

REVIEW BY THE SUPERVISOR OF THE BACHELOR THESIS

Thesis title: Serre's Conjecture on Projective Modules over Polynomial rings
Thesis author: Edward Young

SUMMARY OF THE THESIS CONTENT

The thesis deals with the Quillen-Suslin solution to an important conjecture of Serre from 1955. The conjecture states that any finitely generated projective module over a polynomial ring $\mathbb{K}[x_1, \dots, x_n]$ (where \mathbb{K} is a field) is free. The thesis begins with some basic homological notions, including projective and flat modules and relations between them, as well as a local characterization of projective modules. The author then proves Swan's theorem about ascent of regularity to polynomial rings. Then some K-theory and the theory of stably free modules is discussed, and this is used to prove the Quillen-Suslin theorem. In the last chapter, some more recent developments concerning the Bass-Quillen conjecture are discussed.

OVERALL EVALUATION OF THE THESIS

Thesis topic. The thesis discusses a very important foundational problem in the intersection of commutative algebra, algebraic geometry and K-theory. This is a difficult subject, which far exceeds the undergraduate curriculum.

Author's contribution. Edward wrote several original proofs, including to some problems I suggested that were part of the proof. I also want to mention that Edward was very independent while working on the thesis, and that he needed minimal guidance, proposing several promising directions, and demonstrating ability to absorb complex mathematical notions.


Mathematical level. The level of the thesis is very high, and with the exception of the survey in the last chapter, everything is proved rigorously, and appears to be correct.

Work with sources. The author cites the sources he uses in constructing the proofs, and does not copy anything from these sources.

Formal editing. The thesis is nicely written, in the usual style of mathematical texts.

CONCLUSION

I consider the thesis to be excellent and I recommend that it be accepted as a bachelor's thesis.

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June 5, 2023