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THE THEORY OF DESCRIPTIONS
Bertrand Russell's Road Towards Ontological Austerity

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Abstrakt:

Centrálním rysem filosofie Bertranda Russella od roku 1905 po zbytek jeho kariéry je systematické uplatňování Occamovy břitvy za účelem maximální redukce ontologických závazků. Rok 1905 byl v tomto ohledu pro Russella zásadní, neboť v něm publikoval přelomovou stat' "On Denoting". V této stati poprvé představil tzv. teorii deskripcí. Podle klasické interpretace Russellovy filosofie, kterou reprezentuje zejména stat' W. V. O. Quina "Russell's Ontological Development" (1966), Russell v období před teorií deskripcí tíhl k přijetí tzv. "Meinongináských" ontologických závazků: jednak jde o ontologické závazky vůči koherentním a neaktuálním entitám jako je současný král Francie (posibilia), jednak jde o závazky vůči kontradiktorickým entitám jako je kulatý čtverec. Podle Quina se Russell nemohl osvobodit od nutnosti takové ontologické závazky přijmout, dokud neměl po ruce teorii deskripcí. Cílem této práce je tuto klasickou interpretaci, kterou v posledních letech někteří badatelé zpochybňují, potvrdit a vyložit, odkud se Meinongiánské ontologické závazky v Russellově filosofii před "On Denoting" braly. Prověření otázky Meinonginaismu u raného Russella se neobejde bez detailní rekonstrukce jeho sémantických teorií před "On Denoting". Práce se věnuje zejména výkladu a kritice předchůdkyně teorie deskripcí, totiž kritice tzv. teorie denotačních pojmů, kterou Russell rozpracoval v *The Principles of Mathematics* (1903). Mezi další relevantní témata patří Russellova raná teorie propozice a s ní spojený problém jednoty propozice, Russellův raný logicismus, jeho pojem proměnné a obecnosti a tak dále. Jedním z dílčích cílů práce je prokázat, že Meinongiánský ontologický závazek vůči kontradiktorickým entitám, kterému se Russell bez teorie deskripcí nemohl vyhnout, je zcela neslučitelný s Russellovým logicismem. Závěrečná část práce se plně věnuje výkladu teorie deskripcí. Ten je rozvedený především s důrazem na to, aby bylo co nejvíce zřejmé, jak tato bezesporu revoluční teorie posloužila Russellovi k tomu, aby se problematických ontologických závazků úspěšně zbavil. Pozice, která je v souvislosti s Russellovým raným realismem označována jako "Meinongianismus", je důsledně odlišena od fenomenologické teorie intencionality Alexia Meinonga.

Abstract:

Applying Occam's razor in order to minimize ontological commitments is among the central methods of Bertrand Russell's philosophy after 1905 and onwards. The year 1905 was specially significant for Russell in that respect, as he published in this year the groundbreaking paper titled 'On Denoting'. In this paper, he introduced, for the first time, the today widely acknowledged semantical theory, Theory of Descriptions. According to the canonical interpretation of Russell, which is represented, for the most part, by W. V. O. Quine's paper 'Russell's Ontological Development' (1966), Russell tended to embrace 'Meinongian' ontological commitments: these were ontological commitments to coherent and non-actual entities such as the present king of France (possibilia) and ontological commitments to incoherent entities such as the round square. In Quine's view, Russell could not dodge such commitments until he discovered Theory of Descriptions. This interpretation has been challenged recently and it is the main objective of this essay to defend Quine against his opponents. I provide a detailed account of those parts of Russell's philosophy before 'On Denoting' which precluded him from refuting conclusively the problematic ontological commitments. In order to provide such an account, we dive deep into the early Russell's philosophical thought and reconstruct his then semantical views the primary source for which is *The Principles of Mathematics* (1903). This applies especially to the so-called Theory of Denoting Concepts, the predecessor of Theory of Descriptions. Besides this, we tackle other relevant topics such as Russell's early theory of propositions and the related problem of propositional unity, Russell's early logicism, his notion of variable and generality and so on. One of the secondary objectives of this essay is to demonstrate that any Meinongian ontological commitment to contradictory objects was incompatible with Russell's logicist view of mathematics. In the final part of the essay, an account of Theory of Descriptions will be provided, with special attention to showing how this theory helped Russell in dodging successfully the problematic Meinongian ontological commitments. The position associated with the early Russell's realism which we characterize as 'Meinongianism' is meticulously distinguished from Alexius Meinong's phenomenological theory of intentionality.

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Foreword

The research presented in the current essay is a continuation of my long term academic interest in Bertrand Russell's philosophy and logic, previously manifested in my Bachelor thesis which explored Ludwig Wittgenstein's criticisms of Russell's so-called Multiple-Relation Theory of Judgment and in my Master's thesis which dealt with the development of Russell's theory of mind, especially, then, with Russell's adoption of views hugely influenced by the psychology of William James and the behaviorism of John B. Watson. None of my previous works dealt, in sufficient detail, with Theory of Descriptions. Since this theory has been undoubtedly one of Russell's most influential contributions to (the analytic strand of) 20th century philosophy, and since it has been a theory that has proven so fruitful in various branches of science, the decision to make it the central theme of my PhD thesis came naturally. Moreover, it is a theory that has caused many controversies; and controversies, if well founded, are always a great source for developing one's philosophical acumen. Finally, Russell's logical atomism cannot be reasonably understood without a full grasp of the impact that Theory of Descriptions had upon the eliminative method at the heart of it.

I wish to thank my supervisor, Doc. Jan Palkoska, for his patience, as the research presented on the pages of the current essay undoubtedly took longer than should be appropriate.

Very special thanks belong to Prof. Gregory Landini to whose expert views on Bertrand Russell, and logic and scientific methodology in general, I am indebted to an extent that I am happy to acknowledge and appreciate. I have benefited greatly from our numerous debates about Russell, logic and philosophy. Of course, this does not mean that he is in any way responsible for any lack of cogency in my arguments if that happens to be the case. Neither does that mean that the interpretation of Russell proposed in the current essay matches his in every respect.

Finally, I want to express my thanks to my friend Mícheál Úa Séaghdha who was willing go through this difficult-to-read text and provide me with feedback that helped me improve significantly on both its formulation and its content.

Introduction

Bertrand Russell's 'On Denoting' [OD] published in 1905 is one of the most influential philosophical texts of 20th century. The theory presented on its pages has been intensely discussed in the field of philosophy of language and today it forms part of every rational curriculum students of philosophy are expected to master before they are able to complete their degrees. This is Russell's Theory of Descriptions—a theory that gives an intriguing account of how descriptions or, in Russell's then parlance, 'denoting phrases' contribute to the meaning of the sentences in which they are deployed. F. P. Ramsey called Russell's theory a 'paradigm of philosophy,' referring to its sweeping eliminative analysis based on distinguishing the logical form of a description from its surface form (a rendering that is merely 'grammatical').¹ Russell's analysis stands as a poignant methodological embodiment, within the constraints of the linguistic turn in philosophy, of the fundamental distinction of philosophy *per se* between an appearance of something and the reality of that thing hiding behind the misleading veil of its appearance.

In this essay, I will track down and reconstruct in detail what I take to be Russell's main reason for developing his Theory of Descriptions. The theory did not come to Russell all of sudden. It had a predecessor the knowledge of which hardly reaches beyond the circle of devoted Russellian scholars. This was a Frege-style semantical theory based on the idea that there are so-called denoting concepts.² This theory was explicated in and applied throughout *The Principles of Mathematics* [*Principles*] (1903).

According to W. V. O. Quine's influential version of the narrative concerning the revolution in Russell's thought brought about by the Theory of Descriptions, the ontology of *Principles* was populated by what Quine takes to be unwanted entities such as the present king of France.³ In reference to the Austrian philosopher Alexius Meinong, the contemporary literature calls such entities 'Meinongian'. Quine's account does not put the Theory of Denoting Concepts under scrutiny; it clearly implies, however, that it was an

1 Ramsey (1931), 263.

2 Henceforth, it is called the Theory of Denoting Concepts.

3 Quine presented this view in his 'Russell's Ontological Development' (1966).

attempt to provide a semantical account of descriptions which failed exactly where the Theory of Descriptions succeeded: It failed to purge the unwanted entities from Russell's ontology. The moment Russell embraced his Theory of Descriptions is thus depicted by Quine as a decisive step that put Russell on his path towards ontological austerity.

Quine's account has the advantage of being able to neatly accommodate Russell's own retrospective on the matter. Yet, it has been challenged. The early Russell's thought has received a growingly thorough examination in the last three decades, to a considerable extent due the fact that a large portion of Russell's unpublished works were made available by the editors of *The Collected Papers of Bertrand Russell* series. Quine's account should certainly not be taken for granted without any further inquiry, as Russell's defense of the Theory of Descriptions, in any of his published writings, is based on arguments against the naive semantics attributed by Russell to Meinong and arguments against G. Frege's semantics of *Sinn* and *Bedeutung*, not on criticisms explicitly aimed at his earlier position in *Principles*. Concerning Russell's criticisms of Frege, regardless of whether they are just or not, it is not difficult to see that the same or very similar arguments might apply to the Theory of Denoting Concepts, as denoting concepts are posits akin to Frege's *Sinne*. The notorious Grey's elegy argument is such. When it comes to Russell's arguments against Meinong's (but, more precisely, Meinong-style) naive semantics, the situation is much less clear.

Does the early Russell's semantical treatment of descriptions based on the idea that there are denoting concepts entail an ontological commitment to entities such as the present king of France or the round circle? Quine's answer is 'yes' in the case of possibilia (exemplified by the present king of France) and, as we will see, he remains agnostic in the case of contradictory entities. Some Russellian scholars proposed a different answer. According to them, Russell's theory of denoting concepts was designed to account for genuine referential failures! If this was the case, the superiority of the Theory of Descriptions over its predecessor, if there is any after all, certainly cannot consist in purging the unwanted, Meinongian entities from the early Russell's ontology.

I will argue in favor of Quine's interpretation. There is textual evidence, it must be admitted, that shows that even as early as in *Principles* Russell wanted his then semantical treatment of descriptions to account for genuine referential failures. Most probably, this

occurred at some later phase of writing the book. Could Russell deliver, though? I will argue that he could not.

There are passages in *Principles* that indicate that the so-called puzzle of negative existentials (existential judgements) forced the early Russell to adopt a view that made genuine referential failures impossible to accommodate for him. Russell could not dispense with Meinongian entities. Reading the passages in question carefully reveals that Russell envisaged the puzzle of negative existentials as a piece of reasoning that consists in a semantical application of a notion of intentionality (aboutness). This has been entirely omitted by the scholars who oppose Quine's interpretation of Russell.

I will provide the puzzle, as Russell understood it, with a detailed presentation in order to make explicit how a notion of intentionality was utilized in it. The puzzle will be reconstructed and most importantly it will also be applied to descriptions of contradictory entities. Russell could not dispense with both possibilia and contradictory entities. He could not embrace them either! I will explain in detail why embracing contradictory objects would be detrimental to the early Russell's logic and mathematics. This reveals another whole dimension of the early Russell's struggle to construe a satisfactory semantics of descriptions. His inability to cope with the puzzle of negative existentials in a way that avoided an ontological commitment to entities such as the round circle posed a serious threat to the foundational project of *Principles* as a whole. This might explain Russell's urgent need after the completion of *Principles* to elaborate on his semantical account of descriptions.⁴

4 That Russell felt in need of elaborating, after the completion of *Principles*, on his views concerning descriptions is evidenced in several manuscripts. I will refer to some of them later. (For comparison, see a more comprehensive account in Rodríguez-Consuegra (1989-90).) Russell's motivation to improve on his views was, to a great extent, due to a hope that a proper semantical treatment of descriptions could help him to deal with the paradox of classes named after him. It might be objected that this hope, not really any concern about possibilia and contradictory entities, fueled the progression of Russell's thought towards his Theory of Descriptions. This is a valid point, but only as long as we treat the two topics as sharply separated from each other. In this essay, I don't discuss Russell's paradox, but I believe that there is an important connection between Russell's early attempts to avoid it and the problems emerging with an admission of entities such as the present king of France and the round circle. Russell's hope that a semantical theory of descriptions would help him cope with the paradox consisted in a suspicion that its emergence is due to an illegitimate ontological commitment, the commitment to the existence of classes. Although this wasn't, in the end, Russell's preferred solution, the so-called 'no classes theory' remained part of his theory of mathematics. Russell's idea that there are illegitimate ontological commitments was at the center of the post-*Principles* development of his semantical thought.

