

Abstract: This thesis summarises the author's results in representation theory of rings and schemes, obtained with several collaborators. First, we show that for a quasicompact semiseparated scheme X , the derived category of very flat quasicoherent sheaves is equivalent to the derived category of flat quasicoherent sheaves, and if X is affine, this is further equivalent to the homotopy category of projectives. Next, we prove that if R is a commutative Noetherian ring, then every countably generated flat module is *quite flat*, i.e., a direct summand of a transfinite extension of localizations of R in countable multiplicative subsets. Further, we investigate the relations between the geometric and categorical purity in categories of sheaves; we give a characterization of indecomposable geometric pure-injectives in both the quasicoherent and non-quasicoherent case. In particular, we describe the Ziegler spectrum and its geometric part for the category of quasicoherent sheaves on the projective line over a field. The final result is the equivalence of the following statements for a quasicompact quasiseparated scheme X : (1) the category $\mathrm{QCoh}(X)$ of all quasicoherent sheaves on X has a flat generator; (2) for every injective object \mathcal{E} of $\mathrm{QCoh}(X)$, the internal Hom functor into \mathcal{E} is exact; (3) for some injective cogenerator \mathcal{E} of $\mathrm{QCoh}(X)$, the internal Hom functor into \mathcal{E} is exact; (4) the scheme X is semiseparated.