The aim of this master thesis is to develop the background for point processes on both the linear network and the planar network in $\mathbb{R}^3$. The linear, planar network is formed by the system of edges, faces of a 3D tessellation, respectively. Using the Gibbs-Laguerre tessellation model we can investigate the case of a regular or irregular tessellation. We consider cluster and hard core point processes and we compare two types of distances between points, the Euclidean distance and the shortest path distance within the network. The algorithms for simulation of point processes on both networks are developed. Using the simulated realizations of point processes we estimate some functional summary characteristics. Based on plotted graphs of these estimates, the influence of the choice of tessellations, point process models and distances is discussed.