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Report on the doctoral thesis

Structure and complexity of homomorphisms

by Mgr. Jan Bok

Algorithmic graph theory has been an important sub-discipline of theoretical computer science for many decades. Besides its intrinsic mathematical beauty, it has found applications in social and physical sciences, as well as other areas of computer science like networking, scheduling, optimization. The current thesis explores conjunction of two fascinating aspects of the discipline: complexity of algorithmic computation and existence of homeomorphic mappings between two objects, namely combinatorial graphs. A simple example of a problem in this conjunction (not treated in this research explicitly, but nevertheless worth mentioning) is the Graph Isomorphism Problem: "how difficult computationally is it to determine existence of a bijection between elements of two graphs preserving adjacencies and non-adjacencies?" The place of this problem in the dichotomy of NP (non-deterministic polynomial-time solvable problems) between "polynomial-time" and "NP-complete" has not been determined to this day, which attests to the importance of probing the related research questions.

The current thesis presents three aspects of an extensive body of research by the author and his co-authors. The results of this research extend and often resolve previous results and questions of many other diverse research groups. The unifying criterion for the three research questions is the place in the computational complexity dichotomy of each class of existence problems: homomorphism of signed graphs, covering of graphs, and restricted coloring of classes of graphs (defined by forbidding a specified graph as an induced subgraph.) Each existence problem is modified as follows (i) by adding, for each vertex of the input graph, a list of acceptable target vertices, (ii) by augmenting elements of target graphs by multi- and semi-edges, and (iii) by constraining subgraphs induced by any two color classes.

Two aspects of the results and their presentation may impress the reader the most. One is the breadth of the scientific inquiry: while keeping within the research question of the existence of a certain mapping between vertices of two graphs, the author treats quite disparate combinatorial objects: graphs with signed edges, multigraphs with semi-edges, and subgraphs induced by color classes. The other is the facility of the author in dealing with different tools of the trade: construction of gadgets in NP-completeness proofs, the use of forbidden substructures, the extent of his knowledge of and references to the related work.

College of Arts and Sciences
Computer and Information Science
1202 University of Oregon, Eugene, OR 97403-1202
541-346-4408 | FAX 541-346-5373 cs.uoregon.edu

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As far as the question which of the thesis' results can be referred to as a new scientific result, the answer is easy: all of them. In dealing with list homomorphism of signed graphs, the thesis resolves complexity status for signed trees as target graphs. (The use of a certain ordering of vertices in efficient computations is especially ingenious.) The most significant (and certainly the most elegant) results of the thesis may well be the complexity dichotomy of graphs covering problems, where target graphs have very few vertices (one or two) but include semi-edges. Further results are obtained by extending graph covering problem to the list version and restricting target graphs to regular graphs with semi-edges. Also important for future research is the discussion of the graph covering problems for disconnected graphs. Finally, the thesis provides some new results regarding complexity of acyclic coloring of graphs characterized by a forbidden induced subgraph resolving the complexity dichotomy of the problem for a special class of target graphs ("path-" and "cycle-separable".)

As with any written document, there are few awkward turns of phrase (Table 9.1), typos ("Corneill"), and mistaken references (section 6.4.1) There is no doubt, however, that the thesis proves the author's exceptional ability for creative scientific work and therefore he should be granted the doctoral degree.

Andrzej Proskurowski
Professor of Computer and Information Science
University of Oregon
Eugene, OR, USA