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EXTENDED ABSTRACT OF DOCTORAL THESIS

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Vector-valued integral representation

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Contents

Introduction

The thesis consists of seven research papers. The first two papers deal mainly with the so-called maximum principles for convex functions, the remaining papers present results in the area of isomorphic Banach-Stone type theorems for subspaces of continuous functions.

CHAPTER 1

Maximum principles for convex functions

The first two papers of the thesis mainly deal with the so-called maximum principles for convex functions. We recall that if X is a compact convex set in a Hausdorff locally convex space, then a function $f : X \to \mathbb{R}$ is said to satisfy the maximum principle if $f \leq 0$ on X provided $f \leq 0$ on the set $\operatorname{ext} X$ of extreme points of X. The maximum principle for semicontinuous affine functions and affine functions of the first Baire class is a well-known part of the classical Choquet theory, see e. g. [**31**, Theorem 3.16, Theorem 3.85 and Corollary 4.23]. In a recent paper [**16**] it is shown that the maximum principle is valid for a larger class of affine functions, that is, the class of fragmented affine functions.

We recall that a function $f : X \to \mathbb{F}$, where $\mathbb{F}$ stands for $\mathbb{R}$ or $\mathbb{C}$, is said to have the *point of continuity property* if $f|_F$ has a point of continuity for any $F \subset X$ nonempty closed, and the function f is called *fragmented* if for any $\epsilon > 0$ and nonempty closed set $F \subset X$ there exists a relatively open nonempty set $U \subset F$ such that $\operatorname{diam} f(U) < \epsilon$.

By [**31**, Theorem A.121], a function $f : X \to \mathbb{F}$ on a locally compact space X is fragmented if and only if it has the point of continuity property. Any semicontinuous function $f : X \to \mathbb{R}$ is fragmented (see [**31**, Proposition A.122]) and, if X is a completely metrizable, then the function f is fragmented if and only if f is a classical Baire-one function (see [**31**, Theorem A.127]).

1. On fragmented convex functions

(with J. Spurný, Journal of Mathematical Analysis and Applications 484(2) (2020))

The main result of this paper extends the result of [**16**] from the context of affine fragmented functions to the area of convex fragmented functions. It shows that the maximum principle is true not only for fragmented convex functions, but also for bounded monotone limits of sequences of fragmented convex functions, see [**37**, Theorem 2.6]. A key ingredient for this result is [**37**, Lemma 2.1], a geometric lemma concerning semi-extremal sets (we recall that a subset F of a compact convex set X is called semi-extremal if $X \setminus F$ is convex). Also, in the process of proving [**37**, Theorem 2.6] we generalize a result of Raja [**33**] by showing that the set of points of upper-semicontinuity of a fragmented convex function is a relatively dense set in $\operatorname{ext} X$, see [**37**, Lemma 2.4] and [**37**, Lemma 2.5]. As a corollary of [**37**, Theorem 2.6] we recover the classical Rainwater theorem, see [**37**, Corollary 2.8]. As a subsequent result we verify that fragmented convex functions respect the Choquet ordering, see [**37**, Theorem 3.1]. Next, in [**32**], it was showed that nonempty closed semi-extremal subsets of X intersect $\operatorname{ext} X$. We prove a similar result by showing that any nonempty semi-extremal convex resolvable set intersects $\operatorname{ext} X$, see [**37**, Theorem 4.2] (we recall that a set $F \subseteq X$ is resolvable if its characteristic function is fragmented). Finally, in Section 5 we considered an abstract function space $\mathcal{H}$ on a locally compact space K with the Choquet boundary $\operatorname{Ch}_{\mathcal{H}} K$. We showed that a completely $\mathcal{H}$-affine fragmented function f on K satisfies $\sup_{x \in K} |f(x)| = \sup_{x \in \operatorname{Ch} K} |f(x)|$, generalizing thus a theorem of Phelps, see [**19**, Theorem 2.3.8].

