Describing viscoelastic fluids is a difficult task, as the viscoelastic phenomena are not fully understood. This work follows a method for deriving viscoelastic models that accurately capture the behavior of fluids with polymeric substances, which macroscopically manifest as the stress diffusion, within a consistent thermodynamic framework. We implemented these models using the open-source computing platform FEniCS as a finite element library for Python, and we provide a numerical study of the stress diffusion as a stabilization. By extending our implementation using the arbitrary Lagrangian-Eulerian method, we are able to simulate well-known non-Newtonian phenomena, in particular the Weissenberg effect, demonstrating the effectiveness of our approach in enabling a better understanding of these complex fluids.