The first six chapters of this essay are concerned with ontological and semantical doctrines introduced by Russell alongside his development in *Principles* of the idea that non-applied mathematics is reducible to logic (logicism). In Chapter 1, the early Russell's notion of reality and its relation to language is discussed. Special attention will be given to the related notion of a proposition. As for their structure, propositions are understood in terms of the grammar of sentences. They serve as sentential meanings and stand in logical structures related to one another by material implication. Unlike sentences, propositions are essentially mind-independent. We will distinguish two sorts of propositions: simple cases called 'Russellian propositions' and more complicated ones which serve as meanings of sentences that contain one or more denoting phrases.

Russell's approach to ontology was pluralist. He envisaged reality, in the widest sense of everything that has being, as composed of true and false propositions, where true propositions are identified with what we commonly call 'facts'. Although Russell attributes to propositions sentence-like structures and endows them with the quality of being true or false, a sharp division between the representational realm of ideas and linguistic items and reality, which is essentially mind-independent, must be meticulously respected if we wish to understand the early Russell correctly.

Russell's way of expressing himself throughout his early works does not make it particularly easy for the reader to grasp the sharp division between a representation and the mind-independent reality represented. We will take Scott Soames as an example of a reader who was seduced by Russell's somewhat cryptic parlance into distorting savagely the notion of proposition developed in *Principles*. Our criticism of Soames will help to establish a method of reading Russell's texts in the intended, strictly anti-psychologistic manner. This method proves useful right away. It proves useful in grasping the meaning of the passages in which Russell addresses the notorious problem of propositional unity which is the last topic discussed in Chapter 1.

The problem of unity is tied up with the regress argument proposed by F. H. Bradley whose purpose was to show that relations, as long as they are conceived as independent and external (and, therefore, irreducible), cannot be more than convenient abstractions. This is an argument set forth in Chapter 3 of *Bradley's Appearance and Reality* and referred to in the literature as 'Bradley's regress'. The argument will be

reconstructed and the notion of an external or independent relation clarified as we explore the early Russell's metaphysics, which embraced relations as external in a relevant sense.

Russell knew Bradley. Russell admired him while studying in Cambridge and endorsed his Hegelian views as applied to mathematics until 1899 when G. E. Moore's passionate attacks on idealism changed Russell's mind in favor of embracing a realist position. In *Principles*, Russell makes an elaborated attempt to refute Bradley's regress. We will divide Russell's reply to Bradley into three parts and, in the course of the subsequent discussion, we will examine Russell's idiosyncratic views on propositional unity.

Russell's multi-faceted reply to Bradley was unsuccessful. Instead of providing solid grounds for the idea of how the constituents of a proposition relate to each other as to yield a whole (the proposition), it reveals yet another serious problem. Russell's notion of propositional unity is perfectly adequate with regard to true propositions (facts), but expecting the same sort of unity in the case of false propositions makes false proposition true.

The onset of Chapter 2 is devoted to a question which is rarely addressed in the secondary literature on Russell. Commentators often show fondness for expressing their stance towards Russell's semantical theories of denoting phrases (descriptions), but rarely do they care to clarify Russell's notion of a denoting phrase. What kind of notion is Russell's notion of a denoting phrase? In my view, it was an essentially epistemological notion. Graham Stevens treats it as a grammatical notion, but, as we will see, it is often impossible to identify an expression as a denoting phrase without deploying an intensional sort of equivalence. Drawing on how the early Russell utilizes his then theory of denoting in explaining that our knowledge of infinite collections is possible, we expose his epistemological notion of denoting, whose content can be articulated in terms of the distinction between object-dependence and object-independence. In this I follow the approach adopted by Stephen Neale in his 'Descriptions' (1990).

OD is a fun paper to read. One might think that Russell developed the Theory of Descriptions in order to make elaborated philosophical puns, to turn a classic poem into an insanely cryptic puzzle and to expel the present king of France from his poor Meinongian exile into yet poorer nothingness. The relevance of Russell's groundbreaking semantical theory for his project of reducing non-applied mathematics to logic can be hardly

discernible in the paper. Make no mistake, the Theory of Descriptions bore extreme importance in the project.

If we want to fully appreciate the importance of denoting for Russell's logicism, we need to examine the early Russell's concept of generality. In *Principles*, generality is expressed by two kinds of variables, real and apparent. Apparent variables, analogous to the modern bound variables, belong to the content of wffs called by Russell 'formal implications' (Russell borrowed the term from G. Peano), whereas real variables occur in so-called propositional functions. Russell's notion of a propositional function is controversial and it will be discussed at length, but it is the notion of formal implication which will require special attention. Every wff of logic and, provided Russell's project can be successfully carried out, every statement of non-applied mathematics has, according to the Russell of *Principles*, the form of a formal implication—this is, loosely speaking, the form of a universally quantified conditional which contains logical constants, one or more apparent variables and nothing else. Logic is the science of the synthetic *a priori*, hence our knowledge of it must profess maximum generality.

In *Principles*, Russell attempts to provide apparent variables, linguistic vehicles expressing the required generality, with a semantical account based on his theory of denoting concepts. He assimilates the meaning of a variable to the meaning of the natural language pronoun 'anything'. The meaning of a variable is equivalent to the meaning of 'any term' (in the technical sense of 'term' in the early Russell's terminology).

Chapter 3 provides a detailed account of the early Russell's semantical theory of descriptions and aims to identify and discuss their weak points. The theory was grounded in a belief that there are denoting concepts. Such concepts are akin to Frege's *Sinne*. The function of a denoting concept is to pick out or point to something (the denotation) by means of a descriptive content. Unlike Frege's references (*Bedeutungen*), Russell's denotations do not need to be singular. This comes as a consequence of Russell's decision to apply the theory to all kinds of denoting phrases. The denotations picked out by the phrases other than definite descriptions are called 'combinations'. We will see that the peculiar nature of combinations plunges the theory into serious trouble.

The controversy starts with phrases beginning with 'all'. Russell's inability (or lack of willingness) to apply Frege's quantificational approach to sentences containing such

phrases leads to a dubious distinction between a set as one and a set as many. Sets as many, also dubbed ‘plurals’, are denotations in those cases where an ‘all’-phrase (or an equivalent to it) is used to pick out something as a subject of a number-attribution, e.g., ‘All of us in the room are two’. Notwithstanding the artificiality, the (admitted) immense straining of the usage of the ‘all’-phrases included, we can agree with Russell that the attribution of the number 2 applies neither to the set of all of us in the room, nor to each of us separately. The attribution applies ‘collectively’ (Russell’s term) and this is possible, according to Russell, because the denotation is a peculiar object which consists of entities related to each other by numerical conjunction. The denotation is a set as many.

The phrases beginning with ‘every’, ‘any’, ‘a’ and ‘some’ are submitted to a similar analysis, but in all these cases the denotations are not sets as many, but objects which, in Russell’s view, manifest ambiguity.

Russell admits, in *Principles*, that the combinations are paradoxical. According to P. Hylton, whose interpretation of the early Russell has been widely recognized, the paradox consists in the fact that the combinations are plural in their essence, hence not fitting Russell’s supposedly all-embracing category of terms. Although Hylton is right that all combinations fail to fit the category of terms, he misinterprets Russell when he says that all combinations are plural. Only the denotations of ‘all’-phrases are. Rectifying Hylton’s account will lead us to reveal that the problem of Russell’s theory is not restricted to the consequence of there being entities that fail to fit the category of terms. The concluding part of Chapter 3 explains that the combinations associated with ‘every’, ‘any’, ‘a’ and ‘some’ (Russell’s ‘cases of ambiguity’) fail to play the semantical role for which they were introduced.

An influential interpretive tradition based on Quine’s famous paper ‘Russell’s Ontological Development’ (1966), but also on some statements that came from Russell himself, has it that until Russell arrived at his Theory of Descriptions, he could not purge entities like the present king of France from his ontology. This means, in turn, that Russell’s semantical views before *OD* somehow necessitated endowing such entities with some sort of being. This account has been recently challenged by several scholars who found it unfair towards the semantical doctrines presented on the pages of *Principles*.

Before we inquire into the fate of the infamous french aristocrat in Russell's philosophical thought before *OD*, we open in Chapter 4 a related question of whether the Russell before *OD* embraced contradictory entities. Decisive evidence will be presented to the end that he could not. Unfortunately, in *Principles* Russell does not address the question, at least not directly. This allows for uncharitable attributions or, in a better case, for agnosticism, as in the case of Quine, who professes that he is not sure whether the Russell before *OD* embraced contradictory entities or not.

Russell's theory of cardinal numbers, his notion of the null set and his logic based on the principle of explosion and Law of Contradiction—none of this would be possible, had contradictory entities been allowed by Russell among what there is. This discovery is so clear that it is on verge of being trivial. We can only wonder what prevented Quine, a logician as apt as Russell, to overlook this. Although Russell was ready to dispense with the null class, it is unthinkable that he would sacrifice the rest of the listed doctrines for a bunch of extravagant entities whose value for his logicist project was zero.

It wasn't an option for the early Russell to commit himself to the existence of contradictory entities. It is worth examining, however, whether the semantical theories he was advocating lead to this undesirable ontological commitment or not, regardless of how destructive its effect on the logicist project of *Principles* would be. This is part of the agenda of Chapter 6.

The first installment of my argument in favor of Quine's interpretation of Russell is set forth in Chapter 5. I will focus on the challenge posed to this interpretation in the recent works by Graham Stevens and David Bostock. The first step towards a sufficient degree of clarity in the debate consists in carefully elaborating on the often used term 'Meinongianism'. What does it mean for an ontology to be Meinongian?

First of all, 'Meinongian' and 'Meinong's' is not the same insofar as our debate concerns the early Russell's transcendental realism rather than Meinong's descriptive phenomenology. This does not mean that there was no connection between Russell's philosophy and the Brentanian school to which Meinong belonged. Nonetheless, the above mentioned Russell scholars conceive Meinongianism as a position in which a semantical account of singular terms is adopted which entails an ontological commitment to entities such as the present king of France (possibilia) and the round circle (contradictory objects).

Of course, the ontological commitment here is envisaged in the very sense of Quine's definition of it: to be is to be the value of a bound variable. This is hardly anything Meinong would vote for, but I am convinced that the concept of Meinongianism is accurate as far as the debate concerns Russell.

The notion of Meinongianism that is used by those who interpret Russell seems accurate in what it takes an ontological commitment to be. However, we will see that it is inaccurate, or narrow, short-sighted rather, in its scope concerning semantical matters. Russell's overall discussion of unwanted ontological commitments throughout his career covers all of that which he took to be relevant to theorizing about meaning: the theory of judgment, semantical analysis of whole sentential signs, the theory of truth, and so on. Following Stevens and Bostock, I will address the question whether the Theory of Denoting Concepts commits Russell to include possibilia among what there is. I will do so, however, as part of taking into consideration of a larger portion of Russell's semantical thought.

In his retrospective on the breakthrough that came with the Theory of Descriptions, Russell exemplifies the impact of this theory by a brief exposition of how it solves the puzzle of negative existentials. This is the moment where Russell scholars take issue with Quine's interpretation of Russell as well as with the retrospective itself. We will see that the puzzle can be traced back to certain passages in *Principles*. This evidence will be examined. Stevens' and Bostock's interpretation, although referring to important facts about the Theory of Denoting Concepts, will be shown to be misleading. Their mistake lies in their failure to acknowledge that the puzzle, as Russell understood it, concerns intentionality (aboutness). Implicitly though, Russell was adhering to what I call the 'Intentionality Rule'. With this principle in hand, his version of the puzzle will be formally reconstructed. The puzzle shows that Russell's treatment of denoting phrases as complete symbols leads, if considered in a wider context of taking aboutness or intentionality as a precondition of sentential meaning, to an admission of possibilia into the realm of being.

As mentioned above, the agenda of Chapter 6 includes an answer to the question whether Russell was committed to the existence of contradictory entities. An incarnation of the puzzle of negative existentials will be explicated to show that Russell's semantical thought was plagued by a serious contradiction. Russell needed true denials of being such

as ‘The round circle has no being’ to be available insofar as contradictory entities were not to be found among what there is. However, true denials of being were impossible for him. As long as descriptions of contradictory objects are well formed descriptions, Russell was committed to the existence of contradictory objects.

In Chapter 6, we discuss two more conflicts in the semantical thought of the early Russell. We will discover that there is a passage in *Principles* in which Russell professes very clearly that the Theory of Denoting Concepts should account for genuine referential failures. I take this passage to provide evidence in favor of a view that Russell, at some point of writing the book, decided that TDC should not necessitate the extravagant entities such as the present king of France or the round circle to be. This was an ambition, an unfulfilled one, I contend, until the Theory of Descriptions arrived.