2. Maximum principle for abstract convex functions

(with J. Spurný, Journal of Convex Analysis 28 (2021))

The maximum principles can also be considered for the case of abstract $\mathcal{H}$-convex functions defined with respect to a function cone $\mathcal{H} \subseteq \mathcal{C}(K)$, where K is a compact space. The maximum principle for semicontinuous or Baire $\mathcal{H}$-convex functions is well-known, see [**31**, Section 3.9]. In this paper, we extend the result [**37**, Theorem 2.6] of the first paper by showing that the maximum principle holds for bounded monotone limits of sequences of fragmented $\mathcal{H}$-convex functions, see [**40**, Theorem 4.2]. The generalization of the result from the case of compact convex sets to this more abstract setting was not at all straightforward, as the main geometric idea (see [**37**, Lemma 2.1]) could not be easily used here. To be able to use similar geometric ideas, we needed to transfer this abstract setting to the area of the so-called ordered compact convex sets, see [**40**, Section 3].

CHAPTER 2

Isomorphic Banach-Stone type theorems for subspaces of continuous functions

The rest of the papers contained in the thesis partly shows an application of the above theory. In these papers we obtained results in the spirit of the Banach-Stone theorem for spaces of continuous functions. In four of these research papers, the maximum principle for fragmented affine functions plays a crucial role. We first collect some of the known results in this area.

To start with, the well-known Banach-Stone theorem asserts that, given two (Hausdorff) compact spaces K_1, K_2, they are homeomorphic if and only if the spaces $\mathcal{C}(K_1)$ and $\mathcal{C}(K_2)$ are isometric. Holszctynski [**27**] proved the following one-sided version of the Banach-Stone theorem: Let K_1 and K_2 be compact Hausdorff spaces. Suppose that there exists a linear isometry from $\mathcal{C}(K_1)$ into $\mathcal{C}(K_2)$. Then there is a closed subset L of K_2 and a continuous function of L onto K_1. Since then, there has been a vast amount of results in the spirit of these two results. As it turns out, in most of the results the considered compact spaces and spaces of continuous functions can be replaced by locally compact spaces and spaces of continuous functions vanishing at infinity, respectively. Here we focus on results that could be described as isomorphic versions of the Banach-Stone theorem.

Amir [**1**] and Cambern [**2**] independently proved the following improvement of the Banach-Stone theorem. They showed that compact spaces K_1 and K_2 are homeomorphic if there exists an isomorphism $T\colon \mathcal{C}(K_1, \mathbb{F}) \to \mathcal{C}(K_2, \mathbb{F})$ with $\|T\| \cdot \|T^{-1}\| < 2$. Cohen [**15**] and Drewnowski [**17**] gave alternative proofs of this result. Jarosz in [**28**] proved an extension of the theorem of Holszctynski by working with into isomorphims $T : \mathcal{C}_0(K_1) \to \mathcal{C}_0(K_2)$ with $\|T\|\,\|T^{-1}\| < 2$ instead of into isometries.

These results have been extended to the context of vector-valued functions. In a recent paper [**13**], it was showed that if E is a real or complex reflexive Banach space with $\lambda(E) > 1$, then for all locally compact spaces K_1, K_2, the existence of an isomorphism $T : \mathcal{C}_0(K_1, E) \to \mathcal{C}_0(K_2, E)$ with $\|T\| \cdot \|T^{-1}\| < \lambda(E)$ implies that the spaces K_1, K_2 are homeomorphic. Here

$$\lambda(E) = \inf\{\max\{\|e_1 + \lambda e_2\| : \lambda \in \mathbb{F}, |\lambda| = 1\} : e_1, e_2 \in S_E\}$$

is a parameter introduced by Jarosz in [**29**].

It is easy to check that $\lambda(\mathbb{F}) = 2$, thus this result recovers the theorem of Amir and Cambern. The authors of [**13**] also showed that the constant $\lambda(E) = 2^{\frac{1}{p}}$ is the best possible for $E = l_p$, where $2 \leq p < \infty$. In the paper [**22**] the authors give a vector-valued extension of the Holsztynski theorem based on the above constant $\lambda(E)$.

