Posudek práce

předložené na Matematicko-fyzikální fakultě
Univerzity Karlovy

☐ posudek vedoucího ☑ posudek oponenta
☑ bakalářské práce ☐ diplomové práce

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Název práce:	Counting operators in Effective Field Theories
Studijní program a obor:	Fyzika, Obecná fyzika (FOF)
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Odborná úroveň práce:
☑ vynikající ☐ velmi dobrá ☐ průměrná ☐ podprůměrná ☐ nevyhovující

Věcné chyby:
☑ téměř žádné ☐ vzhledem k rozsahu přiměřený počet ☐ méně podstatné četné ☐ závažné

Výsledky:
☐ originální ☑ původní i převzaté ☐ netriviální kompilace ☐ citované z literatury ☐ opsané

Rozsah práce:
☑ veliký ☐ standardní ☐ dostatečný ☐ nedostatečný

Grafická, jazyková a formální úroveň:
☑ vynikající ☐ velmi dobrá ☐ průměrná ☐ podprůměrná ☐ nevyhovující

Tiskové chyby:
☑ téměř žádné ☐ vzhledem k rozsahu a tématu přiměřený počet ☐ četné

Celková úroveň práce:
☑ vynikající ☐ velmi dobrá ☐ průměrná ☐ podprůměrná ☐ nevyhovující

Slovní vyjádření, komentáře a připomínky oponenta:

The thesis deals with the topic of Hilbert series in the context of effective Quantum Field Theories. These series encode the number of independent operators (invariants) of a certain type in the theory.

In my assessment, the thesis is composed of two logical parts, which I elaborate upon below:

1. The first part develops all the necessary tools for later computation. It does not merely present the formalism to be used, but contains a mathematical exposition of the material, complete with definitions and proofs of (almost all) statements.

   The concepts covered are those necessary to state the master formula for the Hilbert series: the Haar measure as a tool of integration on compact connected Lie groups, culminating in the Weyl integration formula for class functions; representation theory of compact Lie groups, culminating in the use of characters of representations to project out multiplicities; and the construction of representations relevant to the QFT context, i.e. of the graded multi-derivative multi-particle representation of a spin 0 (or spin 1) field and of the graded representation of a singlet operator where certain relations (EOM, IBP, Gram condition) are imposed.

   Although most of this material can be found in (advanced) textbooks and/or recent research papers on Hilbert series, the organization of this material in the thesis makes it clear that its author has clearly spent significant effort in learning and presenting it. I strongly support and encourage this approach, more so since the material itself is quite advanced for the Bachelor level of studies.

2. The second part of the thesis uses the tools from the first part to compute coefficients in the Hilbert series of spin 0 and spin 1 particles. Emphasis is put on the Lorentz invariance of the theory rather than internal/gauge symmetries, and the main intricacies stem from how to correctly account for relations when space-time derivatives are applied to fields; in particular the equations of motion, the equivalence up to total derivatives, and the Gram conditions trivializing certain expressions with derivatives in space-times of a smaller dimension. The main results of the thesis are the coefficients corresponding to powers $n = 4, 5, 6$ (spin 0) or $n = 4, 5$ (spin 1) in the number of fields in the operators for any space-time dimension d and any number of derivatives (powers of D).

   The results partly overlap and partly extend those from the literature. The level of novelty of these results completely suffices for the Bachelor level.

Overall, the thesis is written very compactly and concisely in the mathematical style of "definitions and theorems", which remains very consistent throughout the thesis. The number of typos is remarkably small for a text of this sophistication. The material is self-contained and highly cross-referenced between its various parts. Finally, I would also like to commend the efficiency of presenting the material: the textbook topics in the first part are covered exactly to the degree needed, with no concept missing or redundant for a detailed understanding. This degree of polish of the text could not have been achieved, I believe, without considerable effort and consideration of detail on the part of the author.

Content-wise, my only minor complaint is that the text becomes too terse in some places if intended for pedagogical purposes. One example of this is the proof of Proposition 3.70, where the relation of k_i to $\mathbf{k}$ for the anti-symmetric power cannot be interpreted in the same way as the relation of r_i to $\mathbf{r}$ for the symmetric power; the former are a list of positions where there is a non-zero entry of 1 in $\mathbf{k}$, and the latter are components of the vector $\mathbf{r}$. This compactness of writing also sometimes manifests in the use of the same symbol for different mathematical concepts/objects (often the clash is inevitable if one uses standard mathematical notation), which may become confusing if the different uses appear close together. I can point to the following examples:

- In Definition 3.77, the symbol e is used both for the exponential map and the identity element in the Lie group, see last line.
- In Example 3.93, the symbol t is used for both a torus element conjugate to the group element g, and for the dummy (grading) variable in the graded character.
- The symbol $*$ is used in the superscript as a dual for the vector space in Definition 3.95, and then as the pullback of a function on a manifold in the "Sketch of Proof" of Theorem 3.97.

Although these uses are clear from the mathematical context for the attentive reader, I believe that readability would be improved in such cases if the clash in notation is accompanied by a short reminder or clarification.

Language-wise, the thesis is written very well. My only minor complaints in this regard are the following:

- Occasionally, definite and indefinite articles ("a" and "the") in front of nouns are missing.
- Sometimes the choice of words is of a more colloquial style than the author perhaps intended, e.g. I would suggest to use "completely" rather than "totally" above Table 5.1.

All in all, the thesis is in my opinion of very high quality, and I would recommend the highest grade "excellent" for it.

Případné otázky při obhajobě a náměty do diskuze:

Setting specific questions aside, the broader topics of possible discussion can be the following:

- How does the computation of the Hilbert series change if we include or exclude certain constraints? What if we add internal/gauge symmetry and particles in a reducible representation of this symmetry? What happens if the internal symmetry is discrete?
- Once the Hilbert series is obtained, are there any non-trivial checks that can be performed for the consistency of the result, even if these are merely heuristic?

Práci:
☑ doporučuji
☐ nedoporučuji
uznat jako bakalářskou.

Navrhuji hodnocení stupněm:
☑ výborně ☐ velmi dobře ☐ dobře ☐ neprospěl

Místo, datum a podpis oponenta:

Praha, September 1, 2022

dr. Vasja Susič