Evaluation of a Master Thesis by Supervisor

Anna Mlezivová: Representation theory of gentle algebras

The thesis focuses on a special class of algebras over a field, the so-called gentle algebras. These are not necessarily finite dimensional quotients of path algebras of finite quivers modulo quadratic monomial relations, where both the quiver and the relations are required to satisfy certain conditions. The categories of finite dimensional modules over such algebras are tame, so classification problems are in principle tractable.

The starting point of the thesis is a recent preprint by Chan and Demonet (arXiv:2009.10266) which translates the classification of torsion pairs in the category of finite dimensional modules over a gentle algebra to a (difficult) combinatorial problem on the quiver with relations. In the thesis, the author explains this result in the first of the three sections and illustrates it on examples in the remaining two sections. Section 2 treats "easy" examples: path algebras of linearly oriented Dynkin quivers of type A and the Kronecker algebra.

The main contribution of the author resides in the last long and technical section which is devoted to the classification of torsion pairs for the Markov algebra. This is an explicit gentle algebra studied to some extent by Chan in an unpublished manuscript and for which, intuitively, the combinatorics related to torsion pairs should be controlled by curves on a punctured torus. The author classifies infinite walks in the Markov quiver relevant for the classification of torsion pairs (so-called non-self-crossing infinite strings) and relates them to specific non-self-crossing curves on the punctured torus using techniques close to the theory of continued fractions. All the proofs in this part are original and involve quite heavy combinatorics.

In my opinion, this is an excellent thesis making advance in understanding of more complex structures in module categories of gentle algebras such as torsion pairs. On the example of Markov algebra, it clearly reveals various non-obvious subtleties of the behavior of non-self-crossing infinite strings. This is an extremely helpful starting point if one wishes to understand torsion pairs in terms of some geometric model (and in particular if one wishes to promote the above-mentioned intuition about curves on a punctured torus to a rigorous result), similarly as one understands individual finite dimensional modules thanks to Baur and Coelho Simões and their the paper A geometric model for the module category of a gentle algebra, Int. Math. Res. Not. IMRN (2021), no. 15, 11357–11392.

My only comment about thesis is that the final parts (especially Sections 3.1.3 and 3.1.4) are rather terse and so not very easy to read. When

eventually the work is going to be prepared for publication (and I am convinced it should be at some point), the presentation should be expanded and improved.

In conclusion, I **recommend** the thesis for defence and the suggested grading will be communicated to the committee.

Prague, August 30, 2023 doc. RNDr. Jan Šťovíček, Ph.D.