In [**21**] it was shown how the constant $\lambda(E)$ can be improved, if one moreover assumes that E is a real Banach lattice and $T : \mathcal{C}_0(K_1, E) \to \mathcal{C}_0(K_2, E)$ is a

Banach lattice isomorphism. The constant $\lambda(E)$ may be then replaced by

$$\lambda^+(E) = \inf\{\max\{\|e_1 + e_2\|, \|e_1 - e_2\|\}, e_1, e_2 \in S_E, e_1, e_2 \geq 0\}.$$

It is easily seen that $\lambda(E) \leq \lambda^+(E)$ for each Banach lattice E, and in [**21**] it is shown that for $E = \ell_p$, where $1 \leq p < 2$, the inequality is strict. Moreover, the constant $\lambda^+(E)$ is optimal for $E = \ell_p$, where $p \in [1, \infty)$. In the paper [**23**] the authors investigate positive isomorphisms between $\mathcal{C}(K, E)$ spaces that are not necessarily surjective.

Another way to extend the Amir-Cambern theorem is directed to replacing the number 2 by a larger number. After proving the result, Amir conjectured that the number 2 appearing in the Amir-Cambern theorem may be replaced by 3. Cohen [**14**] showed that this is not true in general by providing a counterexample. However, in [**25**], Gordon proved that it is true in the class of countable compact spaces: If K_1, K_2 are nonhomeomorphic countable compact spaces and $T : \mathcal{C}(K_1) \to \mathcal{C}(K_2)$ is an isomorphism, then $\|T\| \|T^{-1}\| \geq 3$.

The result of Gordon was extended in [**7**, Theorem 1.5], where the authors show that if E is a Banach space having non-trivial cotype, and such that for every $n \in \mathbb{N}$, E^n contains no subspace isomorphic to E^{n+1}, then countable compact spaces K_1 and K_2 are homeomorphic provided there exists an isomorphism $T : \mathcal{C}(K_1, E) \to \mathcal{C}(K_2, E)$ with $\|T\| \|T^{-1}\| < 3$. It is clear that every finite-dimensional Banach space satisfies the above condition, and the authors also show in [**7**, Remark 4.1] that there exist many infinite-dimensional Banach spaces that satisfy it.

Next, we collect some results in this area concerning general isomorphisms T (with the number $\|T\| \|T^{-1}\|$ being possibly large). The first result in this area, also referred to as the weak Banach-Stone theorem, is due to Cengiz [**11**], who showed that locally compact Hausdorff spaces K_1 and K_2 have the same cardinality provided that the spaces $\mathcal{C}_0(K_1, \mathbb{F})$ and $\mathcal{C}_0(K_2, \mathbb{F})$ are isomorphic.

This result was extended to the context of vector-valued functions by Galego and Rincón-Villamizar in [**20**], who showed that if E is a Banach space not containing an isomorphic copy of c_0, then either both K_1 and K_2 are finite or K_1 and K_2 have the same cardinality provided that the spaces $\mathcal{C}_0(K_1, E)$ and $\mathcal{C}_0(K_2, E)$ are isomorphic.

The next property of locally compact spaces that is closely connected with isomorphisms between the respective spaces of continuous functions is the scattered structure. Since it is known that a compact space K is scattered if and only if $\mathcal{C}(K)$ is Asplund (see e. g. [**18**, Theorem 1.1.3]), and it is well-known that the class of Asplund Banach spaces is closed under isomorphisms, it follows that if $\mathcal{C}(K_1)$ is isomorphic to $\mathcal{C}(K_2)$ then K_1 is scattered if and only if K_2 is scattered. This fact was improved in [**4**, Theorem 1.4], where the author shows that if E is a Banach space not containing an isomorphic copy of c_0, K_2 is a scattered locally compact space and $\mathcal{C}_0(K_1)$ embeds isomorphically into $\mathcal{C}_0(K_2, E)$, then K_1 is also scattered.

Moreover, there have been proven estimates of the Banach-Mazur distance of $\mathcal{C}(K)$ spaces from $\mathcal{C}_0(\Gamma, E)$ spaces, where Γ is a discrete set, and from $\mathcal{C}(F)$, where F is a compact space of height 2 (in particular for $F = [0, \omega]$), based on the height of the compact space K. It was proved in [**9**, Theorem 1.2] that if K is a compact space with the n-th derivative $K^{(n)}$ nonempty for some $n \in \mathbb{N}$, F is a compact space with $F^{(2)} = \emptyset$ and there exists an isomorphism $T : \mathcal{C}(K) \to \mathcal{C}(F)$, then $\|T\| \|T^{-1}\| \geq 2n - 1$. Moreover, if $\left|K^{(n)}\right| > \left|F^{(1)}\right|$, then $\|T\| \|T^{-1}\| \geq 2n + 1$.