In our discussion of the third of the conflicts discussed in Chapter 6, we return to the early Russell’s theory of propositions. We will expand on our findings formulated at the outset of Chapter 1. The collapse of Russell’s universe of true and false propositions into a realm of true propositions only is a disastrous consequence; and it is also another root of the early Russell’s Meinongianism.

The arguments gathered demonstrate beyond question that the early Russell, that is, the Russell before *OD*, set up an ontological picture which compelled him to an ontological commitment to the unwanted entities. Quine was right that until Russell hit upon the Theory of Descriptions, he was unable to purge the unwanted entities for good. Chapter 7 is devoted to a detailed exposition of the eliminative analysis of the Theory of Descriptions. We explore the eliminative rules of the Theory clarifying thus what Russell meant when he famously proclaimed that descriptions, both definite and indefinite (ambiguous), are so-called ‘incomplete symbols’.

The concluding chapter is devoted to an exposition of Theory of Descriptions with special attention to the role of this theory in providing Russell with a solution to the problem of Meinongianism associated with his earlier views. After giving a detailed account of the theory, we discuss two arguments that Russell directed, in *OD*, against Meinong, linking them to Alexius Meinong’s *Gegenstandstheorie* as well as to Russell’s semantical views derived from *Principles*.

1. The early Russell's theory of propositions and their unity

1.1 What is a Russellian proposition?

Every declarative sentence is meaningful in as much as and only in as much as it expresses a proposition. Following Moore (1899), Russell took propositions to be complex wholes whose being is not principally dependent on the existence of a mind or consciousness. As Peter Hylton put it, 'a proposition ... has to be something which we do not in any sense make; it has to be something objective in the most simple-minded sense, something *out there*.'⁵ When I know that Obama is human, the object of my knowledge which is strictly separated from the states of my mind is the true proposition that Obama is human. As we will soon acknowledge, true propositions are for Russell the same as facts. An ontological notion of truth, applied to propositions, is another doctrine Russell adopted from G. E. Moore. (The proposition that) <Obama is human> is the meaning of 'Obama is human', and the truth of this proposition is what makes this sentence true.⁶ We distinguish four roles propositions are assigned to in the early Russell's philosophical framework:

- **Ontological:** Reality in itself is composed of propositions.
- **Epistemological:** Propositions are immediate objects of the attitudes expressible by an attitude verb and a that-clause, e.g., 'I know that ...', 'He believes that ...'.
- **Linguistic:** Declarative sentences are meaningful and have a particular truth-value in virtue of that fact that they express certain propositions.
- **Logical:** By standing in a certain manner in the relation of material implication, propositions make up complex propositions that are mapped onto valid inferences. The (transcendental) realist thrust of the early Russell's logic (and

⁵ Hylton (1984/2005), p. 14.

⁶ Henceforth I refer to the proposition expressed by a declarative sentence *s* by means of 'the proposition that *s*' or, in brief, by '<*s*>'.

‘pure mathematics’ which is, according to Russell, reducible to logic) is based on this view.⁷

In *Principles*, Russell distinguishes two fundamental sorts of propositions. This is one of his notable departures from a rather simplistic notion of a proposition presented in Moore (1899). The following conditionals determine Russell’s two-fold notion of a proposition:

(type1) If a proposition is expressible by a sentence containing an expression for a relation or property and names only, all the terms this proposition is about are included among its constituents.

(type2) If a proposition is expressible by a sentence with one or more denoting phrases, it is a proposition which does not contain all the terms it is about.

An example will help to clarify this distinction.

Suppose I am as tall as John, I am not John, and there is no one other than John, besides myself of course, who is as tall as me. The propositions expressed by these two sentences:

(i) I am as tall as John.

(ii) I am as tall as someone else,

are both about me and John. The systematic connection between (i) and (ii), manifested by an inferential pattern, can be expressed in the following way. Given the premise that John is the only person who is both other than me and as tall as me, from (ii) we can infer (i). This possibility of inferring (i) establishes the *indirect* sense in which (ii) is about John in spite of not containing among its component symbols the name ‘John’ or any other name

⁷ Logic is, according to Russell (*Principles*, §11), ‘the study of the various general types of deduction.’ ‘What symbolic logic does investigate,’ he continues, ‘is the general rules by which inferences are made ...’ (ibid.) The rules of inference, expressed for Russell by the theorems of logic and followed in our correct inferential practices, are not, however, the subject-matter of logic. The subject-matter consists in certain propositions, namely those which correspond to these rules and practices. Such propositions form what could be dubbed by a Wittgenstein’s term ‘the scaffolding of the world’ (Wittgenstein 1921, 6.124)—of the world indeed, not just of the discursive thought and language.

of John. The indirect reference to John is effected by ‘someone else’ which is, in Russell’s terms, a denoting phrase.

The phrases Russell calls, in *Principles*, ‘denoting’ are those we today call ‘quantifier’ phrases. However, to use the contemporary term would be anachronistic, for, until the theory of ‘On Denoting’ (1905) was developed, Russell had not used the apparatus of quantification theory for the purpose of giving a semantic analysis of such phrases.⁸ The phrases like ‘someone else’, or, to give another example, ‘a man’, are classed, by the early Russell, as ‘denoting’ without any recourse to the symbolic apparatus of quantification theory.⁹ The early Russell’s theory of denoting will be explained in Chapter 3. At this moment, it suffices to acknowledge that denoting phrases are supposed to function as vehicles of indirect reference.

The phrase ‘someone else’, expandable into ‘some x which is a man and is not identical with me’, expresses a so-called ‘denoting concept’. Such a concept is the vehicle of an indirect reference. Let us symbolize such concepts by putting denoting phrases between slashes.¹⁰ Thus ‘someone else’ expresses /someone else/. The proposition expressed by (ii) is, accordingly, symbolized as follows:

(ii’) <I am as tall as /someone else/ > ,

whereas the proposition expressed by (i) is symbolized in this way:

(i’) <I am as tall as John > .

It is crucial that /someone else/ in (ii’) and John in (i’) function differently despite the fact that they occupy the same position in the form which is shared by (i’) and (ii’). Had the

⁸ Although some unpublished manuscripts and letters to Gottlob Frege show that Russell learnt a modern-like quantifier notation before 1905, only after ‘On Denoting’ was this notation utilized by him for semantic purposes.

⁹ In the next chapter I present an interpretive attempt to define Russell’s notion of a denoting phrase. We will see that the proposed definition is unique in its being neutral between the theory of ‘On Denoting’ and its predecessor. It is an epistemologically grounded notion.

¹⁰ I follow the notation introduced by Nicholas Griffin in his (1980).

denoting concept functioned in the same way as John, by asserting (ii') I would be saying that I am as tall as the denoting concept expressed by 'someone else'! This nonsense is not what I intend to say. My assertion is supposed to be *not* about the concept, but about what the concept denotes, that is, John.

'A [denoting, MS] concept denotes when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept.'¹¹ This statement from *Principles* clarifies the sense in which a proposition with a denoting concept is about a term or terms which are not among its constituents. (ii') is about John. Note that /someone else/ which is a concept denoting John is a constituent of (ii'), but John himself (the denotation) is not. (ii'), as opposed to (i') is, therefore, a proposition which belongs to (type2) above. It is a proposition which does not contain all the terms it is about.

Following David Kaplan, we will henceforth indicate the propositions containing *all* the terms they are about by the epithet 'Russellian'. Kaplan, in his works on the semantics of indexicals, and his followers, assume that there are such propositions, for without them we are unable to construe an adequate theory of indexical terms. They call such proposition 'Russellian' (or, more neutrally, 'singular'), but one should be warned that their notion of a Russellian proposition in some of its aspects does not coincide with the early Russell's notion of a proposition.

Russell scholars¹² have urged that for the purpose of interpreting Russell, an important reservation concerning the modern, Kaplanian, notion of a Russellian proposition must be made if we are not to distort the actual Russell. As Gregory Landini emphasizes, Russell's propositions are intensional entities. For Kaplan and his followers, this is not the case. Their Russellian propositions are modeled as ordered *n*-tuples. For example, Kaplan's analog of (i') is this:

(i'k) <me, being as tall as something, John>

¹¹ *Principles*, §56.

¹² Linsky (1992), Landini (1996), Soames (2010, 2014).

For Kaplan, the truth-value of (i'k) is determined once we take into account the circumstance of evaluation of this proposition, which is the context of the assertion (i), 'I am as tall as John'. Kaplan's 'Russellian' propositions are thus extensional and, furthermore, they are context-dependent semantical posits. Neither of these features holds of Russell's propositions in *Principles*.

Keeping in mind the important difference between the Kaplanian notion of a Russellian proposition and Russell's original theory, we will use the term 'Russellian proposition' for the propositions posited by the early Russell which contain all the terms they are about.

The compositionality of a Russellian proposition is similar to the compositionality of a sentence which expresses that proposition. To implement this idea, Russell developed a system of absolute and relative categories. He first assumes one all-encompassing ontological category. This is the category of 'terms' which is described in *Principles* as follows:

Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call *term*. This, then, is the widest word in the philosophical dictionary. I shall use as synonymous with it words unit, individual, and entity. The first two emphasize the fact that every term is *one*, while the third is derived from the fact that every term has being, *i.e.*, *is* in some sense. [...] A term is, in fact, possessed of all the properties commonly assigned to substances or substantives.¹³

Terms are classified exhaustively into two categories, things and concepts. Things cannot hold of something. They can combine with concepts in a way which amounts to their possessing a property or their standing in a relation to something. *Unlike things, concepts can function in both these ways*: a concept can hold of something as well as be subject of a concept.

13 *Principles*, §47.

The words ‘term’ and ‘concept’ are also employed to invoke *relative* categories which determine contextual roles of propositional constituents. In a proposition, there is always something which functions as a term and something which functions as a concept. To function as a concept is to hold of something. For example, in <*Blackness* differs from *redness*>, *blackness* and *redness* function as terms, although they are concepts in the absolute sense. *Difference* is a concept in the absolute sense and also functions as a concept in this proposition.

Russell’s category of concepts is further analyzed into two disparate sub-categories, properties and relations. Properties apply essentially to just one term. For example, *redness* or *humanity* are such. Relations can apply to two or more terms, depending on their arity (adicity or degree). This difference is codified in natural language by the fact that relations, as opposed to properties, *can be said to stand between something and something*.

Importantly, the differentiation of relations from properties does not entail that the terms a relation can be said to apply to must be *distinct*. The relation of identity, to take an obvious instance, yields a true proposition only in those cases where it applies to entities that are not distinct.

For Russell, recognizing the difference between properties and relations does make the category of properties an unanalyzable concept. It has been widely recognized that after his break with idealism in 1898 Russell favored, although tentatively, to view subject-predicate statements as assertions predicating some quality to a term (e.g., the quality *redness* to this rose). In *Principles*, Russell writes ‘the so-called properties of a term are, in fact, only other terms to which it stands in some relation.’¹⁴ This means that, for Russell, there are no unary concepts, i.e., for every concept, if it holds of something, it holds at least of *two* terms. For example, ‘This rose is red’ does not express what we could call a subject-predicate proposition, namely the proposition which consists of this rose and an unary concept expressed by ‘is red’. The copula ‘is’ is taken by Russell to express a relation which applies to two terms, this rose and the quality *redness*. The quality *redness* is, in his view, no more concept-like than this rose is. In this way Russell achieves to construe an

¹⁴ *Principles*, §216; Cf. *ibid.*, §53; for more details see Levine (2014), 236-8; as Hylton throughout his (1990) emphasizes, Russell was influenced in this respect by the tradition of British idealism represented by T. H. Green and F. H. Bradley.

ontology in which the difference between things and concepts coincides with the difference between things and relations.

For the sake of better comprehensibility, I will henceforth omit this feature of Russell's thought. As his denouncement of properties as self-standing, unanalyzable units does not relate, in any substantial way, to the questions we will be facing, we can retain a more straightforward view according to which properties are unanalyzable units rather than composites of a quality and relation.

Russellian propositions do not represent facts. Russell suggested that a proposition is commonly called 'fact' if it is a true proposition as opposed to a false one. Thus, for him, the notion of a fact is derivative from that of a true proposition (that is, the latter is for him conceptually prior to the former). In a talk delivered in 1904 to *Jowett Society* in Oxford Russell says: 'It is the things which are or may be *objects of belief* that I call *propositions*, and it is these things to which I ascribe truth or falsehood.'¹⁵ After a brief discussion of the Kantian view that relations are produced by mind, Russell expands on his notion of a proposition by saying that

we must admit that things may really have relations; that their real relations [as they apply to their relata, MS] are *facts*, and that these facts are the objects of our judgments when the objects of our judgments are *true*.¹⁶

Russell's definition of 'fact' does not amount to a theory of truth. *It is merely his attempt to make sense of a commonsense view that having a true belief is having a belief in a fact.* Taking facts to be true propositions thus helped Russell to avoid an objection that his theory cannot explain the relation of beliefs to facts—an objection which he, in his 1904 talk, expects to be proposed by those who favor a correspondence account of truth.

