCA matrix with six latent factors and ARMs as instruments. Results are based on the in-sample analysis. Data cover the period between January 1968 and December 2018.

Figure 10: *Alphas of the ARM-IPCA models.*



Note: The figure reports estimated Γ_α vectors for unrestricted IPCA models with numbers of latent factors between one and six and ARMs as instruments. Results are based on the in-sample analysis. Data cover the period between January 1968 and December 2018.

Table 29: *All-IPCA results.*

		IPCA(K)							
		1	2	3	4	5	6	7	8
Panel A: All stocks									
Individual stocks									
Total R^2	$\Gamma_\alpha = 0$	16.54	18.28	19.46	20.13	20.66	21.01	21.28	21.48
	$\Gamma_\alpha \neq 0$	16.94	18.65	19.77	20.42	20.79	21.05	21.32	21.51
Predictive R^2	$\Gamma_\alpha = 0$	0.35	0.35	0.41	0.42	0.65	0.67	0.66	0.67
	$\Gamma_\alpha \neq 0$	0.73	0.72	0.71	0.71	0.70	0.70	0.69	0.69
Managed portfolios									
Total R^2	$\Gamma_\alpha = 0$	89.35	94.89	96.74	97.95	98.29	98.77	99.07	99.22
	$\Gamma_\alpha \neq 0$	89.90	95.29	96.89	98.08	98.57	98.79	99.10	99.24
Predictive R^2	$\Gamma_\alpha = 0$	1.61	1.63	1.77	1.82	2.02	2.03	2.02	2.04
	$\Gamma_\alpha \neq 0$	2.21	2.15	2.13	2.12	2.10	2.08	2.07	2.07
Asset pricing test									
W_α p-value		0.10	0.00	0.00	0.00	3.90	71.80	27.70	61.90

Note: The table reports in-sample estimation results of the IPCA models with varying numbers of latent factors and using 11 ARMs and 32 characteristics from Kelly et al. (2019) as the instruments. The asset pricing test reports p-values of the null hypothesis that $\Gamma_\alpha = 0$. Data cover the period between January 1968 and December 2018.

Chapter 5

Conclusion

The thesis comprises three main chapters that contribute to the empirical asset pricing literature by introducing new approaches to assessing risks that traditional models do not encompass. The chapters demonstrate that non-linear and horizon-specific dimensions of risks capture important features of risk priced across assets.

Chapter 2 demonstrates that quantifying risk solely through contemporaneous dependence averaged over the entire joint distribution of asset and risk factors omits crucial information about risk. Additionally, we report that the short-term component of market tail risk is priced consistently across all asset classes. On the other hand, horizon-specific components of extreme volatility risk are priced heterogeneously. This fact yields significant implications for investors who wish to avoid certain risks and exploit others given their risk tastes.

Chapter 3 introduces a factor pricing model that captures common movements of cross-sectional quantiles instead of focusing on factors that explain common average movements of stock returns. By identifying quantile-specific factors, we conclude that only those capturing downside risk have significant implications for asset prices. We show that the downside common idiosyncratic quantile factors robustly predict the market return in the out-of-sample setting. This observation provides a good option for investors to time their market portfolio strategy to obtain significant gains. Moreover, we show that exposure to these downside factors is robustly priced in the cross-section of stock returns. We also provide possibilities for aggregating information across factors for various quantiles. We conclude that the reported premium cannot be fully attributed to any other previously discovered return anomaly.

Chapter 4 discusses eleven systematic asymmetric risk measures that relax the assumption of expected utility maximizing representative investor or the assumption that some tradable factors can linearly approximate the stochastic discount factor. An efficient investment strategy based on instrumented principal component analysis provides a Sharpe ratio that is more than twice as large as the best strategy based on any systematic asymmetric risk measure. In addition, the pure-alpha portfolio returns do not display any extreme behavior, such as fat tails or negative skewness. Furthermore, none of the investigated factor pricing models fully account for the resulting premium.

