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Doctoral thesis „Asymptotic inference for stochastic geometry models" by Daniela Flimmel

Stochastic geometry is the branch of probability dealing with spatial random structures such as point processes, random sets, random tessellations or random graphs, which arise in many other fields like geology, material sciences or communication networks. Since such observed structures are usually very large, one studies their asymptotic behaviour as the observation window or the intensity is increased. Often one is interested in showing that some quantities converge in distribution to a Gaussian random variable. In the thesis under consideration, three different approaches to this problem are presented and applied to new examples.

The first chapter of the thesis is a very comprehensive introduction to models from stochastic geometry, while the second chapter concerns the Malliavin-Stein method and some of its preliminaries such as Stein's method and Malliavin calculus. Results from Torrisi (2017), which rely on the Malliavin-Stein method, are applied to the normal approximation of some functionals of Gibbs processes (see Flimmel and Benes (2018)).

In the third chapter, first several existing results for the normal approximation of stabilising functionals are presented. The main contribution of this part of the thesis is the application of stabilisation to statistics of weighted Poisson-Voronoi tessellations, a general class of tessellations constructed from an underlying marked Poisson process, which includes Poisson-Voronoi tessellations, Laguerre tessellations and Johnson-Mehl tessellations. For the estimation of the expectation of functionals of the typical cell of a weighted Poisson-Voronoi tessellation so-called minus-sampling estimators are studied. The results concern unbiasedness, asymptotic unbiasedness, consistency, variance asymptotics and asymptotic normality. Apart from the method of stabilisation a crucial ingredient of the proofs are careful computations to show that some of the estimators approximate each other and, thus, have the same limiting behaviour. The application to weighted Poisson-Voronoi tessellations is from a joint paper of the author with Zbynek Pawlas and Joseph E. Yukich, which was published in the Journal of Applied Probability.

The method of stabilisation has been applied to many different problems for the last twenty years. The application presented in the thesis is one of the few applications of a more statistical nature. Using stabilisation in the context of statistics might be a good direction for further research.

The last chapter of the thesis concerns planar cylinder processes. Here, one starts with a marked point process on the real axis from which lines in the plane are constructed and thickened. In this construction the locations of the points in $\mathbb{R}$ represent the distance from the line to the origin, while the marks provide the direction and the thickness. The thickened lines are called cylinders and throughout this chapter the union set Ξ of the cylinders is studied. The random set Ξ is intersected with an observation window of the form ρK and the asymptotic behaviour of the area $|\Xi \cap \rho K|_2$ for increasing ρ is considered. It is shown that the area fraction $|\Xi \cap \rho K|_2/|\rho K|_2$ converges in expectation and in the L^2-sense to some constant if the underlying point process on $\mathbb{R}$ is Brillinger mixing and satisfies some other mild assumptions (see Corollary 4.2 and Theorem 4.3). Under stronger mixing assumptions, an explicit formula for the asymptotic variance of $|\Xi \cap \rho K|_2/\rho^{3/2}$ is derived. The mentioned results are all part of a joint preprint of the author of the dissertation with Lothar Heinrich. The last result of this chapter, which is unpublished so far, is a central limit theorem for $|\Xi \cap \rho K|_2$ (see Theorem 4.5) under quite strict assumptions, in particular on the reduced factorial cumulant measures of the underlying point process. The idea of the proofs in this chapter is to express the cumulants of $|\Xi \cap \rho K|_2$ in terms of the factorial moment and cumulant measures of the underlying point process. The proof of the central limit theorem relies on the method of moments, i.e., one shows that all cumulants of order three and higher vanish as $\rho \to \infty$. Overall, the proofs in this chapter require some rather delicate calculations.

For Poisson cylinder processes in $\mathbb{R}^d$ with underlying flats of dimension k with $k \in \{1, \dots, d-1\}$, where the underlying point process is a Poisson process, similar results as in this chapter were established in two papers by Heinrich and Spiess. It is a very interesting but also difficult question what happens if one replaces the underlying Poisson process by other types of point processes. The results presented in the thesis are a first promising step in this direction and could lead to further research in this area.