In [**5**, Theorem 1.1] it has been showed that if Γ is an infinite discrete space, E is a Banach space not containing an isomorphic copy of c_0 and $T : \mathcal{C}(K) \to \mathcal{C}_0(\Gamma, E)$ is an into isomorphism, then for each $n \in \mathbb{N}$, if $K^{(n)} \neq \emptyset$, then $\|T\|\,\|T^{-1}\| \geq 2n+1$. Similar results for isomorphisms with range in $\mathcal{C}_0(\Gamma, E)$ spaces were proven before in [**6**] and [**8**].

Starting in [**12**], and continuing in [**30**], [**16**], [**35**] and [**38**], the theorem of Amir and Cambern was extended to the context of subspaces. The papers [**12**], [**30**], [**16**], [**35**] deal with the spaces of continuous affine functions on compact convex sets. The paper [**12**] may be viewed as the first generalization of the Amir-Cambern theorem to the context of subspaces. The next papers [**30**] and [**16**] successively improved the result of [**12**] by removing redundant assumptions. The progress to final result for spaces of real continuous affine functions, contained in [**16**], was possible due to the maximum principle for fragmented affine functions.

1. Isomorphisms of spaces of affine continuous complex functions

(with J. Spurný, Mathematica Scandinavica 125 (2019), 270-290)

This paper extends the result of [**16**] to the case of complex continuous affine functions. Let $\mathfrak{A}(X, \mathbb{C})$ stands for the space of affine continuous complex functions on a compact convex set X. It is shown that if X, Y are compact convex sets such that each point of $\operatorname{ext} X$ and $\operatorname{ext} Y$ is a weak peak point, and $T\colon \mathfrak{A}(X, \mathbb{C}) \to \mathfrak{A}(Y, \mathbb{C})$ is an isomorphism satisfying $\|T\|\cdot\|T^{-1}\| < 2$, then $\operatorname{ext} X$ is homeomorphic to $\operatorname{ext} Y$, see [**35**, Theorem 1.1] (We recall that a point x in a compact convex set X is called a *weak peak point* if given $\varepsilon \in (0,1)$ and an open set $U \subset X$ containing x, there exists a in the unit ball $B_{\mathfrak{A}(X,\mathbb{C})}$ of $\mathfrak{A}(X, \mathbb{C})$ such that $|a| < \varepsilon$ on $\operatorname{ext} X \setminus U$ and $a(x) > 1-\varepsilon$). The paper also contains a generalization of the result of Cengiz [**11**]: It is proved in [**35**, Theorem 4.2] that if X, Y are compact convex sets such that $\mathfrak{A}(X, \mathbb{C})$, $\mathfrak{A}(Y, \mathbb{C})$ are isomorphic, and if each point of $\operatorname{ext} X$ and $\operatorname{ext} Y$ is a split face, then the cardinality of $\operatorname{ext} X$ is equal to the cardinality of $\operatorname{ext} Y$. As a corollary, using the transfer to the state space, we obtained analogous results for spaces of selfadjoint complex function spaces, see [**35**, Section 5].

2. Small-bound isomorphisms of function spaces

(with J. Spurný, Journal of the Australian Mathematical Society (2020), 1-18)