Asset prices exhibit complex behavior. This thesis aims to improve understanding of their dynamics by using non-standard tools to measure risk. The results may help investors identify previously hidden risks and either avoid or embrace them to enjoy related premiums. Each chapter has the potential for extensions. Future research could explore theoretical models that encompass the presented risk-return tradeoffs or empirically investigate sources of horizon-specific and non-linear risks.

Chapter 6

Responses to Opponents' Reports

I am very grateful to all three referees for taking the time to review the thesis. Their discussions, suggestions, and points raised certainly have improved the thesis. Below, I summarize my responses to each of their questions and suggestions.

6.1 Dr. Mattia Bevilacqua

6.1.1 General Comments

- **Motivation and linkage:** *"The introduction could be enhanced by a discussion about the main motivation behind the chosen topic in general and of how the candidate developed these three exemplary ideas. Some "glue" between the chapters would strengthen the overall discussion and contribution of the thesis. The author mentioned that a unifying thread in the presented research is the understanding of various deviations from traditional asset pricing models that impose simplifying assumptions that are not met in the data. Expanding upon this statement would help the reader to draw clearer connections between each chapter."*
  Answer: Based on this suggestion, I have significantly expanded the introductory chapter. I motivate the analyses presented by linking them to the shortcomings of traditional asset pricing models. In addition, I provide general unifying ideas that link all three chapters together.

- **Literature review:** *"The candidate states that the thesis focuses on asset pricing questions, with a special emphasis on non-linear risks. Specifically, the first paper focuses on the link between stock return and a (down-*

side) risk factor. The second paper investigates the pricing implications of a quantile factor model applied to the dynamics of the cross-sectional quantiles of stock returns, that is relating asset returns to exposures to common movements that affect all assets in the market. The last paper regards systematic asymmetric risk measures and their relation to the linear factor models, applying dimension reduction techniques from the machine learning toolbox. Including a concise and brief general literature review section that presents key articles from each of these areas would be highly beneficial. Some of the main literature strands to include span the relationship between stock returns and (downside) risk, tail risks and asymmetric risk measures, the use of machine learning in asset pricing, common risk and common factors in asset pricing, characteristics and (non-linear) factors, etc. This suggestion aims to elaborate and compile what the author briefly mentions in each chapter. This section could either be integrated into the introduction or presented as a standalone section."

Answer: I have incorporated this comment by adding a concise literature review to the introduction chapter. I focus there on significant recent developments in the related strands of literature.

- **Conclusion:** *"Given that the conclusions of the individual chapters are relatively brief, the author could consider adding a final Conclusion chapter. This chapter could concisely summarize the results of the papers, discuss how they interconnect, and briefly propose potential direction for further research. For instance, building on the insights from the last chapter, an possible extension could involve exploring the ARM-IPCA results when the relationship between ARMs and stock returns is conditional on whether the market (or stocks variance/tail risk) is below or above its mean. Some implications of the thesis findings for portfolio and risk managers can also be briefly discussed in this section."*

  Answer: I have extended the conclusions of Chapters 3 and 4 by including suggestions for future research. In Chapter 4, I also provide a more detailed discussion regarding the main results. Furthermore, I have added a conclusion chapter summarizing the main ideas and results of the thesis.

6.1.2 Comments regarding Chapter 4

1. *"A brief explanation to interpret the varying patterns of significance in the coefficients of measures when comparing the results from using the entire universe of stocks against the results obtained by excluding penny stocks, as shown in Tables 4.2 and 4.3 can be added. Are some of these measures more sensitive to penny stocks/volatile returns? This discussion could encompass also subsection 4.3.5."*

   Answer: In Subsection 4.2.4, I have included a brief explanation of the varying significances of the ARMs with relation to their risk premia across all- and no-penny-stock datasets. The main observation is that many ARMs have higher and more significant risk premiums in the no-penny dataset, which may be caused by higher estimation uncertainty related to ARMs across less liquid penny stocks. This explanation may also be applied to high-volatility stocks.

