

August 2021

## Report on the doctoral thesis of Vojtech Kika

“Copula-based multivariate association measures and tail coefficients”

As summarized in the title, the two main topics in this thesis are the multivariate association measures and the multivariate tail coefficients of a general  $d$ -dimensional random vector  $\mathbf{X}$ . The author starts from the 2-dimensional case and works out the possible generalizations into higher dimension.

The thesis is well-structured around these two main topics. Vojtech Kika shows that he masters the existing literature and his new results are of a high level and give a substantial broadening into dimensions higher than 2. The new findings are also illustrated by practical applications on environmental quality and Euro Stoxx 50 data.

**Chapter 1** is introductory material with in particular the required tools from copula theory. The copula associated with the random vector  $\mathbf{X}$  describes the complete dependence structure and is the basis for the further chapters of the thesis.

**Chapter 2** provides a discussion on the axioms for multivariate association measures and the verification of their validity. An interesting result is the limiting behaviour of certain association measures when the dimension tends to infinity. Further sections deal with the nonparametric estimation of association measures and the Environmental Quality Index data application.

In **Chapter 3** a set of desirable properties is formulated for ‘reasonable’ multivariate tail dependence coefficients and these are verified for the existing examples. For the case of Archimedean copulas, the limiting behaviour for  $d$  tending to infinity is explored.

**Chapter 4** deals with statistical estimators for various tail coefficients and their asymptotic properties (weak consistency, limit distribution). Important solutions are established for the problem of how to estimate the limits appearing in the

definition of a tail coefficient. The practical use of multivariate tail coefficients is illustrated with the Euro Stoxx 50 data.

**Chapter 5** deals with variable clustering by means of an extremal dependence coefficient.

**Chapter 6** gives a summary of the results obtained and formulates some topics for further research.

Vojtech Kika has already two papers published in peer-reviewed journals (Entropy (2020) and Journal of Multivariate Analysis (2021)). A third manuscript is currently submitted and under review. During the past four years I was happy to attend two interesting seminars of Vojtech where he demonstrated the progress of his thesis and where he discussed his results with the doctoral committee members.

**In conclusion:**

This high-quality thesis demonstrates that Vojtech Kika has the methodological knowledge and the practical expertise to be a mature independent researcher.

I wholeheartedly express a positive recommendation for public defence of this thesis.

Prof. Dr. Noël Veraverbeke

University of Hasselt

Center for Statistics

3590 Diepenbeek (Belgium)

noel.veraverbeke@uhasselt.be