Report on the PhD thesis

Stochastic Equations with Corellated Noise and Their Applications

by Ondřej Týbl

The doctoral thesis of Ondřej Týbl is based on two papers published by him and his coauthors in very good journals. It is a very nice contribution to the theory of the stochastic differential equations with non-continuous noise, in particular the Lévy-driven SDE.

The main contribution of the thesis consists in three chapters. The first chapter is an introduction to Lévy processes, stochastic integration with respect to Lévy processes and Lévy-driven stochastic differential equations. The chapter ends with the definitions of an invariant measure and definition of boundedness in probability in the mean. These properties of a solution of SDE are studied in the next two chapters.

The authors own contribution is summarized in the second and third chapter. In particular, the second chapter presents sufficient conditions for the stabilization properties of the solution of Lévy-driven SDE, namely the boundedness in probability in the mean, and existence of an invariant measure. These sufficient conditions are relatively mild, on the other hand there are several interesting combinations of restrictions on the coefficients of the SDE that must be fullfiled to ensure the desired result. The main results on the boundedness in probability in the mean are summarized in Theorems 2.1.1, 2.2.1, 2.3.1, and 2.3.2. All results are illustrated in several examples. The existence of an invariant measure is studied in Section 2.4, where the existence is shown to be a direct consequence of the previous results under additional assumptions on the coefficients. The third chapter is focused on stochastic approximation procedure. The main result (Theorem 3.1.1) gives conditions under which any solution of the Lévy-driven SDE converges to a constant almost surely.

All the results are based on Lévy-driven SDE, hence the noise is considered stationary and, moreover to be a semimartingale. A natural question is whether the long-term behaviour of the solution of the considered SDE might be "robust" to a violation of the stationarity. By this I mean, e.g., a noise with the Poisson process intensity depending (moderately) on the time variable t. Another natural question is the speed of the convergence to a constant proved in Theorem 3.1.1. Such results might be very interesting for a statistical analysis of the considered SDE.

The thesis is very well written. All assumption are explained and illustrated in examples, the proofs are detailed and there are several remarks that put the results into the context of stochastic analysis.

The presented thesis is based on new and original results published by the author and his coauthors. Ondřej Týbl thus demonstrates his ability to conduct a research work of high quality and high level. Beyond any doubt, the thesis meets the expectations set for the dissertation theses for the study programme Probability and Mathematical Statistics and I recommend its defense at the Faculty of Mathematics and Physics of Charles University.

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