Freie Universität Berlin

Freie Universität Berlin
Institut für Informatik

Prof. Dr. László Kozma
Takustrasse 9
14195 Berlin

Telefon +49 30 838 66213
E-Mail laszlo.kozma@fu-berlin.de
Internet http://page.mi.fu-berlin.de/lkozma/

To:
Office of Student Affairs
Faculty of Mathematics and Physics
Charles University Prague

Report on the doctoral thesis of **Mgr. Michal Opler:**
"Structural and Algorithmic Properties of Permutation Classes"

The thesis of Mr. Opler studies Permutation Classes, subsets of the set of permutations defined primarily through the avoidance of *patterns*. Permutation classes have been a central concept of combinatorics, also of great interest in theoretical computer science, as well as other fields, e.g. non-parametric statistics. Both structural and algorithmic properties have been intensively studied during the past few decades, and this research area remains very active to date.

The thesis of Mr. Opler significantly advances the state of knowledge about these fascinating combinatorial objects, introducing novel tools and bringing fresh insights to the topic, through which new and improved structural relations are established, as well as efficient algorithms and complexity results. Besides the immediate results, the thesis identifies new research directions, that will no doubt strongly influence future research on the topic.

On the structural side, the thesis starts with an essentially complete characterization of how different graph-theoretic complexity measures of permutations relate to one another. Besides the classical measures, a novel *grid-width* and *linear grid-width* are introduced, exhibiting a natural analogy to *treewidth* and *pathwidth*. All measures are shown to be related to *twin-width*, a quantity recently identified in structural graph theory as a unifying concept. Previously some bounds were known for the complexities (in terms of treewidth) of principal permutation classes – those defined by the avoidance of a single pattern. Mr. Opler pushes this to an almost complete asymptotic characterization. Perhaps as interesting as the bounds themselves are the novel structural properties arising in the grid view (long path, cycle, deep tree, bicycle), from which the bounds fall out in a principled, unified way. I am certain that these tools will see further applications in research on permutations, and more broadly, the insights about grid-based decompositions will have lasting impact.

Next, the thesis adopts a logic-based view of permutations that has been used in the literature towards algorithmic results of remarkable generality. Here too, Mr. Opler extends the state-of-the-art, showing that MSO-sentences are tractable under a bounded treewidth assumption for the input. This is complemented by hardness results, showing that FO-model checking on permutations captures the entire complexity of general undirected graphs, and that deciding MSO-sentences in permutation classes with the long path property captures the complexity of general graphs. The thesis gives further insight by identifying properties that separate the expressiveness of FO and MSO.

On the computational side, perhaps the main contributions of the thesis are improved complexity results for the permutation pattern matching (PPM) problem and its counting version #PPM: (1) streamlined and unified proofs ruling out subexponential algorithms for PPM and FPT-algorithms for #PPM, (2) $W[1]$-hardness of PPM w.r.t. the width-parameters introduced earlier in the thesis, (3) ETH-hardness (in classical or in FPT-sense, even w.r.t. width parameters) for PPM even with patterns from all but the most restricted principal classes, (4) assorted algorithms and matching hardness results for variants of PPM, (5) assorted algorithms and hardness results for PPM with grid classes.

These groups of results provide a comprehensive picture of the complexity landscape of PPM. Each relies on delicate constructions; while some of them build on earlier observations from the literature, they reach significantly further, and yield an exceptionally fine and essentially complete (apart from a few remaining concrete cases) understanding that was unavailable before, with parts (2) and (3) being particularly impressive.

Little was known previously about the case of PPM where the text permutation is also restricted to a principal class. Here too the thesis remarkably provides an almost complete characterization through a chain of technically deep arguments, narrowing down the set of patterns where the classical complexity is open to five. Finally, the thesis is rounded off with algorithmic and hardness results for "generalized coloring", a different algorithmic problem still closely related to permutation patterns, with applications to enumeration.

Overall, the thesis is very well-structured and pleasant to read; while research papers about permutation classes tend to drown in notation, Mr. Opler achieves clarity and compactness, with precisely formalized definitions and mathematical arguments, accompanied by helpful high level intuition and beautiful illustrations. Mr. Opler demonstrates a remarkable mathematical creativity and acumen, particularly in proving complexity results, inventing sophisticated and original gadget constructions and carrying out reductions of significant depth. The thesis also showcases a taste for theoretical unity, with streamlined arguments and analogies. It significantly advances our understanding in a specialized but important and natural area of discrete mathematics, also identifying concrete challenges for further research. The author has an impressive track-record of publications – not restricted to the topic of permutation classes. In my opinion the thesis is an excellent scientific work, I therefore recommend the granting of the PhD degree without any reservations whatsoever.

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Suggested questions for the defense:

- Warm-up: what are some examples of natural permutation properties that do *not* define a permutation class?
- What is the intuition that suggests **Conjecture 2.31** and what is the simplest example class where it is not known to hold?
- According to **Table 2.1**, only 5 principal classes have no tight asymptotic treewidth-bounds. Is it plausible that the true answer may be $\Theta(\sqrt{n})$ in some of these cases? Where could such an *upper bound* come from (if not from planarity)?
- Some algorithmic results hold e.g. when the input permutation avoids some pattern π. However, this property is very “fragile” as a few local changes could break it. Could there be some way to make such algorithmic results more “robust” to perturbations?
- What is the conjectured complexity of $Av(\sigma)$-PPM in the remaining five cases?
- What is the intuition for why the bicycle property cannot be used in hardness reductions similarly to the long path and deep tree properties?
- A large body of work studies permutation patterns in a “property testing” framework, i.e. distinguishing inputs that are in a permutation class from those “very far” from it. Could there be some bridges between that area and the investigations of the thesis?

External Referee
Prof. Dr. László Kozma

Berlin, September 15th, 2022