2. *"How do we interpret htcr's coefficients compared to the others in Table 4.2?"*

   Answer: The magnitude of this coefficient is affected by the fact that the hybrid tail covariance risk (`htcr`) is defined by conditional covariance and is not standardized by any measure of variance (unlike, e.g., various versions of downsisde beta). Thus, the average value of `htcr` is smaller than other measures, so the corresponding regression coefficient is larger.

3. *"Any relevant reference you are following for the managed portfolio scheme in subsection 4.2.4?"*

   Answer: Kelly et al. (2019) proposed the managed portfolio weighting scheme, as it naturally emerges in interpreting the IPCA estimation procedure.

4. *"A few lines/footnote to discuss some robustness checks to validate that the naive combination approach is not affected by different initial (moving) periods or lasso/ridge parameters estimation."*

   Answer: I have added/extended short footnotes that state that these results are not significantly altered by changing the settings employed in the presented research.

5. *"The subsequent implementation of the ARM-IPCA, along with all the validating tests, particularly in relation to momentum and other factors,*

is, in my opinion, robust and convincing. Consequently, this chapter could represent a highly significant contribution to the literature on ARMs and asset pricing."

Answer: Thank you.

6.1.3 Minor Comments

- *"Thinking of treating the papers as chapters by moving all the references, some repeating, and appendices at the end of the thesis. This approach could streamline the thesis and improve its readability and coherence. Please disregard this point if the candidates' guidelines for thesis writing state otherwise."*

  Answer: I have decided to keep the chapter-specific lists of references and appendices in the thesis as it is a standard among theses defended at our Institution.

- *"The author has detailed the publication status and journal submissions of some of the chapters. It would also be beneficial to include information about the conferences, workshops, and seminar presentations in parts of the thesis. This is particularly relevant given the author's proactive engagement in presenting this work at prestigious international events, as I previously mentioned."*

  Answer: For each chapter, I have included a list of conferences, workshops, and seminars where the work was presented and discussed.

6.2 Prof. Jeroen Rombouts

6.2.1 The First Paper

1. *"To compute t-test statistics and interpret them, we need some asymptotic normality. Hence, don't we need some existence of moments of the underlying process?"*

   Answer: I believe that we do not need to assume the normality of the underlying processes when interpreting t-statistics in the cross-sectional regressions. We only need an assumption of normality regarding the error term in the second-stage regression. This is a standard assumption that is exploited when evaluating asset pricing models and does not directly affect the robustness related to the QS betas.

Because we use the estimates of the quantile spectral betas over the whole period, the need to correct t-statistics for the fact that the quantile spectral betas are estimated becomes also of less importance.

2. *"The section 2.4.5. Size of the 2-Stage Estimation Procedure is better called "Finite sample size properties of the testing approach" or something like that."*

   Answer: I agree and have changed the section's title and corresponding table accordingly.

3. *"Regarding the empirical application with 70 years of data, is the stationarity assumption you make a reasonable one? I would like to see more motivation for this. Is there empirical work that also considers higher frequency data?"*

   Answer: I agree that the stationarity assumption regarding the history of 70 years for single stock returns is strong. Because we focus on extreme risks over investment horizons, it is difficult to estimate the QS risks accurately with shorter data samples. To show that the results are not specifically driven by the choice of 70 years, we also include results for 50- and 60-year histories of the stocks. These choices do not significantly alter the results.

   Moreover, the statistical features of the stock returns are related to their characteristics, such as size, book-to-market, investment, or other. We investigate the pricing of the QS risks among portfolios sorted on these characteristics. The stationarity assumption among these portfolios is a much weaker one. Thus, the combination of single-stock and portfolio results provides a comprehensive look at the pricing of the QS risks.

   Regarding the results using higher frequency data, at this moment, I do not possess any significant relevant outcomes. However, I may investigate the QS risks in this direction in the future.

4. *"I would not call the other models "competing" models, since they are just less general as yours."*

   Answer: I fully agree, and based on that, I have changed the wording from "competing" to "related".

