Report on Master Thesis "Mapping spaces of algebras over iterated +-construction for polynomial monads" by Maroš Grego

The subject of Maroš Grego thesis is n-fold delooping of mapping spaces between operadic algebras. This is a very active area of research which involves Category Theory (including Higher Categories), Homotopy Theory, Combinatorics and Algebraic Topology. The aim of the project was to find a generalisation of a celebrated Turchin-Dwyer-Hess theorem on double delooping of mapping spaces between infinitesimal bimodules over associativity operad. The associativity operad governs classical associativity law and is the second term in a sequence of operads ζ_n, $n \geq 2$ known as Baez-Dolan opetopic sequence. These operads are complex and contain all information about coherence laws of all dimensions in higher category theory. Turchin-Dwyer-Hess theorem establishes a bridge between (derived) combinatorics of associativity law and homotopy type of spaces of knots in the Eucledian spaces. A generalisation of this theorem to higher terms of opetopic sequence would clarify the connections between higher coherence laws and the embedding spaces of higher dimensional manifolds.

There are several original contributions in the thesis:

1. It is shown that the operad ζ_n has an associated category of k-dimensional bimodules for any $0 \leq k < n$. This series of categories of bimodules includes classical category of ζ_2-bimodules (in case $k = 1$) as well as the category of infinitesimal ζ_2-bimodules (in case $k = 0$).

2. As a direct generalisation of Turchin-Dwyer-Hess theorem the existence of an explicit 2-fold delooping of mapping spaces between $(n-2)$-dimensional bimodules over ζ_n is established for all $n \geq 2$.

3. Moreover, a possibility of a third delooping for $(n-3)$-dimensional bimodules over ζ_n is investigated and a necessary and sufficient condition for the third delooping is given. This condition is new, intersteting and deserves further investigation for its own right.

4. An n-dimensional generalisation of the category of finite ordinals Δ is introduced for any $0 \leq n \leq \infty$ (Δ corresponds to $n = 1$.) Close relationships are demonstrated between these categories and the Moerdijk-Weiss dendroidal category Ω, which is of prime importance in the ∞-operad theory.

On my opinion the thesis is a great success and the results exceeded my expectations as supervisor. The thesis is well written and contains the original new results which deserve to be published in a peer-reviewed journal. I recommend it to be recognised as a diploma thesis on MFF UK with the highest possible grade.

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