This thesis is concerned with combinatorial properties of homogeneous structures such as the Ramsey property, big Ramsey degrees, EPPA, and others. What these properties have in common is that, while being finitary problems on classes of finite structures, they are equivalent to various dynamical properties of automorphism groups of the corresponding homogeneous structures. This thesis consists of an extended introduction to these areas, a list of open problems, and ten papers of which the author is a co-author, seven of which have been published at the time of writing this thesis, the other three have been submitted. The goal is to demonstrate that, at least on the combinatorial side of things, there are many interplays of these properties which can be (and have been) exploited to further each of the areas.