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Report on Adam Vrátný's thesis

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**Report on the thesis entitled "Spacetimes with black holes" submitted by Mr. Adam Vrátný for the Ph. Doctor degree (Physics), Charles University**

The thesis deals with *exact solutions* of the General Relativity (GR) field equations for the vacuum and electromagnetic cases, possibly including a cosmological constant $\Lambda$. More specifically, it deals with exact solutions that describe (pairs of accelerated) black holes with spin and the so-called NUT parameter, electromagnetic charges, and $\Lambda$. Black holes (BH) are one of the most remarkable predictions of GR, and we believe they have already been observed in several instances, leading to the Nobel prize in physics 2020 and 2017. In this sense, and despite the obvious differences between realistic dynamical BHs and purely theoretical (mathematical) ones, the subject of the thesis is well chosen and of indubitable mathematical and physical interest.

The main goal of the thesis is to re-consider the important solution of the GR field equations found years ago by Debever and later by Plebański with Demański (let us call it DPD) in order to improve our understanding of these solutions and of their properties as well as to consider other solutions (surprisingly found in the literature) that describe the same type of physical situations but were not included in the general DPD family of metrics. The starting point for this analysis is an improved form of the DPD solutions as proposed by Griffiths and Podolský, and the thesis intends to optimize this –already very good– explicit structure of the mentioned solutions.

The candidate starts with some introductory material that will be repeatedly used in later chapters and presents the DPD family of solutions specifying all the particular cases that it contains. The general family contains 7 free parameters $\Lambda$ and $m, a, l, \alpha, e, g$ which are interpreted as the mass, specific angular momentum, NUT, acceleration, electric

and magnetic parameters, respectively. It is then noted that a solution with only $m, l, \alpha$ parameters is missing in the family. However, a solution with just those three parameters was found by Chng, Mann and Stelea some years ago, with properties analogous to those of the DPD family. This was a little surprising, and the thesis resolves this puzzle by checking that indeed the extra solution is not part of the DPD family but it does describe accelerated BHs with mass and NUT parameters.

This is the content of the first chapter with new results, and of the attached Paper I. First of all, a direct demonstration that the extra solution is certainly a solution of the field equations is presented, based on algebraic computing. Then, in order to prove that this metric does not belong to the general DPD family a known algebraic property of the latter is used: all DPD metrics have a Weyl tensor of algebraic type D (meaning two principal null directions). Again using algebraic computing techniques the non-type D character of the extra solution is explicitly shown —it is type I, meaning four distinct null principal directions—, implying that it cannot belong to the DPD family, as expected. A new form of the metric, in a better coordinate system, is provided. This is useful to check the standard form of the particular cases included, as well as to perform the very detailed and exhaustive study of the geometrical and physical properties of the Chng-Mann-Stelea solution. In particular, the global structure, pathological regions, horizons, maximal analytical extension, conformal infinities and (non)-regularity of the axes are properly identified and thoroughly analyzed. Of particular interest is the absence of curvature singularities —whenever the NUT parameter is non-vanishing.

The remaining two chapters deal with a yet improved form of the DPD family, first for the case with $\Lambda = 0$ (chapter 2 and Paper II) and then with $\Lambda$ included (chapter 3 and Paper III). These two papers are technically correct, with possible nice applications, and perform an extremely detailed analysis of the general DPD family and the geometric and physical properties of the solutions therein contained. In the former case, with $\Lambda = 0$, the new chosen parametrization allows one to factorize the main functions in the metric ($P(\theta)$ and $Q(r)$), something which is very convenient to deal with these solutions. Furthermore, the new explicit form has the virtue that all sub-cases are simply obtained by setting the free parameters to zero, enhancing the interpretation of these free constants. A similar improved form is achieved for the second case, with $\Lambda \neq 0$, but now only one of the two main functions of the metric –namely $P(\theta)$– can be factorized. Of course, by setting $\Lambda = 0$ one recovers the factorization of the previous case. Again, a complete analysis of the geometrical and physical properties of the solutions is performed in a very detailed manner. In particular, the Killing horizons are identified, all of them at constant values of the coordinate $r$ that can be easily given, and their type shown, as well as their thermodynamic properties; the existence of ergo-regions is also resolved; the cases with curvature singularities found; the global structure of the solutions, their maximal extensions, the conformal infinities and the Penrose conformal diagrams are provided in all relevant cases; the (non)-regularity of the axes analyzed; and the possible existence of chronology violating zones studied.

Thus, the candidate has made a substantial contribution to the understanding of this important class of BHs solutions, and the results presented on the published papers

will surely be useful for other researchers. I myself consider these results very useful and, in my opinion, they constitute a good Ph D thesis in theoretical physics and/or in mathematics. In the printed dissertation made available to me, the candidate shows familiarity with the subject, and with the pertinent literature. The aims of the thesis are made plain in the Introduction. The methodology is appropriate, and the needed tools used are well based or even developed in the thesis, which is a plus.

All in all, I consider the thesis a good work on the whole, containing interesting and original results relative to the field of exact BH solutions in GR.

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