This paper futher extends the results for affine functions on compact convex sets to general subspaces of $\mathcal{C}_0(K, \mathbb{F})$ spaces. We recall that if $\mathcal{H}$ is a subspace of $\mathcal{C}_0(K, \mathbb{F})$ then the Choquet boundary $\operatorname{Ch}_{\mathcal{H}} K$ of K with respect to $\mathcal{H}$ is defined as the set of points $x \in K$ such that the evaluation functional defined for $h \in \mathcal{H}$ as $i(x) : h \mapsto h(x)$ is an extreme point of the compact convex set $B_{\mathcal{H}^*}$ endowed with the w^* topology. If $\mathcal{H}$ is the space of affine continuous functions on a compact convex set X, then $\operatorname{Ch}_{\mathcal{H}} K = \operatorname{ext} X$, and if $\mathcal{H} = \mathcal{C}_0(K, \mathbb{F})$, then $\operatorname{Ch}_{\mathcal{H}} K = K$. The final result in the spirit of the Amir-Cambern theorem for subspaces of scalar functions (see [**38**, Theorem 1.1]), reads as follows. For $i = 1, 2$, let $\mathcal{H}_i \subseteq \mathcal{C}_0(K_i, \mathbb{F})$ be closed subspaces such that all points in their Choquet boundaries are weak peak points. If there exists an isomorphism $T\colon \mathcal{H}_1 \to \mathcal{H}_2$ with $\|T\|\,\|T^{-1}\| < 2$, then their Choquet boundaries $\operatorname{Ch}_{\mathcal{H}_i} K_i$ are homeomorphic (we recall that $x \in K_i$ is a *weak peak point* (with respect to $\mathcal{H}_i$) if for a given $\varepsilon \in (0,1)$ and a neighborhood U of x there exists a function $h \in B_{\mathcal{H}_i}$ such that $h(x) > 1 - \varepsilon$ and $|h| < \varepsilon$ on $\operatorname{Ch}_{\mathcal{H}_i} K_i \setminus U$). It turns out that this result is in a sense optimal since the assumption of weak peak points cannot be omitted (see [**26**]), and the bound 2 is optimal even for $\mathcal{C}(K)$ spaces (see [**25**]). The theorem [**38**, Theorem 1.2] gives a one-sided version of [**38**, Theorem 1.1], and thus provides an extension of the theorem of Holsztynski to this context. The paper [**38**] also contains a weak Banach-Stone theorem for subspaces: If for $i = 1, 2$, $\mathcal{H}_i \subseteq \mathcal{C}_0(K_i, \mathbb{F})$ are closed subspaces such that all points in their Choquet boundaries are weak peak points and there exists an isomorphism $T\colon \mathcal{H}_1 \to \mathcal{H}_2$, then their Choquet boundaries $\operatorname{Ch}_{\mathcal{H}_i} K_i$ have the same cardinality (see [**38**, Theorem 1.3]).

3. Isomorphisms of subspaces of vector-valued continuous functions

(with J. Spurný, Acta Mathematica Hungarica (2020))

This paper gives vector-valued extensions of [**38**, Theorem 1.1] and [**38**, Theorem 1.3] in the spirit of papers [**13**] and [**10**]. The first main result, see [**36**, Theorem 1.1], shows that if for $i = 1, 2$, $\mathcal{H}_i$ is a closed subspace of $\mathcal{C}_0(K_i, E_i)$ for some locally compact space K_i and a reflexive Banach space E_i over the same field $\mathbb{F}$ satisfying $\lambda(E_i) > 1$, such that each point of the Choquet boundary $\operatorname{Ch}_{\mathcal{H}_i} K_i$ of $\mathcal{H}_i$ is a weak peak point and if $T\colon \mathcal{H}_1 \to \mathcal{H}_2$ is an isomorphism satisfying $\|T\| \cdot \|T^{-1}\| < \min\{\lambda(E_1), \lambda(E_2)\}$, then $\operatorname{Ch}_{\mathcal{H}_1} K_1$ is homeomorphic to $\operatorname{Ch}_{\mathcal{H}_2} K_2$. The second main result [**36**, Theorem 1.2] shows that if for $i = 1, 2$, $\mathcal{H}_i$ is a closed subspace of $\mathcal{C}_0(K_i, E_i)$ for some locally compact space K_i and a Banach space E_i over the same field $\mathbb{F}$, such that for $i = 1, 2$, E_i does not contain an isomorphic copy of c_0, each point of $\operatorname{Ch}_{\mathcal{H}_1} K_1$ and $\operatorname{Ch}_{\mathcal{H}_2} K_2$ is a weak peak point and $T\colon \mathcal{H}_1 \to \mathcal{H}_2$ is an isomorphism, then either both sets $\operatorname{Ch}_{\mathcal{H}_1} K_1$ and $\operatorname{Ch}_{\mathcal{H}_2} K_2$ are finite or they have the same cardinality. Since the vector-valued Choquet theory has not been studied extensively, in the process we needed to come up with a new definition of a Choquet boundary and weak peak points of a vector-valued space $\mathcal{H}$, and we proved some auxiliary results, that are well-known in the classical Choquet theory, such as distribution of the Choquet boundary in the dual ball of $\mathcal{H}$ (see [**36**, Lemma 2.1]), or representation of functionals on $\mathcal{H}$ by vector measures (see [**36**, Lemma 2.2]).