Henceforth, I call the early Russell's theory of truth 'primitivist'. This is a theory according to which truth and falsehood are unanalyzable (primitive) features of propositions. Russell's taking true propositions to be facts does not aim to reveal anything

15 *CP4*, p. 494.

16 *ibid.*, p. 495.

new about what truth is. At best it reveals something about how the notion of truth is accommodated in our language and thought.

Russellian propositions cannot represent facts. But, according to Soames, they are representational, they are ‘*about* certain things, which they *represent* as being one way or another, and so are properly characterized as *true* (false) iff those things are (or are not) the way they are represented to be.’¹⁷ This is supplied with two qualifications. Soames first says that the supposed representational character of propositions is not derived from ‘cognitive activities of the agents who entertain them’.¹⁸ If propositions are representational, this must indeed hold for Russell, for otherwise propositions could not be mind-independent. They must be what they essentially are regardless of whether someone entertains them or not. ‘On the contrary,’ Soames adds the second qualification, ‘since propositions are the *primary* bearers of intentionality, the secondary intentionality and truth conditions of cognitive acts or states must, for Russell, be explained in terms of substantially passive acts of perception-like awareness of propositions.’¹⁹ (my emphasis) I will argue that this interpretation is confused.

Russellian propositions are *about* something only in a derivative, non-substantial sense. E.g., <Socrates is human>, is said by Russell to be *about* Socrates in the sense that Socrates occurs in it as term.²⁰ There is nothing more to Russell’s use of ‘about’ (and any other associated expressions of intentionality like, for example, the verb to predicate)! This is closely analogous to Russell’s invoking the term ‘fact’; ‘is a fact’ is *just* another label for ‘is true’ (as applied to propositions). By analogy, ‘y is about x’ is just another label for ‘x occurs as term in y’, where y is a proposition.

Contra Soames, Russellian propositions are not representational in any substantial sense. In turn, they cannot be primary bearers of intentionality. The most one can possibly

17 Soames (2014), 282.

18 Ibid.

19 Ibid.

20 For an arbitrary term a, to occur in a proposition p as term means that, in p, a has a certain property or stands in a certain relation to something. The early Russell’s notion of a term, therefore, does not presuppose any notion of aboutness.

assume is that Russell sometimes talks as if propositions were representational in a substantial sense. Later in this chapter, it will be explained why Russell tends to talk in this peculiar way.

With the truth-conditions the situation is opposite. Every proposition is substantively either true or false, and, consequently, it is in possession of certain truth-conditions.²¹ The truth-conditions of our cognitive states indeed derive from those that belong to propositions.

The intentionality of cognition, i.e., the directedness of a cognition towards an object, is for Russell a purely psychological/linguistic phenomenon. As long as Russellian propositions are concerned, the early Russell expels anything psychological/linguistic from the nature of reality.

Soames' misapprehension of the notion of Russellian propositions comes from his mistake of taking the peculiar terminology of *Principles* in its face value. I will argue that such a reading renders Russell's statement of his theory unintelligible. By principle of charity, and also by taking into account a methodological remark in Chapter 4 of *Principles*, we will adopt an alternative reading.

Russell distinguishes linguistic symbols from what such symbols stand for. What the symbol stands for is its meaning in the 'logical' sense as opposed to the linguistic sense (see the passage cited below). The meaning in the logical sense is not essentially linguistic or psychological. For example, the sun exists and is what it is independently of the linguistic fact that 'the sun' is its name and independently of people's recognition of it as the sun. Since the sun is the logical meaning of 'the sun', it is its meaning in a non-linguistic and non-psychological sense.

The general language/mind independence of what is symbolized applies to propositions as well as to things like the sun. It applies to all terms. Propositions are not to be identified with sentences in the first place. Consequently, they are not essentially composed of words. Russell writes:

21 '[...] I require a name for the true or false as such, and this name can scarcely be other than proposition.' (*Principles*, ix)

To have meaning, it seems to me, is a notion confusedly compounded of logical and psychological elements. *Words* all have meaning, in the simple sense that they are symbols which stand for something other than themselves. But a proposition unless it happens to be linguistic, does not itself contain words: it contains the entities indicated by words. Thus meaning, in the sense in which words have meaning, is irrelevant to logic.²²

Logic in Russell's distinctive sense is concerned with propositions, not with the sentences that express them (types) or the actual sentential signs (tokens). Of course, logic is concerned only with the propositions of a certain sort (those that contain logical constants only). Nonetheless, propositions, logical or not, should not, according to Russell, be confused with linguistic items.

A problem arises with the fact that Russell's language in *Principles* seems to be notoriously unfaithful to the sharp division between a symbol and what that symbol stands for. Let us take, e.g., the following passage from §46:

In every proposition [...] we may make an analysis into something asserted and something about which the assertion is made. A proper name, when it occurs in a proposition, is always, at least according to one of the possible way of analysis (where there are several), the subject that the proposition or some subordinate constituent proposition is about, and not what is said about the subject.²³

Let us first focus on that part of Russell's statement which concerns the role of a proper name. As we saw above, Russell insists that 'a proposition unless it happens to be linguistic, does not itself contain words.' How it comes, then, that propositions in general contain a proper name among their constituents?

²² *Principles*, §51.

²³ *Ibid.*, §46.

Had Russell's statement in §46 been confined to linguistic propositions (sentences), our question would have to be dismissed. But the statement clearly concerns propositions in general. This can be shown as follows. Let us pretend, for the sake of argument, that it concerns linguistic propositions only. Notice that if we take a sentence (linguistic proposition) which has a proper name among its constituents, such a sentence is not about that proper name. This goes against what Russell states in §46, provided we take him to address linguistic propositions only. Russell states that '[a] proper name, when it occurs in a proposition, is always ... the subject that the proposition or some subordinate constituent proposition is about'. For example, 'Obama is mortal' has 'Obama' as a constituent, but surely it is not a sentence about this name. And if we take a sentence which actually is about this name, e.g., "'Obama' is a proper name', we get an asymmetric result: this sentence is about 'Obama', but it does not have this name as a constituent. Moreover, "'Obama'" is a description of 'Obama'. So, not only that "'Obama' is a proper name' does not contain 'Obama', but it is of a different type from 'Obama is mortal'. It contains a description (denoting phrase), not a name, in its grammatical subject.²⁴

'Obama' is not what 'Obama is human' is about. Drawing on what is said in §46, it remains to ask whether it is what a subordinate constituent of 'Obama is human' is about? Apparently not, since the sentence has no such constituents. The name 'Obama' is simple as well as 'is' and 'human' are. By principle of charity, Russell's statement in §46 concerns propositions in general. So, we need to ask again: How it comes that propositions in general contain a proper name among their constituents?

Let us hold on for a moment. A related puzzle concerns Russell's talk, in §46 of *Principles*, of an assertion and what that assertion is made about. How can *every* proposition contain an assertion? The solution to both puzzles is to read 'proper name' and 'assertion' in a modified sense as ontological, rather than linguistic, categories. Let us deploy this reading in the case of the following proposition:

<Obama is as tall as Putin>.

²⁴ This is a sentence expressing a (type2) proposition, while 'Obama is mortal' expresses a Russellian proposition, that is, a (type1) proposition.

By analysis, we can take Obama to be the subject and the fragment <... is as tall as Putin> to be the assertion. In an analogous way, Putin can be conceived as the subject. The full analysis presents both Obama and Putin as the subjects (the terms of the proposition) while <... as tall as ...> is presented as the assertion. Despite the used terminology, the assertion <... as tall as ...> should be treated as something mind-independent.

Let us now return to Russell's untoward use of 'proper name' in the passage of §46 cited above. In this passage, Russell actually intends to say this:

If a proper name a is a constituent of some sentence s , s expresses a proposition, call it p , and p is about the bearer of a or a subordinate constituent of p is about the bearer of a .

So, 'Obama dislikes Putin' deploys 'Obama' and expresses a proposition about Obama. By 'proper name' Russell means 'bearer of a proper name'.

In an analogous way, we need to read the passages in which Russell talks about relations by means of the term 'verb'. §54 of *Principles* is an often-cited example. At the end of this paragraph, Russell concludes that '[the] verb, when used as a verb, embodies the unity of the proposition, and is distinguishable from the verb considered as a term ...' Here, Russell utilizes his doctrine of the twofold nature of concepts to identify the source of the propositional unity. The actual verb, a linguistic item, is irrelevant to the unity of something which is not essentially linguistic. What Russell intends to say is the following:

If a verb v is used as a verb in a sentence s , the relation expressed by v is that constituent of the proposition expressed by s which is responsible for the unity of that proposition.

So, in 'Obama dislikes Putin' the verb to dislike is used as a verb (as opposed to how this verb is used, e.g., in 'disliking someone sometimes happens to me'), and the relation indicated or expressed by 'dislikes' is, according to Russell, responsible for the unity of <Obama dislikes Putin>.

Russell's point in §54 of *Principles* is that the linguistic distinction between the nominal and assertional form of a verb maps onto an ontological distinction between two ways how a relation occurs in a proposition. Relying on the *structural* overlap between language and reality (the realm of sentences and the realm of propositions), Russell uses linguistic categories in an ontological sense. This is present, though less confusingly, in his later works as well. In his 1918 lectures on logical atomism, Russell writes:

You will perceive that it [= 'I believe that Socrates is mortal', MS] is not only proposition that has the two verbs, but also the fact, which is expressed by the proposition, has two constituents corresponding to the verbs. I shall call those constituents verbs for the sake of shortness, as it is very difficult to find any word to describe all those objects which one denotes by verbs. Of course, that is strictly using the word 'verb' in two different sense, but I do not think it can lead to any confusion if you understand that it is being so used.²⁵

For the Russell of 1918, propositions are identified with declarative sentences and reality is taken to be composed of facts, not of propositions. Despite these and other important differences separating the Russell of 1918 from his earlier self, he still is, as we can see, in habit (adopting explicitly this habit as a convention) to refer to the entities symbolized by verbs (in their assertional form) as 'verbs'.²⁶

The main reason for Russell's peculiar terminological choices in *Principles* seems to be methodological. At the outset of Chapter 4, he explains that exploiting grammar can help to discover 'philosophical differences'. We read:

Although a grammatical distinction cannot be uncritically assumed to a genuine philosophical difference, yet the one is *prima facie* evidence for the other, and may

²⁵ *LA*, 49.

²⁶ The passage from *LA* shows that Russell's ontological use of the linguistic categories was adopted for its practical value. Had he used the accurate language of our paraphrases, he would end up publishing a tome much bulkier, probably more tiresome to read too. In this sense, Russell's use of the linguistic categories amounts to a notational convention.

often be most usefully employed as a source of discovery. ... in what follows, grammar, though not our master will yet be taken as our guide.²⁷

Genuine philosophical differences are those that determine the nature of reality. Grammatical distinctions are taken to be ‘a source of discovery’ of such differences and grammar is taken to be ‘a guide’ for philosophizing. Not everything grammatical, or linguistic in a wider sense, is relevant to philosophy. A critical consideration for separating what is relevant from what is not is always called for. Based on this method, Russell’s use of grammatical and linguistic categories was governed in the following way.

Suppose that the linguistic distinction between the sorts A and B is philosophically significant. Then the distinction between A and B is correlative to an ontological distinction between the sorts A^* and B^* . This establishes a structural overlap between the nature of language and the nature of reality. In Russell’s view, you may, then, keep using the terms ‘ A ’ and ‘ B ’ in reference to the ontological sorts A^* and B^* as long as it is understood that the the terms ‘ A ’ and ‘ B ’ are used in abstraction from the linguistic aspect of the distinction between the sorts A and B . Only the structural aspect of this distinction is retained in such a use. That Russell assumed this method is suggested in Hylton (1990) as follows:

Language becomes, as it were, a transparent medium through which propositions may be perceived. The transparency of the medium makes it possible to ignore it. Russell sometimes appears to be talking about language, or moving back and forth between the linguistic and non-linguistic. This is not, however, because he is at all unclear or ambivalent about what his real subject-matter is. It is, rather, because the assumed symmetry between the linguistic and the non-linguistic means that it is not important to keep the distinction clear in practice.²⁸

In Linsky (1988), we find a similar interpretation. Linsky writes:

²⁷ *Principles*, §46.

²⁸ Hylton (1990), 171; see also Makin (2000), 12.

