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To whom it may concern:

This is a report on the PhD thesis by **Matěj Konečný**.

The thesis consists of a sequence of papers preceded by an extended general introduction to the topics covered by the papers. The thesis lies in the area of combinatorics that concentrates on issues around Ramsey theory. Two themes in it relate directly to Ramsey theory—big Ramsey degrees and structural Ramsey theorems; one theme relates to Ramsey theory through proof methods—extension property for partial automorphisms (EPPA). Additionally, the thesis contains a chapter connecting combinatorics with certain structural properties of Polish groups.

The thesis is part of a discourse, whose perimeter was established about a quarter-century ago after the papers of Nešetřil, Rödl, Hrushovski, Herwig, and Lascar had been published. Some important and difficult technical issues have remained unresolved by this pioneering work. They revolve around dealing with particular cases not covered by the general theory, extending the theory to infinite structures, incorporating infinite (as opposed to finite) languages, etc. The papers in the thesis contribute to these investigations.

The thesis contains two remarkably general theorems. The first one is on the EPPA theme and is contained in the paper *All those EPPA classes* (Chapter 4). The theorem allows one to construct a finite EPPA witness over whose structure one exerts considerable control. This special witness is constructed from an arbitrary finite EPPA witness. This is a very nice result. On the theoretical level, it isolates the notions with which the structure of EPPA witnesses can be controlled; on the more practical level, it deals with disparate particular cases. The second general theorem is contained in the paper *Big Ramsey degrees in infinite languages* (Chapter 9). Finiteness of big Ramsey degrees is proved there for a broad class of structures over certain infinite languages. It is likely that this theorem will form the background to much of the subsequent research on this topics.

Further, the thesis deals with some specific classes of structures both in the context of EPPA (Chapters 5 and 6) and Ramsey-type theorems (Chapter 7, 8, and 10–12). Of those, I would like to single out the very interesting theorem on the extension property for switching automorphisms of finite graphs in Chapter 5. The theorem tackles a

natural special situation that cannot be handled by general methods. Additionally, the theorem leads to an intriguing question on the coherent version of EPPA for switching automorphisms.

The research presented in the thesis gives rise to a number of good questions that, I am sure, will guide future research. I find the questions related to coherent EPPA particularly appealing, for example, Questions 2.0.5 and 4.1.7.

One should add that the thesis is based on a collection of papers, many of which are authored by large groups of people. However, as, aside from his advisor Hubička, Konečný is the only author present on all these papers, I am convinced that his contributions were substantial, perhaps even crucial to the projects.

**Conclusion:** The results of the thesis lie in a rapidly developing area of mathematics. Matěj Konečný's work, as exposed in his PhD thesis, contributes in a real way to this development. The thesis warrants granting him a PhD degree, which I am happy to recommend.

Sincerely yours,

Sławomir Solecki