

## THE OPPONENT REPORT ON THE DISSERTATION THESIS OF DR. DALIMIL PEŠA

This is to express a strong endorsement of the Ph.D. thesis submitted by Dr. Dalimil Peša at the Faculty of Mathematics and Physics at Charles University.

The thesis contains results of six papers either authored or co-authored by Dalimil Peša. It consists of a brief introduction which describes the main results, the motivation for the research and the approach taken towards the questions tackled, and the six said papers in form of attachments, and an addendum to one of the papers. Two of these papers already appeared in respectable international journals, and four are submitted for publication (further three also appeared on arXiv).

### THE DETAILED OVERVIEW OF CONTENTS OF THE THESIS

The papers are divided into three sections, namely Function spaces in the abstract, Lorentz–Karamata spaces, and Applications of function spaces. Each of the three sections contains two papers.

In Paper I, written jointly with Aleš Nekvinda, the authors concentrate on certain specific questions concerning quasi-Banach spaces. Basic functional properties of such spaces are studied, including separability, completeness, the Riesz–Fischer property, and also relations to other such spaces. The (single-authored) Paper I-I provides an extensive treatise of the so-called Wiener–Luxemburg amalgam spaces. This concept extends the classical idea of Wiener amalgam spaces to the rearrangement-invariant world. The basic philosophy of an amalgam space is to provide separate description of the behavior of a function near zero and near infinity, motivated by applications for example to the signal transmission. Properties of these spaces are studied such as normability, duality properties, the Hardy–Littlewood–Pólya principle, and more.

A common feature of Papers III and IV, gathered in the next section, is the involvement of slowly-varying functions, a concept which was introduced in classical works of Karamata in the 1930s. Paper III offers a detailed study of the so-called Lorentz–Karamata spaces, which appeared around 2000 in connection with optimal Sobolev embeddings. These spaces had been studied before, but in all cases under some (unnecessary) restrictions. The paper by D. Peša brings a comprehensive array of results concerning these spaces including their several characterizations, associate spaces, fundamental functions, positioning on the fundamental level, relations to other spaces, and more. Paper IV then focuses on the question of smoothness of the slowly-varying functions which generate the Lorentz–Karamata spaces.

In Paper V, boundedness of Copson type integral operators involving kernels is studied on rearrangement-invariant spaces. This research is motivated by the well-known indispensable role of such operators in the so-called reduction principles which turned out to be extremely handy in the study of optimal Sobolev embeddings. The author solves the notorious question whether one can restrict oneself

to the cone of monotone functions. The result had been partly known before, but only for spaces of a finite measure.

Finally, paper VI, written jointly with A. Kalamajska and T. Roskovec, establishes some interesting relations, mostly in integral form, for elliptic differential operators. The result is motivated by the investigation of eigenvalue problems with nonlinearities.

#### THE RECOMMENDATION

The results contained in the papers that constitute the core of the dissertation thesis of Dr. Dalimil Peša show that the author is an extraordinarily talented young mathematician who can produce deep results either on his own or as a member of an international scientific team. His results bring a number of new interesting new pieces of information, and they substantially broaden the existing knowledge in the field. Moreover, his approaches to open problems as well as the proofs of the principal theorems are quite innovative. The thesis was prepared with care and it is written in a reader-friendly way.

I am convinced that the thesis of Dr Peša surpasses the requirements for a Ph.D. thesis and I strongly recommend that the Ph.D. title is awarded to him.

prof. Angela Alberico (Consiglio Nazionale delle Ricerche, Napoli)