Proper names, adjectives, and verbs, occur in propositions, but a proposition (unless it is about words) does not contain words. This is beginning to seem bizarre. But so far we have nothing more to be alarmed about than a rather non-standard use of words. The motivation for Russell's non-standard use is not difficult to discover. Since grammar is to be our guide, and, in fact, our master, it is entirely natural to ontologize grammar. If every grammatical distinction marks a rift in nature, we can make sure not to lose sight of ontological differences by just transferring the grammatical categories from language to the world.²⁹

Linsky's commentary is correct, except the remark that grammar 'is to be our guide, and, in fact, our master'. Here, Linsky seems to misread Russell, who puts his point as follows: '... in what follows, grammar, *though not our master* will yet be taken as our guide.'³⁰ Earlier in this paragraph, Russell warns that 'a grammatical distinction cannot be uncritically assumed to a genuine philosophical difference.' Linsky's saying 'every grammatical distinction marks a rift in nature' must, then, be conceived as a misinterpretation.

Linsky's comment is not an insignificant slip. It is a part of his interpretation of the development of Russell's philosophy. We read:

[...] Russell adheres to grammar at almost every point and in almost every detail. He treats this guide, in fact, as a master. He treats the most superficial grammatical differences as revealing fundamental logical distinctions. Underlying logical form is exactly reflected, in *Principles*, by surface grammar. It is the rejection of this master that marks the real gulf between *Principles* and "On Denoting".³¹

The theory of descriptions of 'On Denoting' surely presents a distinctive departure from grammar as a guide of philosophizing. The whole of Chapter 7 is devoted to a detailed

29 Linsky, L. (1988), 625.

30 *Principles*, §46, (my emphasis).

31 Linsky, L. (1988), 624.

exposition of this. However, Linsky goes too far when he generalizes this particular case in order to account for a difference in method between *Principles* and ‘On Denoting’.

MacBride (2013) pointed out that there are, in *Principles*, several cases in which Russell refuses to ontologize a linguistic feature. A notable of such cases is his treatment of converse relations. Although not decisive, Russell is reluctant to assume that, e.g., ‘*a* is bigger than *b*’ and ‘*b* is smaller than *a*’ express distinct propositions. Already in *Principles*, Russell considers a possibility endorsed later in his (1913) that such pairs of sentences contain alternative expressions for one and the same proposition.

Let us now return to our criticism of Soames. Sentences, statements and attitudes such as belief are about something in a substantial sense. For instance, a statement of ‘Socrates is mortal’ is, in a substantial sense, about Socrates, not about mortality. Mortality is here, in a substantial sense, predicated to Socrates. The distinction between what the statement is about and what is predicated is, for Russell, philosophically vital because it is correlative to the distinction concerning the occurrence of Socrates and mortality in <Socrates is mortal>. In this proposition, mortality *holds of* (or belongs to) Socrates. Russell resists using the term ‘holding of’ or any other unambiguously ontological idiom. Instead he exploits the original linguistic distinction between what a statement is about and the predicate. Thus, he says that

- <Socrates is mortal>
- is about Socrates,
- asserts/predicates mortality of/to Socrates.

Intentionality and predication, both taken in a substantial sense, belong to the linguistic aspect of the distinction and as such they are abstracted from in Russell’s ontological inquiry. <Socrates is mortal> is not directed towards (is not about) Socrates in the same sense in which <Socrates is mortal> does not contain the proper name ‘Socrates’ and the adjective ‘mortal’.

1.2 External relations

One way to grasp the notion of a Russellian proposition is to assimilate the unity of such a proposition to the unity of a fact in Russell's later logical atomism. Unity is a manner of the interrelatedness between entities in virtue of which these entities make up the whole of a certain sort. The unity of a Russellian proposition is not a product of any act or process, and, in particular, it is not a product of mental acts. It is a sort of unity we can call 'objective' as opposed to 'representational'. <Socrates is mortal> embodies the property of mortality as *holding of* Socrates.³² The unity of facts in Russell's logical atomism is the same. However, we should keep in mind that Russellian propositions differ from facts in having a truth-value. There are true and false propositions, and every proposition, true or false, is extra-mental. As we saw, for the early Russell facts are true propositions.

The unity of a Russellian proposition consists in the application of an external relation to its terms. Facing Bradley's regressive argument against external relations (the so-called 'Bradley's regress'), Russell was compelled to defend his idea of propositional unity by an external relation. Before we approach Bradley's argument and Russell's defense, a clarification of what the epithet 'external' or 'independent' (the latter is Bradley's original term) means is in order.

In the context of interpreting Russell, the term 'external' or 'independent' is ambiguous. It can mean at least two things. First, it can mean the existential independence of a relation from its terms. Second, it can mean the existential independence of a relation from the whole it forms in application to its terms. Let us start with the first form of externality.

For example, in <Obama dislikes Putin> Russell treats the relation of disliking independent from the existence of Obama and Putin. This is a consequence of his rejection of the theory according to which relations should be explained away in terms of the attributes of its supposedly apparent relata—the so-called theory of *internal* relations. To

³² The proposition that Socrates is mortal does not represent Socrates as having wisdom. It is Socrates' having wisdom.

expand on this theory, take again the true sentence ‘Obama dislikes Putin’. This sentence, according to the theory, does not express the fact that Obama and Putin stand in a relation. Despite appearances, it expresses two facts, one being that Obama has in its nature the attribute *disliking Putin*, the other being that Putin has in its nature the attribute *being disliked by Obama*.³³ Russell takes this approach to relations to lead to monadism. He notices that, under the theory of internal relations, reality is envisaged as composed of monads because it forces us to dissolve every relational statement into statements about the natures of the supposed terms (monads).

It should be said that both Russell and Bradley take issue with monadism. They do so, however, in opposite directions. While Bradley reduces monadism to monism and, consequently, assumes that reality must be, at the end of the day, the indivisible one (the Hegelian Absolute), Russell, again, follows Moore’s early strand of thought by embracing a pluralist ontology which is based on a theory that relations are irreducible entities.³⁴

Russell rejects the monadist analysis of relational statements, that is, the doctrine of internal relations, taking instead relations to be entities attaching to their terms. Knowing that Obama dislikes Putin is knowing about Obama and Putin that they stand in the relation of disliking (in the order <Obama, Putin>). Had Obama liked Putin, nothing would change in the natures of the two men. This is the first and primary sense in which relations are said by Russell to be external.

The second and secondary sense in which Russell treats relations as external concerns their status with regard to the wholes they form together with their terms. We can assume that the relation of disliking in <Obama dislikes Putin> is an irreducible, self-standing entity, and yet we might insist that the relation is particularized in a way that its identity criteria involve the fact that it stands between Obama and Putin in the given order.

33 At the time of Russell’s early realist phase, a notable proponent of this theory was Harold Joachim. Russell exposes Joachim’s position as grounded on the ‘Axiom of internal relations’ which he formulates in the following way: ‘Every term is grounded in the natures of the related terms.’ (*ONT*, 37)

34 In the preface to *Principles*, Russell explains that the theory that relations are mind-independent, irreducible entities is part of the views he learned from G. E. Moore. He adds: ‘Before learning these views from him, I found myself completely unable to construct any philosophy of arithmetic, whereas their acceptance brought about an immediate liberation from a large number of difficulties which I believe to be otherwise insuperable.’ (xlvi)

Such relations are today called ‘tropes’. Simmons characterizes a (non-transferable) trope as a relation which is borne *essentially* by the things it relates and which could not have been borne by anything else.³⁵ According to the trope theorist, it is erroneous to suppose that <Obama dislikes Putin> and <Putin dislikes Obama> share one relation, the universal *disliking*. The trope theorist insists that the propositions in question, or rather their substitutes in one’s favored ontology,³⁶ contain *two different* relations.

In §55 of *Principles*, Russell conceives a trope theory combined with a view that there actually are shareable relations (universals). Russell first offers the following outline of this theory:

It may be doubted whether the general concept *difference* occurs at all in the proposition “*A* differs from *B*,” or whether there is not rather a specific difference of *A* and *B*, and another specific difference of *C* and *D*, which are respectively affirmed in “*A* differs from *B*” and “*C* differs from *D*”. In this way, *difference* becomes a class-concept of which there are as many instances as there are pairs of different terms; and the instances may be said, in Platonic phrase, to partake of the nature of difference.³⁷

The point of this theory is not that there are no universals. Rather it is that universals, while being admitted into one’s ontology, are excluded from the content of propositions. The particularized relation (trope) of disliking, not the associated universal, occurs in <Obama dislikes Putin>.

35 Simmons (2002), 6.

36 Due to the classical paper on tropes, Williams (1953), tropes are envisaged as parts of objects, typically of those that are perceptible. To use one of Williams’ examples, the taste of a particular lollipop is a candidate for being a trope. This position does not fit Russell’s framework. For example, if *A* and *B* are two lollipops, their difference is not part of either, *A* and *B*, but rather part of a proposition of which *A* and *B* are parts. The same applies to the tastes of the lollipops. Thus, if we consider implementing the category of trope to Russell’s framework, propositions, not objects, must be treated as bearers of the tropes. In our discussion, we hold to Russell’s ontology of propositions, and, accordingly, we imagine a trope theorist who treats tropes as parts of propositions.

37 *Principles*, §55.

Russell argues against the outlined view as follows. He begins by proposing a desideratum that even if there are particular differences involved in the propositions and, in consequence, these differences differ among themselves, there must be some sense in which these propositions have something in common. $\langle A$ differs from $B \rangle$ and $\langle C$ differs from $D \rangle$ must have something in common, according to Russell, which, for example, $\langle A$ precedes $D \rangle$ does not have in common in any of them. In Russell's terms, particular differences must be in some sense instances of the universal *difference*. 'The most general way,' says Russell, 'in which two terms can have something in common is by both having a given relation to a given term.'³⁸ For brevity, call the two propositions we are currently considering ' p ' and ' q '. Then p and q have something in common if and only if they both possess a certain relational property. This is the property *instantiating the universal difference*. Now, if *all* relations are particularized, this cannot be the case. Russell writes:

[I]f no two pairs of terms have the same relation, it follows that no two terms can have anything in common, and hence different differences will not be in any definable sense *instances* of difference.³⁹

Russell's argument relies on the consequence that, as long as the trope theory holds for all relations, it also applies to the *instantiation* relation. The *instantiation* relation of p to *difference* must differ from the *instantiation* relation of q to *difference*. Call the former relation 'I' and the latter 'I*'. Then $I(p, \textit{difference})$ and $I^*(q, \textit{difference})$. Where ' x ' is a free variable, the proposition p has the property $I(x, \textit{difference})$, whereas q has the property $I^*(x, \textit{difference})$. This prevents us, Russell concludes, making sense of saying that both p and q are instances of *difference*. From the point of view of the considered theory of relations, the way how *difference* is instantiated in p and the way how *difference* is instantiated in q are no more associated with each other than any of them and the way I sit on my chair. This is, for Russell, an unacceptable consequence because the desideratum that p and q must have something in common is left ungrounded. According to him, only by replacing I and I* (the particularized instantiation relations) by a shareable universal we

38 Ibid.

39 Ibid.

can establish the desired result. But, due to the supposed generality of the trope theory, this is impossible.

The trope theory fails to explain how p and q are instances of the universal *difference*, and thus, by Russell's measure, it fails to give a satisfactory account of relations. 'I conclude, then,' Russell continues:

that the relation affirmed between A and B in the proposition " A differs from B " is the general relation of difference, and is precisely and numerically the same as the relation affirmed between C and D in " C differs from D ". And this doctrine must be held, for the same reasons, to be true of all other relations; relations do not have instances but are strictly the same in all propositions in which they occur.⁴⁰

The relations which occur in Russellian propositions are therefore external in two ways. Firstly, they are existentially independent of their terms or, more specifically, they are not reducible to any attributes of their terms. Secondly, they are external in the sense of being existentially independent of the fact that they relate some particular terms.

In relation to the second sort of independence, we can say that Russell was realist about universals. There are universals, and although their ontological status (being or subsistence) differs from what exists, they are in principle as extra-mental as existing entities. The table in front of me is out there, not in my mind, and its difference from the chair I am occupying is likewise out there.

Russell rejected to treat relations as tropes under two suppositions. First, the trope theory must be general. Second, the trope theorist cannot ground the saying that two propositions like $\langle A \text{ differs from } B \rangle$ and $\langle C \text{ differs from } D \rangle$ have something in common. If the later holds, not only the hybrid trope theory considered by Russell, but also any pure trope theory without universals must be deemed erroneous. Both assumptions, however, can be challenged. Russell's standpoint in *Principles* favoring pure realism about universals is thus rather dogmatic.

