Review on the Bachelor thesis: "Free Boundary Problems" handed in by Terézia Ferková.

Content. The thesis is about the classical free boundary problem known as the one-phase Bernoulli problem. Even so very classical a satisfying theory has only been developed in the 80's and was at that time one of the most important and most challenging contributions in the analysis of PDE. The very sophisticated analysis developed there has been most influential on the research afterwards and the methods are very relevant for nowadays research. An important feature of the solution is its support region which is the free boundary. Terezia focussed on the global regularity of the solution. For that she introduces archetypical examples namely half-plane solutions and radial solutions. While to show that for radial solutions the equations are satisfied is computing basic ODE solutions to show that the constructed solutions are actual minimizers is mores sophisticated and in paritulcar depends on the Polya-Szego inequality, a rather delicate estimate. In a second chapter the existence of solutions by the direct method in the calculus of variations is performed. The last chapter shows that all solutions are globally Lipschitz continuous. This neatly closes the circle, as the examples do provide exactly solutions, which have linear growth at the free boundary. Hence the Lipschitz regularity is sharp.

Challenge of the topic. The subject was and is very technical, even for a person who is well accomplished with calculus of variations and the analysis for Sobolev spaces. In my opinion the reason, why it is so hard to study is due to the fact that the proofs even so on they are at the utmost technical level, they permanently rely on geometrical intuition. Certainly the here used lecture notes by Velichkov try to make the subject as accessible as possible. However the notes are far from building on the knowledge of a Bachelor student.

Quality of the presentation. Terezia fought her path through the notes learning a lot of Sobolev theory and theory from the calculus of variation, which allowed her to accomplish the task to a great extend. The resulting presentation is then as I was expecting it. Some parts are worked out in great detail others are more superficial, with inaccuracies and relying on citations. I think the result is quite remarkable and acknowledgeable for a Bachelor student.

Conclusion. I definitely recommend to accept the thesis with a good grade. In my opinion the subject treated here could as well be a Master thesis or alternatively put one of the three scientific chapters could already make a decent Bachelor thesis. The Terezia managed to replicate three pieces of rather independent theory to a large extend makes some inaccuracies more then forgiveable. I believe Terezia has learned a huge amount of an important part of theory, as she demonstrated well by the presented thesis.

Prague 3.9.2023 **Sebastian Schwarzacher.**