In this thesis, we study the relationship between the structure of permutation classes and the computational complexity of different decision problems. First, we explore the structure of permutation classes through the lens of various parameters, with a particular interest in tree-width. We define novel structural properties of a general permutation class $\mathcal{C}$, the most notable being the long path property. Using these properties, we infer lower bounds on the maximum tree-width attained by a permutation of length n in $\mathcal{C}$. For example, we prove that any class with the long path property has unbounded tree-width.

The main decision problem we consider is known as Permutation Pattern Matching (PPM). The input of PPM consists of a pair of permutations τ (the 'text') and π (the 'pattern'), and the goal is to decide whether τ contains π as a subpermutation. After briefly considering general PPM, we focus on its pattern-restricted variant known as $\mathcal{C}$-Pattern PPM where we additionally require that the pattern π comes from a fixed class $\mathcal{C}$. We derive both classical and parameterized hardness results assuming different structural properties of $\mathcal{C}$. For example, we show that $\mathcal{C}$-Pattern PPM is NP-complete whenever $\mathcal{C}$ has the long path property.

Furthermore, we focus on an even more restricted variant of PPM where the text is also assumed to come from a fixed class $\mathcal{C}$; this restriction is known as $\mathcal{C}$-PPM. We present a new hardness reduction which allows us to show, in a uniform way, that $\mathrm{Av}(\sigma)$-PPM, where $\mathrm{Av}(\sigma)$ is the class of σ-avoiding permutations, is NP-complete for any σ of length at least four that is not symmetric to one of 3412, 3142, 4213, 4123 or 41352.

Permutations can also be viewed as models over a signature consisting of two binary relation symbols. We investigate the expressive power of monadic second-order (MSO) logic and the complexity of MSO model checking in this setting. Among other results, we show that MSO model checking is hard not just for general permutations, but also within any class with the long path property.

Finally, we consider, for fixed permutation classes $\mathcal{C}$ and $\mathcal{D}$, the complexity of determining whether a given permutation π can be colored red and blue so that the red elements induce a permutation from $\mathcal{C}$ and the blue ones a permutation from $\mathcal{D}$; this problem is known as generalized coloring. As a consequence of more general results, we can provide nontrivial instances of both tractable and intractable generalized coloring involving commonly studied permutation classes.