40 Ibid.

1.3 The unity of a Russellian proposition: Bradley's regress

F. H. Bradley was among Russell's most notable adversaries. His main contention against Russell which falls under the rubric generally referred to as the unity of proposition is suggestively set out in the following quote from Bradley (1910):

Mr. Russell's main position has remained to myself incomprehensible. On the one side I am led to think that he defends a strict pluralism, for which nothing is admissible beyond simple terms and external relations. On the other side Mr. Russell seems to assert emphatically, and to use throughout, ideas which such a pluralism surely must repudiate. He throughout stands upon unities which are complex and which cannot be analysed into terms and relations. These two positions to my mind are irreconcilable, since the second, as I understand it, contradicts the first flatly.⁴¹

Russell of the period under our consideration (1903-18) was a 'strict' pluralist in the sense of holding to the view that reality consists of an infinite number of self-standing entities. In Section 1.1, we acknowledged that the early Russell assumes all-embracing ontological category of termhood. He says, in *Principles*, that every term is 'possessed of all the properties commonly assigned to substances or substantives.'⁴² The defining property of a substance is that its being does not depend on the being of anything else than itself. Due to generality of the category of terms, relations (concepts) are as substance-like as those terms that are not relations (e.g. instants of time, propositions). As a consequence, every relation must be external in the two senses discussed above: it must be independent from its relata and also from the whole it forms together with its relata. We have also acknowledged that Russell admits complex entities into his ontology. The circumstance that a relation holds of a term or terms amounts to a complex entity which Russell calls 'complex', more rarely 'unity' and alternatively also 'proposition' (until 1910) or 'fact' (from 1910 on).

⁴¹ Bradley (1910), 179.

⁴² *Principles*, §47.

According to Bradley, Russell's overall standpoint is incomprehensible. The admission of complexes, it is said by him, flatly contradicts the externality of relations. Talking of a contradiction, Bradley (1910) refers to his often-discussed argument against external relations exposed in the pages of *Appearance and Reality* (1893/1897) [*AR*], the so-called 'Bradley's regress'. This argument is set forth in chapter 3 of Bradley's book. Bradley asks whether our knowledge of the type 'something is somehow related to something' gives us truth or appearance. It is beyond doubts that we commonly claim knowledge of relational sort. However, is such knowledge faithful to reality? In other words, is reality *in itself* relational?

It is a *conditio sine qua non* of answering in the positive that a relation is something (has being). This is involved in Russell's assumption that relations are terms. Assuming for *reductio* that the relation *R* is something, what makes *R* not only to be something, but also to be something to the relata *a* and *b*? If it is to be something to them, then 'clearly we now shall require a *new* connecting relation ... [the original relation, MS] being something itself, if it does not itself bear a relation to the terms, in what intelligible way it succeed in being anything to them?'⁴³

Attempting to fill the gap between *R*, *a* and *b* with a new relation reintroduces the problem. Being something itself, in what intelligible way this new relation succeed to relate *R*, *a* and *b*? This establishes a regress which is said by Bradley to preclude relational unity. So, as Bradley famously concluded, 'a relational way of thought - any one that moves by the machinery of terms and relations - must give appearance, and not truth.'⁴⁴

The rest of this chapter is devoted to an exposition of Bradley's regress, the early Russell's multifaceted reply to Bradley and, finally, to my defense of Bradley against Russell. We begin with a couple of preliminary remarks.

My talk about Bradley's regress should not imply that there is just one argument in Bradley's writings which deserves the often used title. As Perovic shows in her (2014), there are, at least, three regress arguments in *AR* which commentators usually neglect to

43 *AR*, 32.

44 *Ibid.*, 33.

distinguish. As I made clear above, we confine ourselves to the regress argument presented in Chapter 3 of *AR* against external relations.

Whenever I can, I use, in what follows, the less theory-laden term ‘complex’ instead of ‘proposition’. The problem of unity posed by Bradley’s regress can be formulated in the context of Russell’s logical atomism developed at around the period 1910-13. Since Russell of this period no longer believes in Russellian propositions, endorsing an ontology of facts instead, we should better separate our exposition of the problem of unity and some of the related comments from the peculiarities of the ontology of Russellian propositions as much as possible.

For a better grasp of Bradley’s regress, we will gradually elaborate on a similar argument which we often find in the current debate on relations and unity. This is an argument which aims specifically at showing that contingent relational unities cannot exist. Elkund puts this argument as follows:

Take a supposed fact: that *a* is (contingently[fn. 4]) *F*. What is the nature of this fact? If we think that the predicate stands for a property, the property *F*, then arguably *a* and this property are constituents of the fact. But, the regress argument goes, the fact cannot simply consist of *a* and *F*. For *a* can exist and *F* can exist even if *a* is not *F*. For it to be a fact that *a* is *F*, *a* must instantiate *F*. But adding the talk of instantiation just gets us another constituent of the fact: the relation of instantiation, call it *R*. But *a* can exist, *F* can exist, and *R* can exist even if *a* is not *F*. Trying the same strategy as before we can say that *a*, *F* and *R* must stand in the right relation for it to be a fact that *a* is *F*. But it should be clear that we are off on a regress.⁴⁵

The initial question after the ‘nature’ of the fact that *a* is *F* requires us to provide grounds for the existence of that fact—‘What is necessary for the fact that *a* is *F* to be?’ The constituents *a* and *F* do not suffice, it is argued, because ‘*a* can exist and *F* can exist even

45 Elkund (2019), 1227.

if a is not F .⁴⁶ The vicious regress, dubbed by Elkund as ‘constitution regress’, follows under the assumption that we need to supply an additional constituent of the fact.

Bradley’s argument is not restricted to contingent complexes in the first place. Elkund says that his restriction is ‘in principle eliminable,’⁴⁷ he does not add, though, that the elimination results in an argument differing *substantially* from the restricted case. For, if the fact that a is F is necessary, we cannot argue that ‘ a can exist and F can exist even if a is not F .’⁴⁸ Something else is required here. I assume that the argument covering complexes *in general* proceeds along the following lines:

What is necessary for $\langle a$ is $F \rangle$ to be? The constituents a and F do not suffice unless a instantiates F . Therefore, $\langle a$ is $F \rangle$ is (exists, obtains etc.) because a instantiates F . But adding the talk of instantiation (or its converse, the *relating-ness* relation) just gets us another constituent of the complex: the relation of instantiation, call it R . The constituents a , F and R , however, do not suffice unless a and F instantiate (or, are related by) R . Hence, the infinite regress.

This is not Bradley’s original argument yet. We need to be more specific. What we require when asking about what is necessary for $\langle a$ is $F \rangle$ to be? We are not asking how $\langle a$ is $F \rangle$ was brought about, caused in some manner or created, out of its constituents! What a and F do not suffice for is not the creation of a fact.⁴⁹ If we want to follow Bradley, the point of the question is meta-systematic. Provided we treat relations (and properties) as external and intend to introduce complexes into our ontology, it requires us to give non-trivial

46 MacBride puts this point as follows: ‘[A] relational universal is something that could have failed to relate its actual terms and related different terms instead. A relation R of this kind cannot account for the fact (for example) that a and b are related thus and so.’ (MacBride 2011, 168)

47 Elkund (2019), 1227, fn. 4.

48 Consider the early Russell’s ontology of propositions. Bradley’s regress was supposed to apply in this metaphysical framework—both Russell and Bradley took it for granted, disagreeing though on the validity of the argument. But, for Russell, every relational unity (proposition) is necessary; whether ‘ a is F ’ is true or not, F holds of a in a similar sense in which Meinong takes an object a to possess F even if it is false that a is F . Arguing, then, that a can exist and F can exist even if a is not F loses its force, for the unity $\langle a$ is $F \rangle$ exist (or, rather: is, has being) even if a is not F . Thus if we want to assess Bradley’s regress in the context of the early Russell’s propositional realism, clearly we need to eliminate the restriction to the contingent relational unities.

49 Lebens (2008) and Perovic (2014) emphasize this point.

grounds for the introduction of complexes. This is *the* problem of unity posed by Bradley. Commentators sometimes formulate this problem in terms of searching the difference between a complex and the sum of its constituents.⁵⁰ Such formulations are unfortunate because they invite a confusion between the object-linguistic question concerning the creation of a complex and Bradley's meta-systematic question. To avoid this confusion, I propose to put the core part of Bradley's regress as follows. If we assume that *a* is, *F* is, and that *F* is external, what entitles us to assume that $\langle a \text{ is } F \rangle$ is? The minimal answer is that $\langle a \text{ is } F \rangle$ is because *a* instantiates *F* (*F* holds of *a*). At this point, it all depends on what we take the clause after 'because' to express. It is assumed by Bradley that the explanatory clause affirms the relation of instantiation or relating-ness (since now on, I stick to the later) as holding between *a* and *F*. Under this premise the vicious regress unfolds.⁵¹

Besides the given clarification, our modification of Elkund's argument also needs to be slightly changed. It is not that the instantiation talk just get us *another constituent* of the complex. Whether what we get is another constituent or not is irrelevant. We will see throughout the following discussion that Russell's insistence that the unifier is not another constituent of the complex unified helps him little to tackle Bradley's regress. What the instantiation talk gives us is another appeal to the capacity of a relation to relate, no matter whether this capacity is exercised *within* or *outside* of the complex unified.

Bradley's regress is established on the following premises:

/1/ Relations are external.

For an arbitrary relation *R*,

/2/ *R* can apply to its terms as to form a complex.

/3/ if /2/, there is a non-trivial account of how *R* applies to its terms as to form a complex.

/4/ how *R* applies to its terms can be non-trivially accounted for only in terms of another relation's holding between *R* and its terms.

50 See, e.g., Griffin (1993).

51 Lewis (2002) and MacBride (2011) also endorse this reading. The regress argument does not show that a complex could not be brought about. It shows that a non-trivial account of how a complex is grounded in the exercised ability of a relation to relate cannot be completed.

Bradley's regress is a *reductio ad absurdum* of /2/. Given the (opponent's) supposition that there are complexes (this comes as an ontological commitment), Bradley's regress indirectly refutes /1/. Thus it is supposed that /3/ and /4/ hold, and that /2/ and /1/ should be refuted.

The current interpretation makes clear that Bradley's regress cannot be resolved by a mere introduction of complexes into one's ontology. As MacBride (2011, 172) argues against Armstrong and Hochberg, although the being of $\langle aRb \rangle$ suffices for a 's bearing R to b while the joint being of a , b and R does not, introducing $\langle aRb \rangle$ into one's universe does not resolve Bradley's argument, but rather it presupposes that some resolution has already taken place.

It is an inseparable part of Bradley's regress that the advocates of external relations are required to justify their introduction of complexes. It is required that they give a non-trivial account of how relations and terms combine as to form complexes. This is involved in /3/. Bradley's opponent can attempt to refute this premise. If this strategy succeeds, one's introduction of complexes comes directly with one's introduction of relations and their terms. Importantly, this should not be confused with an introduction of complexes *simpliciter*.

1.4 The early Russell's response to Bradley's regress

Clearer about the underpinnings of Bradley's regress, we can now place it in a particular context of the early Russell's attempts to evade it. At around the time of writing *Principles*, Russell favored a view that ' R relates a and b ' is not supposed to give a non-trivial account, if it is supposed to give any account at all, for the role of R in $\langle aRb \rangle$. The metaphysical part of Russell's intended resolution to Bradley's regress consists in treating the unifier as a primitive property of relations when they relate. The main tenet of this position is as simple as Blanshard once put it: 'The business of a relation is to relate.'⁵² Relations relate because they are relations, not non-relations. Period. This is a refutation of /3/.

⁵² Blanshard (1983), 215.

According to Lebens (2008) and Elkund (2019), the early Russell held to primitivism about unity coherently. Leerhoff argues in a similar vein, proposing in his (2008) that ‘Russell’s solution [...] reminds one of Frege’s: According to Frege, relations are not really independent objects (like their relata) but unsaturated entities [...].’⁵³ I will argue to the contrary. Evidence presented later reveals that, in *Principles*, Russell rejected the Fregean approach outright. I will explain how this rejection prevented Russell to hold coherently to primitivism about unity. In what follows I distinguish three parts of the early Russell’s response to Bradley and introduce them in the respective order:

1.4.1 consideration about analysis,

1.4.2 ontological regress,

1.4.3 primitivism about unity.

