Evaluation of a Master Thesis by Supervisor

Dominik Krasula: The Gabriel-Roiter measure in representation theory

The thesis focuses on the so-called Gabriel-Roiter measure, which is a concept distilled by Gabriel from Roiter's proof of the first Brauer-Thrall conjecture for finite dimensional algebras. In the most basic form, it is a method to assign to any object M in an abelian length category a rational number $\mu(M)$ such that

1. The composition length of M can be determined from $\mu(M)$ and
2. $\mu(M)$ is monotonous with respect to inclusions and behaves well with respect to direct sum decompositions.

However, the author also studies in detail a generalized version due to Krause where the composition length is replaced by an arbitrary length function (which, effectively, gives simple objects arbitrary positive real lengths).

The thesis consists of three parts. It starts with a long section on preliminaries which introduces a number of concepts and existing results, puts them into context and explains the relation to the thesis. In the second part, the author studies the above-mentioned generalized Gabriel-Roiter measures which have been almost entirely neglected in the literature. He concentrates on the special situation of thin representations, which is rather convenient as it allows to compute down to earth examples illustrating various subtleties of the notion. He also obtains a nice theoretical result saying that any indecomposable filtration of a thin representation whose support is a tree is a Gabriel-Roiter filtration for a certain length function. In the final section, he explicitly computes the classical Gabriel-Roiter measure for all indecomposable finite dimensional representations of a specific orientation of the Euclidean diagram $\tilde{A}_3$. This is one of the simplest tame hereditary cases for which such a computation does not appear in the literature.

The thesis is very nice an well written, contains original computations, interesting examples and a new result in the second section. Only the presentation is a little marred by imprecise arguments or formulations and by misprints. These are never essential and can be easily fixed, but they are slightly more frequent than they should be. Some such places are listed here:

1. Page 5: The uniqueness part in the Jordan-Hölder and Krull-Schmidt theorems is formulated in a rather weak form, which is equivalent to the standard one only if the composition series in question has pairwise

non-isomorphic factors or the direct sum decomposition has pairwise non-isomorphic summands. Furthermore, the definition of indecomposable algebra A is probably not the intended one: The definition on page 5, which asks for A being indecomposable as a right A-module, is equivalent to A being local. This is a way to strong assumption for Theorem 27 on page 26—one should rather ask that A is not a product of non-trivial subalgebras, which is a weaker property.

2. Page 6, Remark in Section 1.1.1: The kernel of f vanishes if f is a GR-inclusion.

3. Page 8, statement of Theorem 1(2): At least in [3], one assumes in addition that A is finite dimensional.

4. Page 11: If the preprojective representations of Euclidean type quivers are to be characterized by negative defect, one should better define the defect as $\langle \delta, X \rangle$ (as opposed to $\langle X, \delta \rangle$).

5. Page 13, proof of Lemma 6: It is not true that a finite length module has finitely many submodules, not even up to isomorphism. Indeed, if $R = KQ$ where K is algebraically closed and Q is the Kronecker quiver, then any quasi-simple regular module embeds into the non-simple indecomposable injective module I.

6. Page 14, Example 7: The mentioned map $Ch(\mathbb{N}) \to \mathbb{R}^+ \cup \{0\}$ is surjective, not an embedding. For example, the chains $\{1\}$ and $\{2, 3, 4, \dots\}$ are both mapped to $\frac{1}{2}$.

7. Page 15, statement of Theorem 8: The symbol U is not defined there.

8. Page 23, Definition 11: The space $Irr(X, Y)$ should be defined more carefully. It is not a subspace of $Hom(X, Y)$ as the sum of two irreducible maps may not be irreducible. It better be defined as a certain subfactor.

9. Page 27: Why does Lemma 28 hold? There is neither a proof nor a reference there.

In conclusion, I **recommend** the thesis for defence and the suggested grading will be communicated to the committee.

Prague, August 30, 2023 doc. RNDr. Jan Šťovíček, Ph.D.