6.2.2 The Second Paper

1. *"I didn't see the link with Bollerslev et. al (2009) VRP results. More generally, the previous chapter focused on different horizons. This chapter seems to ignore this, while in general return predictability changes over horizons but not necessarily in a decreasing way as shown in the Bollerslev paper. In the latter paper, they find no predictability at short horizons, some predictability at medium term horizons, and again less predictability at longer horizons (i.e. inverse U-shaped pattern)."*
   Answer: I agree that there is no clear theoretical link between CIQ factors and VRP. This fact is particularly true because the VRP is related to the aggregate S&P 500 composite index (rather than to the value-weighted return of all CRSP stocks), which it significantly predicts only over a medium-term horizon. We have included it in the investigation based on a reference report that suggested it. Moreover, the observation that the VRP and CIQ factors share predictive information for the market return may lead to establishing a relationship between these two phenomena in the future.

   A clarifying footnote has been added to the main text to explain this rationale.

2. *"The conclusion should ideally conclude some directions for future research, similarly as what is done in the first chapter."*
   Answer: As suggested, I have included a paragraph of suggestions for future research in the Conclusion subsection.

6.2.3 The Third Paper

1. *"My question stationarity question is recurrent throughout the different chapters, and here I wonder how Figure 4.1 correlation structure across ARMs differs over time, and business cycles."*
   Answer: I plot the correlations between *values* of asymmetric risk measures over distinct times in Figure 6. The first row reports these correlations over two separate periods – the figure on the left captures the years between 1968 and 1993, and the figure on the right period covers 1994 to 2018. No clear pattern would suggest that this splitting should lead to a different conclusion than the conclusion obtained over the whole period. Moreover, in the second row, I plot these correlations separately for

non-recession and recession periods as identified by NBER. The results do not suggest a significant difference between these two regimes.

To further investigate the stability of the dependence structure, in Figure 8, I plot the correlation between the *returns* of the managed portfolios corresponding to the asymmetric risk measures. The first row shows that the correlations exhibit relative stability across two time periods with one noticeable exception – the managed portfolio corresponding to the downside common idiosyncratic quantile beta (`ciq_down`) became much less correlated with other portfolios in the second period. The second row reports similar results for recession and non-recession splits.

Overall, these results support the value of combining the asymmetric risk measures into an investment strategy, as correlations of these strategies do not peak during the bad times. This fact contributes to the statistical features of the pure-alpha portfolios that do not exhibit heavy tails and negative skewness. Moreover, the stability of the dependence structure is evidenced by the fact that the investment strategies work better with expanding-window estimation.

2. *"When Kelly et al (2019) is used, I missed the motivation for the bootstrap. Is it because of sample size or complex limiting distributions? It seems to me that the same procedure is reexplained on page 161."*
   Answer: Using the bootstrap to draw the inference is mainly driven by its robustness features. I exploit the fact that bootstrap enjoys favorable statistical properties in finite samples. Furthermore, I can perform statistical testing without making strong distributional assumptions regarding the model residuals. I have included this reasoning in the main text to make it clear.

   I perform two major bootstrap tests. The first one tests whether there is a significant improvement in the model fit if we use the unrestricted version of the model compared to the restricted version. The second one tests whether a specific variable significantly improves the fit of the restricted model if we include it in the model. These procedures share a similar rationale, but I have decided to include a thorough explanation of both for completeness.

3. *"The sections contain long lists of subsections which are all very interesting but it is a lot of information to digest. Is it possible to focus first on*

the key findings?"

Answer: I have reorganized the chapter so that the sections on robustness checks, the time-varying nature of risk compensation, and the relation to the momentum factor follow after the two main sections regarding pure-alpha portfolios and the relation between asymmetric risk measures and latent factor structure.

4. *"The conclusion should explain in more details the main findings of the paper. It also ideally conclude some directions for future research, similarly as what is done in the first chapter."*

   Answer: I have extended the Conclusion to provide more details regarding the results obtained. I have also proposed a potential trajectory for generalizing the ARM-IPCA model there.

Furthermore, I have adjusted the tables across the thesis so that the parentheses and number of decimal places are consistent throughout each chapter. I also tried to erase typos that emerged in the text.

6.3 Dr. Deniz Erdemlioglu

There were no additional questions or comments raised that I should incorporate in the final version of the thesis.