1.4.1 Failure of analysis

In §54 of *Principles*, Russell puts forward an argument which is a modified version of Bradley’s regress. The argument is concluded as follows:

A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition. The verb, when used as a verb, embodies the unity of the proposition, and is thus distinguishable from the verb considered as a term, though I do not know how to give a clear account of the precise nature of the distinction.⁵⁴

⁵³ Leerhoff (2008), 256; It should be emphasized that Frege’s metaphorical talk about gaps that make concepts unsaturated is not adopted in Russell’s formulation of primitivism about unity. Russell says in his formulation that relations are asserted (in a non-linguistic sense) or that they are relating essentially. Since criticisms specific to Frege’s metaphor were proposed in the literature (see, e.g., Vallicella 2000, 242), it is better to avoid this metaphor in characterizing Russell’s intended primitivism about unity.

⁵⁴ *Principles*, §54.

Something similar to Bradley's point, Russell admits, applies to analysis. Let us take $\langle a \text{ differs from } b \rangle$. By analysis we itemize the members of the class of all constituents of this proposition. Thus we get $\{a, \textit{difference}, b\}$. Since this class is not the proposition analyzed, Russell asks: What more there is to the proposition which is not involved in the class yielded by its analysis? If we answer that this additional element is one or more of its constituents we have forgotten to mention we end up caught in an endless regress. Suppose the omitted constituents are two relations, one between a and *difference* and the other between b and *difference*. Call them R and R' respectively. Joining them to $\{a, \textit{difference}, b\}$, we get $\{a, R, \textit{difference}, R', b\}$. Now, is this class identical with $\langle a \text{ differs from } b \rangle$? Obviously not. No class in general can be a proposition.

The element distinguishing $\{a, \textit{difference}, b\}$ from $\langle a \text{ differs from } b \rangle$ is propositional unity, the exercised capacity of *difference* to relate. Russell's regress argument shows that propositional unity cannot be a constituent of the proposition. It cannot be something we can arrive at by analysis. This is reaffirmed in the first installment of Russell's review of Meinong which was written shortly after *Principles*. We read the following: '[What] distinguishes our complex [from the class of its constituents, MS] is not any constituent at all, but simply and solely the fact of relatedness in a certain way.'⁵⁵

In his reply to Bradley's 1910 paper, 'Some Explanations in Reply to Mr. Bradley' (1910), Russell suggests that his observation about the failure of analysis to capture unity amounts to a refutation of Bradley's regress. He writes:

Mr. Bradley finds an inconsistency in my simultaneous advocacy of a strict pluralism and of "unities which are complex and which cannot be analysed into terms and relations". It would seem that everything here turns upon the sense in which such unities cannot be analysed. I do not admit that, in any strict sense, unities are incapable of analysis; on the contrary, I hold that they are the only objects that can be analysed. What I admit is that no *enumeration* of their constituents will reconstitute them, since any such enumeration gives us a plurality not a unity.⁵⁶

⁵⁵ Russell (1904a), 210.

⁵⁶ Russell (1910a), 373.

For the Russell of 1910, Bradley's regress is just a confused version of the consideration about analysis. Unity is a non-constitutive part of the complex unified—this is what 'everything here turns upon,' he declares.

1.4.2 Ontological regress

The consideration about analysis is not Russell's only attempt to identify Bradley's regress with something less challenging. In §55 of *Principles*, Russell acknowledges that every sentence can be transformed in the following way. Take, e.g., '*a* differs from *b*'. This can be transformed into 'difference relates *a* and *b*' (or the like). Such a transformation always yields a sentence which is materially implied by the original sentence. As Russell observes elsewhere (*Principles*, §§46, 57), this is a pair of materially equivalent sentences. Such pairs of sentences express pairs of materially equivalent propositions.⁵⁷ Since the transformation is available for every sentence, it applies recursively, yielding, in our example, the following regress of sentences:

a differs from *b*
difference relates *a* and *b*
relating-ness relates* difference, *a* and *b*
relating-ness* relates** relating-ness, difference, *a* and *b*,
and so on *ad infinitum*.

Russell insists that each member of this series of sentences expresses a distinct proposition. Hence the infinite series of sentences maps onto an infinite series of propositions. Does this pose any issue? In §§55, 99, Russell answers 'no'. Insofar as the notion of actual infinity is coherent, we can take infinite series of propositions to be real.

⁵⁷ The Russell of PoM holds that reality is composed of mind-independent propositions which possess truth-values and stand in logical relations.

1.4.3 Primitivism about unity

The final ingredient of the early Russell's multifaceted reply to Bradley consists in taking unity to be indefinable. We already encountered Russell's saying in his (1904a) that unity is 'simply and solely the fact of relatedness in a certain way.' This is already present in *PoM*. In §54, we read that 'the verb, when used as a verb, embodies the unity of the proposition' ('verb' is here synonymous to 'relation'). Although §54 closes with a confession that Russell is at loss to give an account of the distinction between the verb used as a verb and a noun verb, the discussion resumes in §99. There Russell states that 'a relating relation is distinguished from a relation in itself by the indefinable element of assertion which distinguishes a proposition from a concept.'⁵⁸

1.5 In defense of Bradley

According to Russell, Bradley describes confusedly one of the arguments presented in Subsections 1.4.1 and 1.4.2. The first which includes a construction of what Russell calls in §55 of *PoM* 'regress of meaning' shows that analysis essentially fails to reconstitute the complex. Its *only* point is this: firstly, if we demand analysis to give us too much, it can never yield a complete result; the excessive, and in Russell's opinion completely unnecessary, demand threatens to stuff every complex with an infinite number of constituents. The second argument can be considered a thread only to those who believe that the concept of actual infinite is contradictory, a Hegelian opinion which Russell deems outdated in the wake of Cantor's set-theoretical research. I will neither defend, nor dispute, the cogency of Russell's arguments themselves. May they be cogent or not, why should Bradley's regress be a confused version of any of them in the first place?

Russell does not offer any error theory to explain Bradley's alleged confusion. Drawing on our interpretation in Section 1.3, Bradley's regress clearly differs from both Russell's arguments. Any consideration about analysis is impossible until we assume that complexes - *qua* possible objects of analysis - are among what there is. Since Bradley's

⁵⁸ *Principles*, §99.

argument disputes something Russell's argument has to start with, they cannot be identical. This applies to the second argument also. Whether there is an infinitude of complexes describable in terms of a recursive operation or not is a question we can ask only if we already suppose complexes to be.

Being unclear about what Bradley's regress involves and what it does not, one might seek to establish Russell's defense on the conclusion of his first argument. This is the conclusion that the unifier is a non-constitutive part of the complex unified. In Section 1.3 I explained that arguing in this way is to no avail. This is obvious once we expand on Russell's statements about unity in §§55, 99 of *Principles*. Let us take $\langle a \text{ differs from } b \rangle$ and $\langle \textit{difference} \text{ relates } a \text{ and } b \rangle$ as an example. *Relating-ness* is, for Russell, a relation which links the relating relation of the former (*difference*) to its terms (a, b). But *relating-ness* is not among the constituents of $\langle a \text{ differs from } b \rangle$! It is a constituent of $\langle \textit{difference} \text{ relates } a \text{ and } b \rangle$. In this proposition it occurs as relating while *difference* is on a par with a and b ; *difference*, a and b are the relata. Russell's main point is negative. *Relating-ness* is as good a relation as any other, and indeed it links *difference* to a and b , but it is excluded from the constitution of $\langle a \text{ differs from } b \rangle$. Now, however elaborated this view might be, it has no force against Bradley's regress. Bradley does not challenge the view that *relating-ness* is a constituent of $\langle a \text{ differs from } b \rangle$. He challenges the view that this relation is responsible for the relating of *difference* in this proposition. Whether this responsibility is carried out from within or from outside of the unified complex is not relevant.

It is very probable that Russell did not understand Bradley's regress properly (for which, no doubt, Bradley's loose presentation of it is partly to blame). Nothing prevents us, however, to find out, on Russell's behalf, whether his views are formidable enough to evade Bradley's regress as we understand it. Russell's attempts to identify Bradley's regress with less challenging arguments might have failed, yet his theory of unity can appear holding well in the face of Bradley's regress. In the following two subsections, I will argue that it does not hold well. I will put forward two arguments showing that Russell's position was untenable.

My first argument will show that Russell could not assume primitivism about unity without falling victim to a version of Frege's 'concept horse' paradox. My second argument is more specific to the theory of Russellian propositions. It shows that Russell's

theory of unity led to an absurdity that all propositions, and in consequence all beliefs and sentences, are true.

1.5.1 The two-fold nature of relations

The doctrine according to which relations differ from other entities by possessing a peculiar two-fold nature is one of the most distinctive theories endorsed by Russell throughout the period 1903-1918. Its importance is bolstered, e.g., by the fact that only with this doctrine at hand could Russell develop in 1910 his multiple-relation theory of judgment and the associated correspondence theory of truth. In *Principles*, the doctrine is introduced by giving an instance as follows:

[H]uman and *humanity* denote precisely the same concept, these words being employed respectively according to the kind of relation in which this concept stands to the other constituents of a proposition in which it occurs.⁵⁹

The occurrence of a relation R in a proposition p as relating (or ‘as concept’ in Russell’s terms) is due to R ’s *being related in a certain way to the other constituents of p* . In other words, the exercised capacity of a relation to relate is not grounded in the relation itself, but in an external factor—the exercised capacity of another relation to relate. This is reaffirmed in §49 of *Principles*. The difference between, e.g., what is referred to by ‘humanity’ in ‘humanity belongs to Socrates’ and what is expressed by ‘human’ in ‘Plato is human’ ‘lies solely,’ says Russell, ‘in external relations, and not in the intrinsic nature of the terms.’⁶⁰

The doctrine relies on the multiple-relation which can be specified as follows. Where n is the arity, we have

the relation y^n relates the terms $x_1, x_2 \dots x_n$.

⁵⁹ *Principles*, §46.

⁶⁰ *Ibid.*

Let R be an arbitrary relation. If R is one of $x_1, x_2 \dots x_n$, it occurs as term in a complex. If $R = y^n$, it occurs as the relating relation of a complex. In this way the Russell of *Principles* conceives the manner in which *difference* occurs in $\langle a \text{ differs from } b \rangle$.⁶¹ He assumes that *difference* possesses this relational property:

y^2 relates a and b

The multiple-relation which is the basis of the properties such as this one is what we have called so far ‘relating-ness’. Its converse is the instantiation relation.⁶²

In §49 of *Principles*, Russell also explains his reason for adopting this view. He considers an alternative view that relations are relating essentially and argues that this view is incoherent. If relations are relating essentially, they cannot be referred to, because reference depends on the possibility that the term we wish to refer to occurs as non-relating (or ‘as subject’, ‘as term’ in Russell’s parlance). For instance, If I state that *difference* is a relation, I express $\langle \text{difference is a relation} \rangle$. *Difference*, here, occurs as subject to which

61 An alternative reading of Russell offered in Korhonen (2013) is that ‘[t]o characterize a Russellian term as a relating relation is to indicate the position it occupies in a given proposition.’ (p. 119) Korhonen proceeds by putting forth the following objection against Russell. ‘The difficulty is to reconcile the fact that a relating relation is what it is because it occupies a certain position in a proposition with the fact that a relating relation is also supposed to be the source of the unity of the proposition. The question arises: How can the position that an entity occupies in a proposition be responsible for the unity of that proposition?’ (ibid., p. 120) This would indeed be troublesome for Russell, but there is no evidence he held to the positionalist view sketched by Korhonen. Korhonen over-interprets Russell’s saying that a relation is relating not in itself, but if it occurs as such in a complex (proposition). From what Russell says it does not follow that the relating of R is due to the position of R in the complex. And certainly it does not follow that complexes have positions. The relating of a relation is, in *Principles*, sometimes referred to as the ‘assertion’ of the relata (e.g., §99). However uninformative this notion of assertion is, it is, in the least, clear that relating was treated by Russell as relative to the relata, not to the whole complex. The idea of positions in a complex came to Russell later in *Theory of Knowledge* (1913). Importantly enough, it came not because Russell was hoping to identify, in such a way, what makes a relation to relate in a complex, but as a response to his struggle to account for the order of the relata in the so-called permutative complexes with non-symmetric relating relations.

