This thesis studies the dynamics of geodesic motion within a curved spacetime around a Schwarzschild black hole, perturbed by a gravitational field of a far axisymmetric distribution of mass enclosing the system. This particular spacetime can serve as a versatile model for a diverse range of astrophysical scenarios. At the beginning of the thesis, a brief overview of the theory of classical mechanical systems and properties of geodesic motion are provided. A brief introduction to the theory of integrability and non-integrability, along with essential tools for analysis of non-integrable systems, including Poincaré surface of section and rotation numbers, is provided as well. These methods are subsequently applied to the under study spacetime through numerical methods. By utilising the rotation numbers, the widths of resonances are calculated, which are then used in establishing the relation between the perturbation parameter and the parameter characterising the perturbed metric.