Supervisor's report on the dissertation thesis by

Vojtěch Kika: Copula-based multivariate association measures and tail coefficients

The aim of the thesis is to investigate various ways how the dependence between more than two random variables can be summarized. The main research questions at the beginning of the research were as follows:

- What associations measures have been already suggested and what are their properties?
- Does it make sense to improve/modify the existing association measures? Or is it possible even to come with some new useful measures?
- How these association measures can be estimated and what are the properties of the corresponding estimators?
- What are the possible applications where estimates of the association measures are useful?

From the beginning the research concentrated on measures that can be expressed in terms of copulas. The advantage of such measures is that there are invariant to the increasing transformation of the univariate margins. The thesis contains the results of seeking answers to the questions given above. While Chapter 1 is introductory, the main contribution can be found in the following four chapters.

In Chapter 2 the author reviews the (overall) multivariate association measures and discuss their properties with the emphasis on the properties that have not been investigated yet. It contains also many new illustrative examples and also application to the real data analysis.

Chapter 3 deals with multivariate tail coefficients that focuses on the dependence of extreme events. Such measures are of interest in particular in finance and environmental sciences. Similarly as in Chapter 2 the author discusses in detail the various interesting properties of these estimators and provide illustrative examples as well as counter-examples.

Chapter 4 then focuses on estimation of multivariate tail coefficients. This is not so straightforward as for multivariate association measures as the estimation of the most tail coefficients requires choice of some kind of a smoothing parameter. That is why, after proving the consistency of the estimators, the author concentrates on one of the coefficients. For this particular coefficient he provides a fined asymptotic result and suggests how this can be used for finding a reasonable value of the smoothing parameter in the estimation procedure.

Finally in Chapter 5 the author illustrates the use of the extreme dependence coefficient in variable clustering. He suggests a clustering procedure and illustrates its use in the real data analysis.

In my opinion the thesis proves that the author is able to do a solid research in mathematics. That is why I recommend the thesis to be accepted as a dissertation thesis for the PhD degree in the study programme Probability and statistics, econometrics and financial mathematics.

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