REPORT ON *SERRE'S CONJECTURE ON PROJECTIVE MODULES OVER POLYNOMIAL RINGS* BY EDWARD YOUNG

This bachelor's thesis provides an expository account of the Quillen-Suslin theorem, which states that every projective module over a power series ring is free. The mathematical content relating to the proof is based on the relevant sections of Lam's '*Serre's problem on projective modules*'.

The first half of the thesis is entirely background material in the mathematics required to approach the problem: the notions of free and projective modules are discussed, before basic categorical constructions are introduced. Following this the construction of the tensor product is given, as is flatness. It is then illustrated how the tensor product interacts with certain functors, and that localisation in commutative algebra is flat. Along the way, proofs are given. Overall they are well written and the information is concisely presented.

The second section contains the proof of the main theorem. As stated in the introduction to the section, the proof is broken down into four stages. The first stage deals with left regular rings, which are noetherian rings that have left finite global dimension. The main content of this stage is a discussion about projective resolutions and Swan's theorem, which shows a polynomial ring over a left regular ring is left regular.

The second stage introduces stably free modules and the Grothendieck group of projective modules, and shows how they relate. The main result relates the Grothendieck group of a left regular ring and the Grothendieck group of a polynomial extension, recalling Swan's theorem. The third stage discusses integral ring extensions. The student includes some additional lemmas and proofs from Matsumura's text, with the aim of providing additional details for the final proof of the Quillen-Suslin conjecture.

The fourth stage is on hermit rings - those over which stably free implies free. Much of this section contains preparatory results about row operations with the goal of proving the result that a polynomial ring over a field is hermite, from which the Quillen-Suslin theorem follows as an immediate corollary.

The third part of the thesis discusses the Bass-Quillen conjecture, which states that every finite rank free module over a polynomial ring arises from the base ring. The content of this section is more reflective of Quillen's method of proving Serre's conjecture, and indeed this new conjecture appears as a comment at the end of his paper. Finally, there are some details about the state of the conjecture.

By and large, the thesis is well written, and the mathematics appears to be correct. Moreover, a wide scope of ideas appear in the thesis. All in all, the desired outcome, namely providing an exposition of a proof of the Quillen-Suslin theorem, was achieved. It is also worth saying that conceptually the mathematical content of the thesis is at a good level, with many of the concepts in homological algebra and category theory being beyond the contents of a bachelor's degree.

However, there are a number of comments worth raising, not to detract from the thesis, but to highlight how it could be improved.

Throughout, citations are poor to non-existent. Even though it is stated that the guiding book is Lam's text, precise references to the corresponding results would be appropriate, not only to see the distinction between the Student and Lam's expositions.

Secondly, there are points when statements or definitions are incorrectly or incompletely given. For example, at Lemma 2.1.2 it is stated, without assumptions on the ring, that one can construct a finite projective resolution of a cokernel. This is not true. Presumably the modules are over a left regular ring, but this is not stated at any point. Similarly, at Definition 2.2.1 the given definition of a stably free module is incorrect without the assumption that k is finite (something which is not stated, but is essential). Indeed, even in Lam's book he explicitly states that k must be finite and includes the finiteness in his definition.

As the thesis is expository, one might expect there to be illustrating examples throughout. However, there are none, and the thesis is lacking for it. For instance, an example of a stably free but not free module would be appropriate, as would (non) examples of the types of ring (beyond polynomial rings) which are considered. These examples would improve the exposition.

Much less significantly, there are a couple of latex errors. Firstly 'Corollary' is misspelt in the macro, hence this spelling error repeatedly appears. The enumerations for the environments do not always align; for example, all of lemma, proposition, theorem and corollary have independent counters. This sometimes makes it different to track where the correct statement is.

This final point is not of any real consequence, but the tools of Quillen's proof, which provide motivation to the Bass-Quillen conjecture, are not really discussed in depth. Given the relatively short nature of Quillen's paper, as well as Lam's detailed exposition of his approach, it would have 'rounded out the picture' to have had slightly more focus on this.

Irrespective of these things, the thesis was of a high level and was well written overall. The exposition is clear, as is the goal of the thesis which was met.