FACULTY
OF MATHEMATICS
AND PHYSICS
Charles University

doc. Mgr. David Kubizňák, Ph.D.

Charles University
Institute of Theoretical Physics
Faculty of Mathematics and Physics
V Holešovičkách 2, 180 00 Prague 8, Czech Republic
david.kubiznak@matfyz.cuni.cz

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Report on Master Thesis of Eliška Klimešová

This is a report on Master Thesis of Eliška Klimešová titled *Multi-black-hole gravitational field*, written under a supervision of doc. Martin Žofka at Charles University, submitted in Prague 2023.

The main objective of this Thesis is to analytically study a 3-body (black hole) problem in general relativity, employing the so called *moduli space approximation*. Namely, the idea is to consider a static configuration of extremally (Maxwell) charged black holes, which is slightly perturbed, and slowly evolves under the influence of magnetic and gravitomagnetic forces. The advantage of this approximation is that it is applicable in the full strong gravity regime, although a limitation on slow evolution has to be imposed (and breaks down in the final stages of the black hole merger).

In particular, the thesis focuses on a case of small black hole moving in the background of a *binary black hole* system consisting of two extremally charged black holes on a circular trajectory. This is then compared to the approximation where the black hole is modelled as a test (potentially charged) particle and its motion is described by (electro-)geodesics in the same background.

The Thesis is *self contained* and presents *many results* and several rather *extended calculations*. Namely, the static Majumdar–Papapetrou metric is re-derived, the derivation of the effective moduli space action for the perturbed slowly evolving metric is reproduced from the literature (up to several remnant terms that do not seem to cancel out), the effective 2-body action is then re-derived, and, most importantly, a novel analytic approach to tackle the 3-body problem is developed. All these analytic derivations are highly non-trivial – they are very tedious and involve many various tricks – and clearly demonstrate *high technical ability*

of the candidate. At the same time, the Thesis contains a larger number of typos, is not ideally structured, and more attention could have been paid to explaining the big picture and outlining the course of the calculation. In any case, I believe that the manuscript *fulfils all the criteria* for the (rather large in scope) Master Thesis.

I have several questions for a potential discussion.

1. On page 15, there is a brief discussion of the moduli space approximation – in particular the dependence of various physical effects on the powers of (small) velocity parameter is estimated. I would like this to be explained a bit more in detail by the candidate, together with the justification for the stated powers. In particular, the discussion seems to imply that the magnetic effects are more important than the gravitomagnetic ones. Why is that?

2. One of the main results of the Thesis is that there exists a circular motion of a binary for which the perturbations vanish. What is the physical reason as to why such a configuration exists at all, and why are the (gravito-)magnetic parts of the metric and vector potential zero in this case? Why is the ADM mass unchanged in this case?

3. Can the stationary binary black hole system studied in the Thesis survive the presence of another black hole beyond the test particle approximation?

To summarize, I believe this is a very good, rather extended Thesis. I am very happy to recommend it to *be recognized* as Master Thesis.
Kind regards,

David Kubizňák