4. An Amir-Cambern theorem for subspaces of Banach lattice-valued continuous functions

(with J. Spurný, Banach Journal of Mathematical Analysis 15 (2021))

The paper [**39**] provides a generalization of results of [**21**] to the context of subspaces of Banach lattice-valued continuous functions. The main result, see [**39**, Theorem 1.1], reads as follows. For $i = 1, 2$, let $\mathcal{H}_i$ be a closed subspace of $\mathcal{C}_0(K_i, E_i)$ for some locally compact space K_i and a real reflexive Banach lattice E_i satisfying $\lambda^+(E_i) > 1$. Assume that each point of the Choquet boundary $\operatorname{Ch}_{\mathcal{H}_i} K_i$ of $\mathcal{H}_i$ is a weak peak point and let $T\colon \mathcal{H}_1 \to \mathcal{H}_2$ be an isomorphism satisfying

$$\|T\| \cdot \left\|T^{-1}\right\| < \min\{\lambda^+(E_1), \lambda^+(E_2)\}$$

such that T and T^{-1} preserve positive elements, that is,

$$f \in \mathcal{H}_1^+ \quad \text{if and only if} \quad T(f) \in \mathcal{H}_2^+.$$

Then $\operatorname{Ch}_{\mathcal{H}_1} K_1$ is homeomorphic to $\operatorname{Ch}_{\mathcal{H}_2} K_2$.

We were able to extend the result of [**21**] to this general setting only in the case where the Banach lattices are reflexive, which is not needed in [**21**]. The reason for this is that meanwhile operators from $\mathcal{C}(K, E_1)$ spaces to E_2 may be represented by Borel measures on K with values in $L(E_1, E_2^{**})$, the space of bounded linear operators from E_1 to E_2^{**} (see [**39**, Remark 5.1]), such a representation is not available for operators from $\mathcal{H}_1 \subset \mathcal{C}(K, E_1)$ to E_2. On the other hand, we were able to replace the assumption that the isomorphism T is a Banach lattice isomorphism by the weaker condition that T and T^{-1} preserve positive elements.

The general strategy of the proof of [**39**, Theorem 1.1] was very similar to the one of [**36**, Theorem 1.1], but we needed to make some adjustments to control the positivity of the elements considered. In [**36**], we used as an important ingredient of the proof the fact proved in [**3**] that if E is a reflexive Banach space and K is a compact space, then the space $\mathcal{C}(K, E)^{**}$ is isometrically isomorphic to the space $\mathcal{C}(Z, E_w)$, where Z is a compact Hausdorff space depending on K, and E_w denotes E equipped with its weak topology. Here we used a simpler approach that required less theory.

5. On the Banach-Mazur distance between continuous function spaces with scattered boundaries

(submitted, available at https://arxiv.org/abs/2012.00334)

This paper contains an extension of the results of papers [**24**] and [**7**] to the context of subspaces. It shows that for some class of subspaces, the constant 2 appearing in the Amir-Cambern theorem may be replaced by 3, see [**34**, Theorem 1.2]. Also, it contains an extension of some of the results from [**4**] and [**9**]. We recall that in those papers, the authors obtained estimates of the Banach-Mazur distance of $\mathcal{C}(K)$, where K scattered compact space, from c_0 and $\mathcal{C}(F)$, where F is a scattered compact space of height 2. In [**34**, Theorem 1.3], we proved a more general result for subspaces of continuous functions. As a consequence of this result, we obtained the estimate of the Banach-Mazur distance between spaces $\mathcal{C}(K_1)$ and $\mathcal{C}(K_2)$, where K_1 and K_2 are scattered compact spaces of finite height, see [**34**, Corollary 1.5]. From this result it also follows that if $\mathcal{C}(K_1)$ embeds isomorphically into $\mathcal{C}(K_2)$ and height of K_2 is finite, then also height of K_1 is finite (see again [**34**, Corollary 1.5]).

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