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OPPONENT'S REPORT ON THE DISSERTATION THESIS OF MS. MANVI GROVER

1. Overview of the contents of the thesis

The dissertation thesis of Ms. Manvi Grover (Charles University, Prague), supervised by doent. B. Opic, contains results of three papers co-authored by Ms.Grover. The thesis itself consists of the three said papers and a brief introduction which describes the main results of the thesis, the motivation for the research and the approach taken towards the questions tackled. The first two papers have already appeared in good international journals and the third one is also to appear..The questions studied in the thesis fall into the area of the theory interpolation theory, which is an independent subdiscipline of functional analysis. Itw started in the late 1920s and early 1930s mainly through advances of Riesz and Marcinkiewicz and whose importance was fully understood later in the 1950s and 1960s thanks to classical works of Aronszajn, Gagliardo, Calderón, Lions, Peetre and others. The research in the theory of interpolation thrived ever since, classical monographs (by Krejn– Petunin–Semenov, Bergh–Löfström, Bennett–Sharpley, Brudny-Krugljak) appeared, and even today the topic constitutes a rapidly developing discipline with wide array of applications.

The thesis focuses on several specific questions concerning properties of spaces obtained by a limiting method of real interpolation involving weight functions of slowly-varying type involved, and also properties of operators acting on these spaces. The central notion is the space $(X0, X1)_{0,q,b,K}$, in which $(X0, X1)$ denote the classical Calderón compatible couple, the '0' indicates the limiting state of the method, b is the slowly-varying function (a concept which had been studied in the classical works of Karamata in 1930s), and 'K' (which can be replaced by 'J') stands for the Peetre's K-method (or J-method) of real interpolation. It is worth to be pronounced that my own method with interpolation with a parameter function of course do not cover such a limiting case.

The first paper contains two principal results. Firstly, it is shown that the K- and J-spaces typically have equivalent norms, and hence the resulting spaces coincide in the set-theoretical sense. Secondly, a density theorem was established, showing that, under appropriate circumstances, the space $X0 \cap X1$ is dense in $(X0, X1)_{0,q,b,K}$.

The second paper concentrates on compactness of operators acting on spaces of type $(X0, X1)_{0,q,b,K}$, and also on the lack of such compactness. Compactness is of course one of the most important features an operator can have, but in many important cases, especially in various critical states, the appropriate operators lack this property. In this situation, a useful quantitative tool called the ball measure of noncompactness, introduced in 1930 by Kuratowski, can become handy. In the second paper questions concerning the ball of noncompactness of operators acting on limiting interpolation spaces with weights of slowly-varying type are investigated. The main results give conditions under which this measure of noncompactness is zero.

The third paper focuses on duality properties (more precisely on the description of the associate spaces in the sense of Köthe) of the spaces (X0, X1)0,q,b,K. The main results of this paper provide description of such spaces in terms of the associate spaces of separate spaces X0 and X1.

2. Conclusion

The results contained in the dissertation thesis of Ms. Grover and in the attached papers show that the applicant is a talented and hardworking young mathematician which is capable to both collaborate with real experts on the subject and do own contributions. Obviously, she is asking herself sensible mathematical questions and come up with solutions of some rather difficult and interesting open mathematical problems. The content of the thesis brings a number of new interesting knowledge, the proofs are correct adequately innovative. The thesis is well written in a reader-friendly way. It is also quite clear that the thesis was very carefully supervised.

I judge that the thesis of Ms. Grover fulfils the requirements for a Ph.D. thesis and I recommend that the Ph.D. title is awarded to her.

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