In this thesis, we study exact black hole spacetimes of algebraic type D, which are a part of much wider Plebański–Demiański class of solutions. We reformulate the wellknown form of this metric and obtain new improved representation of this black hole family with simplified, explicit and (at least partially) factorized metric functions. This new form of the spacetimes allows us to gain the standard expressions for the well-known solutions such as the Kerr–Newman–NUT–(anti-)de Sitter black hole, accelerating Kerr– Newman–(anti-)de Sitter black hole, (possibly charged) Taub–NUT–(anti-)de Sitter black hole, accelerating Kerr–NUT–(anti-)de Sitter black hole, and their special cases in asymptotically flat universe, just by putting the appropriate parameters to zero. We also provide a thorough physical and geometrical analysis of this new form of spacetimes. Furthermore, we analyze a solution corresponding to the accelerating Taub–NUT black hole, which was originally found by Chng, Mann and Stelea in 2006. We perform an in-depth analysis of this solution, and study its relation to the Plebański–Demiański class.