In this thesis, we study the structural and algorithmic properties of graphs embedded or represented in surfaces and with constraints on their faces or cycles.

We derive tools to quantify properties of embedded flows and use these to design an algorithm to decide the extendability of a precoloring of the boundary cycles of near-quadrangulations of the cylinder. We then develop methods to reduce 3-coloring of triangle-free graphs embedded in the torus to 3-coloring near-quadrangulations, obtaining practical algorithms for deciding 3-colorability in linear time, and obtaining a 3-coloring in quadratic time.

We also investigate connection between geometric graph representations and the induced odd cycle packing number (iocp) parameter. We show that wide variety of representable graphs exhibit limited iocp and show that graphs with limited iocp are χ-bounded. We derive an EPTAS for maximum independent set of graphs with limited iocp and linear independence number, as well as QPTAS assuming only limited iocp.