The thesis consists of seven research papers. The first two papers study the properties of fragmented convex functions, mainly the so-called maximum principle. The first paper deals with convex functions defined on compact convex subsets of locally convex spaces, the second one with the abstract convex functions defined on general compact Hausdorff spaces. The next four papers present results in the spirit of the well-known Banach-Stone theorem in the area of subspaces of continuous functions. The first of those four papers deals with the spaces of affine continuous complex functions on compact convex sets. The second paper extends the results of the first one to the context of general subspaces of continuous functions defined on locally compact spaces. The other two papers further extend the previous results to the case of Banach space-valued and Banach lattice-valued functions, respectively. The last paper is devoted to the study of the Banach-Mazur distance between subspaces of vector-valued continuous functions that have scattered boundaries.