This work is concerned with analysing and exploiting mixed precision arithmetic in uncertainty quantification methods with emphasis on the multilevel Monte Carlo (MLMC) method. Although mixed precision can improve performance, it should be used carefully to avoid unwanted effects on the solution accuracy. We provide a rigorous analysis of uncertainty quantification methods in finite precision arithmetic. Based on this analysis, we exploit mixed precision arithmetic in uncertainty quantification methods to improve runtime while preserving the overall error.

We begin by stating the model problem, an elliptic PDE with random coefficients and a random right-hand side. Such a problem arises, for example, in uncertainty quantification for groundwater flow. Our focus is on approximating a quantity of interest given as the expected value of a functional of the solution of the PDE problem. To this end, we use the conforming finite element method for approximation in the spatial variable and the MLMC method for approximation of the expected value. We provide a novel rigorous analysis of the MLMC method in finite precision arithmetic and based on this we formulate an adaptive algorithm which determines the optimal precision value on each level of discretisation. To our knowledge, this is a new approach. Our theoretical results are then verified on numerous examples including an elliptic PDE with lognormal random coefficients, achieving a theoretical speedup of 4–8$\times$ compared to the reference double precision. The modeled speedup is achieved under the assumption that when single, half or quarter precision is used instead of double precision, the runtime is improved by the factor of 2, 4 or 8, respectively.