PARIS
LODRON
UNIVERSITÄT
SALZBURG

Dr. Sebastian Fuchs
Senior Scientist
Head of IDA Lab Team Dependence Modeling
Department for Mathematics
University of Salzburg
Hellbrunnerstrasse 34
5020 Salzburg, Austria
sebastian.fuchs@sbg.ac.at

Salzburg, 25 August 2021

Report on the doctoral thesis

Copula-based Multivariate Association Measures and Tail Coefficients

by Vojtěch Kika

In his thesis Vojtěch Kika studies multivariate extensions of some popular bivariate copula-based association measures including tail dependence coefficients and measures of concordance. Various properties of theses extensions are discussed, with special attention to those properties that illustrate the measures' behaviour with increasing dimension. For each discussed multivariate extension estimators are provided and their asymptotic properties are studied.

Below the different chapters are evaluated.

In the first chapter the author presents a concise overview of the content and structure of his work. He further provides definitions and basic results about bivariate association measures, copulas and their nonparametric estimation.

Chapter 2 is devoted to multivariate extensions of the bivariate measures of concordance Kendall's tau, Spearman's rho, Gini's gamma and Blomqvist's beta. A set of axioms for multivariate measures of concordance that is based on and includes all the axioms proposed by Taylor (2007, 2016) is established. However, in contrast to Taylor, the author adds another axiom (A_8) which compares the value of a random vector with the value obtained by adding an independent random variable to the random vector. Further subtypes of transitional behaviour are examined in more detail by means of two properties that compare the value of a random vector with the value obtained when

(P_1) adding a conical combination of all components of this random vector, and

(P_2) adding a set of arbitrary random variables.

For Archimedean and meta-elliptical copulas the investigation is complemented by revealing the measures' behaviour when the dimension tends to infinity. The results obtained provide valuable insights into the measures' transition behaviour and complement Taylor's work to a considerable extent. The chapter concludes with results on nonparametric inference and a data application.

By contrast, Chapter 3 focuses on multivariate coefficients of tail dependence including Frahm's extremal dependence coefficient, Li's tail dependence parameter, the tail dependence measure of Schmid and Schmidt and a tail dependence coefficient for extreme value copulas. A set of axioms is established and various properties of the coefficients are studied, including monotonicity with respect to pointwise order of copulas (TF_2) and linearity with respect to convex combinations (TF_3). Special attention is paid to those properties that compare the value of a random vector with the value obtained when

(T_4) adding an independent random variable,

(TF_1) adding an arbitrary random variable.

Again, for Archimedean copulas, the investigation is complemented by revealing the coefficients' behaviour when the dimension tends to infinity.

In Chapter 4, for each tail dependence coefficient introduced in Chapter 3 the author proposes estimators and proves consistency. At this point it is worth mentioning that estimating tail dependence is a challenging task as limits need to be estimated from the data. The author focuses on the estimation of Frahm's lower extremal dependence coefficient and proves various asymptotic properties of this estimator. The finite-sample performance of the estimator is investigated via a simulation study and a real data example.

In Chapter 5 the theoretical results obtained in the previous chapters are then used to build a hierarchical clustering procedure using an extended dissimilarity measure based on Frahm's extremal dependence coefficient. The procedure is illustrated by real data applications.

The thesis concludes with a summary of the main thesis contributions and ideas for possible future research directions (Chapter 6).

Vojtěch Kika's thesis is comprehensive and one immediately recognizes its clear and systematic structure. He provides a solid mathematical study of multivariate extensions of existing bivariate measures of concordance and tail dependence coefficients. The text is a coherent piece of work and shows a deep understanding of the subject.

To conclude, this work undoubtedly fulfils the requirements of a doctoral thesis.

The contributions of the thesis are to some extent published in the following articles

- Gijbels, I., V. Kika, and M. Omelka (2020). Multivariate tail coefficients: Properties and estimation. *Entropy 22*(7), Article ID 728.
- Gijbels, I., V. Kika, and M. Omelka (2021). On the specification of multivariate association measures and their behaviour with increasing dimension. *Journal of Multivariate Analysis 182*, Article ID 104704.
- Gijbels, I., V. Kika, and M. Omelka (2021). Choice of smoothing parameter in multivariate copula-based tail coefficients. Submitted and under review.

Sincerely,

Dr. Sebastian Fuchs