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Report on the doctoral thesis *Convergence in Banach spaces*
by Zdeněk Silber

The thesis of Zdeněk Silber was written under the supervision of prof. Ondřej Kalenda, at the Faculty of Mathematics and Physics of Charles University. The essential body of the thesis consists of more than 60 pages and is based on two already published articles and a preprint, all written by the author of the thesis.

The first two parts are devoted to *weak** derived sets in dual Banach spaces. Given $A \subseteq X^*$, the (first) derived set $A^{(1)}$ consists of all *weak** limits of all bounded *weak** convergent nets of elements of A . This notion found a number of applications in studying Banach spaces; in particular, it gives a characterization on reflexive and quasi-reflexive Banach spaces. By an obvious iteration, one can define the α -derived set for every ordinal number α ; the order of a set A is then the first ordinal for which this operation stabilizes. To give a flavour of the content of that part, one of the main results states that for every nonreflexive space X , the dual space contains convex sets of order n for every natural number n as well as such sets of order $\omega + 1$. This is a natural continuation of the research done by M.I. Ostrovskii who mostly concentrated on the separable case.

The final part of the dissertation is devoted to iterated versions of the Banach-Saks property. The original property states that every bounded sequence has norm-converging averages. The iterated version refers to some inductively defined summability method; those properties can be localized in a obvious manner. Recall that a quantitative version of some Banach space property typically assigns some real parameter(s) to the object in question so that the property we have in mind can be expressed either by inequalities or the asymptotic behaviour of those parameters. Such an idea is applied here to iterated Banach-Saks properties and the author presents generalizations of the results from [7] (which is a 2015 article coauthored by Kalenda).

My overall assessment of the doctoral thesis presented by Zdeněk Silber is very positive and I do not hesitate to state the following.

1. The subject of the thesis has its roots in some classical properties of Banach spaces that are still being investigated and applied by the specialists in the field.
2. The main results of the thesis provide a natural continuation of existing lines of research. Their proofs are technically quite involved and require subtle ideas. In

particular, the result of the first part mentioned above builds on theorems by of Michail Ostrovskii; in turn, Silber's publication in JFA found a continuation in Ostrovskii's recent article (one can say that the young mathematician started a scientific dialogue with the well-established expert).

3. The first two parts of the thesis have been published in very good journals devoted to functional analysis, with Silber as their only author — this apparently indicates his substantial contribution and is a good starting point for an independent mathematical career.

To summarize, Mr. Silber has proven his ability for creative scientific work; I am rather convinced that the mathematical level of his dissertation considerably exceeds the usual expectations from the mathematical community.

Let me add that in Polish terms, granting 'PhD with distinction' would be seriously considered (which opens the possibility of nomination for nationwide prizes for young researches).

A handwritten signature in blue ink, reading "J. Pietsch". The signature is written in a cursive style with a large, stylized initial "J" and "P".