

Posudek diplomové práce

Matematicko-fyzikální fakulta Univerzity Karlovy

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Thesis title Distance magic labelings

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Program Computer Science **Specialization** Discrete Models and Algorithms

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Review text:

A *distance magic labeling* of an n -vertex graph is a bijective assignment of integers from 1 to n to its vertices so that the sum of numbers at neighbourhood of each vertex is constant. Motivated by application for design of incomplete tournaments, it has been widely studied which graphs admit distance magic labeling and its variants. Previously, a notion of *neighbour balance* was introduced to fully answer this question on hypercubes with means of elementary linear algebra. Since then, several similar results were obtained for other hypercube-like graphs or for generalized D -distance magic labelings. Typically, these graphs admit these labelings if and only if their dimension meets some divisibility condition.

At the beginning, the student was given an idea (on a 5-page draft) how these results could be unified with a general notion of neighbour balance that could be applied for all Cayley graphs on \mathbb{Z}_2^n . Essentially, to find a distance magic labeling of $\text{Cay}(\mathbb{Z}_2^n, S)$ it then suffices to consider the generating set S as columns of a matrix S and to find an invertible binary $n \times n$ matrix M such that MS (over \mathbb{Z}_2) has a balanced number of 1's in each row. This has been successfully developed in Chapter 2, along with an algorithm to find such a matrix M and a pairing method to count the contributions of columns of S in the product MS .

This general framework allows to elegantly reprove several recent results in Chapter 3: a characterization when a hypercube admits a d -distance magic labeling for odd d (Proposition 7), an existence of D -distance magic labelings of hypercubes (Proposition 10), a characterization when a folded cube admits a neighbour balanced distance magic labeling (Proposition 11), an existence of distance magic labelings of halved cubes (Proposition 12) and halved folded cubes (Theorem 13). Chapter 4 then considers disconnected Cayley graphs of \mathbb{Z}_2^n and shows that finding a component-wise distance magic labeling in this case can be reduced to a single component by applying a prefix matrix obtained from a base of the orthogonal space to the space generated by S . This is illustrated on fixed halved cubes.

The thesis is well-written and pleasant to read. It took the student many thorough editing rounds to get the grasp on formal academic writing. Overall, the thesis has fully achieved its goals although there is still some potential for further new results. Hopefully, it will result in a publication.

I recommend the thesis for defense.

I suggest to not consider the thesis for the annual award.

27th of August, 2023

Signature: