Opponent's oppinion of the bachelor's thesis

Title: Free boundary problems
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Summary of the work's content

The work deals with the study of the classic free boundary problem

$$\begin{cases} \Delta u = 0 & \text{in } \{u > 0\} \\ u = g & \text{in } \partial\Omega \\ |Du| = \sqrt{\Lambda} & \text{in } \partial\{u > 0\} \cap \Omega \end{cases}$$

as well as part of its corresponding regularity theory. This problem was famously studied by Alt and Caffarelli, as well as many others, but the main source are the respective notes by Bozhidar Velichkov (see the thesis for citations). Specifically, the thesis shows existence of solutions to this problem as well as their Lipschitz-regularity.

Overall evaluation of the work

Work topic

The topic is relatively advanced for a bachelor's thesis, using methods from PDE and calculus of variations which are usually only taught during later stages of education. As such the main task was to understand these methods using secondary sources and to make the primary result understandable to a more general reader. This task has been achieved.

Originality

Given by the nature of the task, the general outline of the thesis is mostly following that of the primary source. Nevertheless the author was able to contribute to this by providing additional details and explaining some of the more advanced mathematical techniques. However, while it is a matter of personal style, I believe that there could have been more exposition and elaboration of these techniques instead of following only the mathematics.

Level of mathematics

The mathematics in the thesis is sound. Its presentation however is plagued by minor inconsistencies and errors (see the comments for examples). While some of these can be explained by the difficulty of the topic and intricacies which are easy to miss for a non-expert, other errors are sadly a bit more obvious. In general the thesis could have benefited greatly from more proofreading.

Working with resources

The thesis stays relatively close to its sources. However these sources are always referenced correctly and in particular the proofs are often rewritten with additional details and in the authors own words.

Some comments and questions

The following are some examples of the type of issues that were found when reading the thesis. None of them are major, but all of them would have been avoidable with more care during the preparation of the thesis.

1. For some reason Lemma 7 is Young's inequality for convolutions, not that for products used later in the text.

2. There are some minor technical issues in Lemma 9: The statement unnecessarily assumes C^2-minimizers, yet the primary feature of the problem is a jump of the derivative, so minimizers are generally not even C^1. Additionally $\{u + \varepsilon\varphi > 0\} = \{u > 0\}$ for small ε small enough only holds if $\varphi \in C_c(\Omega_c)$, not for generic $u \in H_0^1(\Omega_c)$, so a density argument is required.

3. The case distinction in Lemma 24 is missing the trivial case of $x_0 \notin \Omega_u$ and would have benefited from some more detailed discussion of how the different statements interact.

4. The notation in Lemma 26 is rather confusing, since t is both used for a scalar and for a vector. This is compounded by the issue that variables are often not fully declared. Similar sudden changes in notation are sadly common in this thesis.

Conclusion

In conclusion while the work could still be greatly improved, it should be kept in mind that it was covering a very advanced topic, far beyond the level normally expected of a Bachelor-student. In recognition of these facts, while there are some issues, I recommend to recognize it as a bachelor's thesis.

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