While reading the thesis, the following scientific questions came to my mind:

- In the application of the Malliavin-Stein method so-called innovations of some Gibbs processes are considered. The innovations consist of two parts, a sum over the points of the point process and an integral involving the Papangelou intensity (see Definition 2.13). Considering the first sum is very natural and I can think of many statistics of this form, but I do not have an interpretation for the integral that is subtracted. Is there a motivation why one has this integral term involving the Papangelou intensity? Is there a way to replace the second term just by the expectation of the first term?

- For the results on the weighted Poisson-Voronoi tessellations it is assumed that the marks of the points are almost surely bounded, which is a rather strong assumption. From the proofs I guess that this assumption is necessary since the problem does not fit into the framework of stabilisation otherwise. This leads to the question whether stabilisation is the best approach to this problem. For example, one could try to apply second-order Poincaré inequalities such as Theorem 2.13 or Theorem 2.14 directly to the problem and hope that this leads to weaker assumptions on the underlying marks.
- Since the random set Ξ seems to be stationary, I guess that one has $\mathbb{E}|\Xi \cap \rho K|_2 = \mathbb{P}(0 \in \Xi)|\rho K|_2$. How does this fit together with Corollary 4.2 where one has a limit? Does it even hold with equality?
- The computations for the area of the planar cylinder processes are rather long and tedious. Nevertheless I would like to ask about possible generalisations. Can the approach be extended to higher dimensions or is there a point in the arguments that leads to the restriction to the plane? In their paper from 2013 on Poisson cylinder processes Heinrich and Spiess were able to deal with the surface area. Can something similar be also done for the situation studied in the dissertation?

The author has spent a lot of efforts on introducing preliminaries and on presenting existing results. It might be a matter of taste, but in my opinion this part of the thesis could have been shorter and stronger focused on the authors own research.

Attached to my report is a list with some minor comments (mostly typos), which is not very long for a work of this length.

The thesis is clearly written and well-structured. The applications of the described methods to Gibbs processes, to weighted Poisson-Voronoi tessellations and to planar cylinder processes are all new and interesting in my opinion. Some of the techniques applied in the thesis are rather technical and challenging. I would like to mention two noteworthy points. In the thesis two completely different methods are applied, namely on the one hand side the Malliavin-Stein method and stabilisation and on the other hand the method of cumulants. While one often studies only models with underlying Poisson or binomial point processes in stochastic geometry, a significant part of the thesis deals successfully with different point processes, which is known to be a difficult problem.

Overall, Daniela Flimmel has written a PhD thesis with many nice and interesting results, which demonstrates her ability for creative scientific work on a high level. Thus, I recommend the thesis for the defense.

Minor Comments

- I think it is Malliavin-Stein method and not Malliavin-Stein's method
- P. 14, line next to (1.6): P should be $\mathbb{P}$.
- P. 15, l. 10: process $\rightarrow$ processes
- p. 17, l. 12: i.e $\rightarrow$ i.e.
- P. 19, l. -11: (1.1) $\rightarrow$ 1.1
- P. 30, (1.14): The line must end with a period.
- P. 53, Definition 2.8: This pathwise definition can be only used if $f \in L^1(\lambda^n)$.
- P. 56, l. 15: The period at the end of the line is wrong.
- P. 66, l. -2: Lemma $\rightarrow$ lemma
- P. 78, l. 26: cause $\rightarrow$ causes
- P. 80, l. -10: $\mathbf{x} \in \mathbb{R}^d$ is not a point set.
- P. 82: The relation between the Poisson and the uniform moment condition is discussed twice, next to Definition 3.8 and next to Definition 3.9.
- P. 86, Definition 3.12: probabilities ... decays
- P. 88: l. -13: was proved $\rightarrow$ was used
- P. 92, l. 9: to be $\rightarrow$ be
- P. 95, l. 8: of vice versa $\rightarrow$ or vice versa
- P. 96, l. 24: carry $\rightarrow$ carries
- P. 103: Throughout the proof it seems that μ is the maximal size of the marks, which was denoted by a on page 96.
- P. 126, l. 7: The link to (2.17) does not make sense.
- P. 129, l. 1: The reference to Lemma 3 does not make sense since there is no Lemma 3.
- P. 131, (4.27): How can the right-hand side involve ρ if one lets $\rho \rightarrow \infty$?