62 Griffin asks in his (1993) commentary: ‘What holds the proposition together?’ and proceeds to describe the early Russell’s answer as follows: ‘It cannot, so Russell thought at this time, be anything outside the proposition, and yet anything which is itself part of the proposition (e.g. a relation or copula) is just another terms to be combined with the others.’ Russell’s exposition of the two-fold nature doctrine in §§46, 49 of *Principles* proves this interpretation wrong. However tentatively, Russell maintained that unity is due to a relation which is not constitutive of the complex unified.

the property of being a relation applies. Therefore, *difference* is not relating in <*difference* is a relation>. In the view that relations are relating essentially, this proposition is false. In general, it is principally impossible to say anything true of a relation, and, more to the point, this very statement must be false too because it involves reference to relations. So, not only can we say anything true of a relation, we also cannot say that we cannot say anything true of a relation. The theory that relations are essentially relating is thus self-refuting, hence false. This is a version of Frege's 'concept horse' paradox specific to the early Russell's theory of propositions.⁶³

One way out is to refrain from talking of relations—no reference to them, no quantification over a domain which includes relations. For Russell whose metaphysics of logic is based on exploiting the possibility that relations appear as subjects this solution won't do. Consequently, he comes to conclude that

terms which are concepts [= relations, MS] differ from those which are not, not in respect of self-subsistence [this means: they are not capable of relating essentially or as themselves, MS], but in virtue of the fact that, in certain true or false propositions, they occur in a manner which is different in an indefinable way from the manner in which subjects or terms of relations occur.⁶⁴

Besides the provided statement of the two-fold nature doctrine, we can see again that Russell appeals to indefinability of unity. It is tempting to take this repeated appeal to be the key to Russell's evasion of Bradley's regress.

Lebens writes in his (2008) that Russell's 'substantive non-solution' to the problem posed by Bradley consists in treating the property that a relation can have of actually relating as 'unanalyzable and primitive.'⁶⁵ The word 'non-solution' is used because, instead of providing what Bradley requires (solution), Russell sweeps the requirement

63 In Appendix A of *Principles*, Russell acknowledges close similarity of this incoherence to Frege's well-known paradox. However, as Griffin (1993, 173-6) shows, Russell discovered this incoherence long before he learned about Frege.

64 Ibid.

65 Lebens (2008), 19.

aside (non-solution). We saw that this amounts to a refutation of /3/. In Lebens' opinion, Russell presents a consistent and compelling response to Bradley. According to Elkund, 'Russell's way out of the regress [...] involves taking relatedness as primitive.'⁶⁶ However, the evidence we gathered so far points to the opposite.

We need to first clarify Russell's notion of definition. The purpose of a definition of some X - more specifically, of what Russell calls 'philosophical' as opposed to 'mathematical' definition - is to articulate the complexity involved in X .⁶⁷ So, the definition of X is possible if and only if X is complex. To be primitive means, for something, that it is not complex and, in turn, not definable in Russell's sense. Indefinability, to give an obvious example, is not indefinable because, we have just articulated, informally though, the complexity involved in its notion.

Russell's doctrine of the two-fold nature of relations renders relating a complex ontological phenomenon. Recall that 'relating of R with regard to x and y ' is informatively expandable into 'relating of *relating-ness* with regard to R , x and y '. The general claim that the relating of R in p is due to the relating of another relation, called 'relating-ness', 'instantiation', etc., in another proposition is a (philosophical) definition in Russell's sense.

More to the point, we can deem relating indefinable only alongside a view that relations relate essentially—a view Russell could not maintain. Why? If an *external* factor is responsible for the relating of a relation in a proposition p , we must assign this factor an ontological category (that is, we must make clear what kind of entity the given factor is) and determine how it relates to p .⁶⁸

An alternative reading of Russell can be suggested that what he wanted to be primitive is not the relating of an *ordinary* relation (like *difference* or *love*), but the relating of *relating-ness*. The latter, one might object, is that which Russell calls in §99 of *Principles* 'an indefinable element of assertion.' But, despite bringing in a sort of scholastic distinction, this reading does not help anything. As long as *relating-ness* is a

66 Elkund (2016), 9.

67 See *Principles*, §§30, 108. Philosophical definitions, Russell claims, are restricted to the analysis of an idea or concept into its constituent parts.

68 The opposite does not seem to hold. If unity is due to an essential property of a relation, I do not see a priori reasons why this property cannot be complex, hence definable.

relation, it is subject of Russell's question in §49 of *Principles* whether it relates essentially or in virtue of another relation. The latter must be the case for Russell otherwise the Fregean paradox is invited to destroy his metaphysics. The relation responsible for relating of *relating-ness* is *relating-ness* of higher order. Based on the reasons explained in this section, the notion of relating as applied to *relating-ness* cannot be primitive. Due to generality of the two-fold nature doctrine, this applies to the notion of relating in general.

Russell was stuck between Scylla and Charybdis, the Fregean paradox and Bradley's regress. Each of the arguments shows, in its own way, that the early Russell's metaphysics based on external relations fails to be consistent. Since the indefinability thesis about relating is compatible only with a view that relations relate essentially, any indefinability thesis about relating, general or restricted, makes Russell's metaphysics subject to the Fregean paradox. Russell, therefore, could not maintain that unity is indefinable. The doctrine of the two-fold nature of relations avoids the Fregean paradox if it is applied to all relations without exception. But, in the same breath, it invites Bradley's regress.

1.5.2 All propositions are true

The problem of unity posed by Bradley's regress is closely related to another problem which is specific to the theory of propositions endorsed in *PoM*. In Subsection 1.4.2, we acknowledged the early Russell's ontological regress of materially equivalent propositions. The series of propositions in our example started with the pair $\langle a \text{ differs from } b \rangle$ and $\langle \textit{difference} \text{ relates } a \text{ and } b \rangle$. According to the doctrine of the two-fold nature of relations, *relating-ness* which is relating in the latter is responsible for the unity of the former. This leads to an absurd consequence that every proposition is true.

$\langle a \text{ differs from } b \rangle$ is materially equivalent to $\langle \textit{difference} \text{ relates } a \text{ and } b \rangle$. Although Russell does not go so far as to explain why this has to be so, it seems to be a conclusion based on the brute, linguistic fact that '*difference* relates *a* and *b*' (or alternatively '*a* and *b* bear *difference*') is true if and only if '*a* differs from *b*' is. If we combine this ingredient of the early Russell's thought with his theory of propositions, we

arrive at the absurd conclusion announced above. The early Russell held to the following general principles about propositions:

1. False propositions have being (as well as the true ones do).⁶⁹
2. A proposition cannot have being unless it is a unity.

Suppose, for an indirect proof, that

3. $\langle a \text{ differs from } b \rangle$ is a false proposition.

By Russell's theory of unity, we have this:

4. $\langle a \text{ differs from } b \rangle$ is a unity because *difference* relates a and b .

The explanatory clause after 'because' provides the grounds for the unity of $\langle a \text{ differs from } b \rangle$. An implicit assumption here is that there are no other grounds for the unity of this proposition. So, if difference does not relate a and b , $\langle a \text{ differs from } b \rangle$ is not a unity, which, by (2), means that such a proposition is not involved in the inventory of reality. From the content of 4, we can derive the following

5. if $\langle a \text{ differs from } b \rangle$ is a unity, then *difference* relates a and b .

From 1 and 3, we get this:

6. $\langle a \text{ differs from } b \rangle$ has being.

From 2 and 6, we derive this:

7. $\langle a \text{ differs from } b \rangle$ is a unity.

⁶⁹ In §427 of *Principles*, we read: 'Being is that which belongs to every conceivable term ... in short to everything that can possibly occur in any proposition, true or false, and to all such propositions themselves.'

And, by Modus Ponens, from 7 and 5, we arrive at the conclusion that

8. *difference* relates *a* and *b*.

Russell held to the principle that the sentences of the forms ' $R(x_1, x_2, \dots, x_n)$ ' and ' $\text{relating-ness}(R, R(x_1, x_2, \dots, x_n))$ ' are materially equivalent. By the law of universal instantiation, we derive the following:

9. *a* differs from *b* if and only if *difference* relates *a* and *b*.

From 9 and 8, derive this:

10. *a* differs from *b*.

10 and 3 are incompatible. If *a* differs from *b*, $\langle a \text{ differs from } b \rangle$ cannot be a false proposition, but a true one. The problem seems to lie in premise 4. Once the unity of a proposition is explained in terms of the truth of a proposition which is materially equivalent to it, then no proposition, in general, can be false. Thus, as long as Russell's theory of unity holds, false propositions are impossible.

Another way of looking at this argument is to take issue with premise 1, expelling thus false propositions from reality. This is what Russell did in 1910 when he assumed that reality is not composed of true and false propositions, but of facts which are neither true, nor false—they simply obtain or subsist. In this way, the Russell of 1910 onwards escapes this argument.

2 The importance of denoting for the early Russell's logic

2.1 Russell's notion of a denoting phrase

The Theory of Denoting Concepts is proposed in Chapter 5 of *Principles* to account for the meaning (in Russell's distinctive sense of this word) of sentences which contain a denoting phrase. Which phrases count as denoting? Examples are 'a man', 'the current president of the US', 'some proofs'. In §58 of *Principles*, Russell takes any phrase beginning with one of these six words: 'all', 'every', 'any', 'a', 'some' and 'the', to count as denoting. In linguistics we call such words determiners. The determiner 'no' is left out by Russell, possibly due to omission, possibly because it can be conceived as a composite of 'not' and 'a'. 'No man survived the battle' can be analyzed as 'Not a man survived a battle'. Unfortunately, Russell did not give much thought to the question of combining a determiner with the negation sign. If 'no' is a composite determiner in the suggested sense, should, then, the 'not' which is used in the theory of denoting concepts be differentiated from the sentential negation? And, should we not conceive, e.g., 'some' as a composite determiner as well, namely 'not every'? This reveals one of many lacunas in Russell's exposition of his theory of denoting concepts.

Denoting phrases are, for Russell, construed by prefixing a determiner to a word standing for a class concept.⁷⁰ For example, by joining 'a' with 'proof', we get the denoting phrase 'a proof'. In some cases, the word for a class concept must be turned into its plural form, e.g., in 'some proofs'.

⁷⁰ Russell introduces class-concepts as follows: 'It is to be observed that the class must be distinguished from the class-concept or predicate by which it is to be defined: thus men are a class, while man is a class-concept.' (*Principles*, §21) The opposition between a class and the related class-concept belongs to Russell's naïve-realist reading of the opposition between extension and intension. (See *ibid.* §§65, 66) Although Russell is unclear about how class-concepts fit into his broader ontological views, they do not seem to create an irreducible ontological category. The class-concept man is the concept of humanity. The qualification 'class-' indicates that the concept is considered as determining a class. This proceeds as follows. Take, e.g., 'x is a man'. Every constant which yields a true sentence when substituted for 'x' is a member of all men. Russell takes the expression which follows immediately the article 'a' ('man') to pick out the concept (humanity) qua a class-concept.

We should resist confusing Russell's grammatical characterization of a denoting phrase in terms of determiners for a definition. Russell's intended notion of a denoting phrase covers also phrases without determiners. In §67 of *Principles*, 'men' is treated as a denoting phrase whose extension is the plurality of all men. In 'On Denoting', Russell proposes 'my only son' as an example of a definite description. In §73 of *Principles*, to give one more example, he argues that 'nothing' expresses a denoting concept which does not denote anything.⁷¹ Russell's characterization is, therefore, too narrow to amount to a definition of a denoting phrase.

Observing that denoting phrases without determiners can always be transformed into equivalent determiner noun phrases shows how to deal with the narrowness of Russell's characterization. The equivalent determiner noun phrase can be treated as canonic. Thus, for example, 'men' can be transformed into 'all men' which is a canonic denoting phrase, 'my only son' into 'the only son of mine',⁷² 'nothing' into 'no thing' or 'not a thing'. Accordingly, the notion of a denoting phrase can be defined as follows:

(D1) A phrase is denoting =def

it is a phrase beginning with a determiner or a phrase which can be transformed to an intensionally equivalent phrase with a determiner.

In order to make Russell's characterization wide enough, we need to use an intensional notion of equivalence. Why the extensional equivalence does not suffice? Suppose that an arbitrary phrase *d* denotes something which is also the bearer of the name *n*. Then, *d* and *n* are extensionally equivalent. Since names are opposed to denoting phrases, we need a stronger notion of equivalence. In Russell's semantics, this is achieved by means of his notion of a proposition. Two denoting phrases are equivalent intensionally if and only if swapping one phrase for another in a sentence does not change which proposition is expressed. For instance, 'My only son is educated' and 'The only son of mine is educated'

⁷¹ See also 'Points on Denoting', *CP4*, p. 307.

⁷² In his (1990), on p. 34, Neale remarks that 'with a little work, (and a definite degree of unnaturalness) we can transform genitive noun phrases into expressions beginning with "the" [...].'

express the same proposition. Then, 'my only son' and 'the only son of mine' are equivalent in the required sense.

The narrowness of Russell's original characterization of a denoting phrase is not its only issue. The characterization is also too wide. As Strawson pointed out, 'the' is commonly used to form names of generic terms, in which case we do not get a definite description. His example is 'the whale' in the true sentence 'the whale is a mammal'. 'the whale' here is not a definite description, for the species *whale* surely is not itself a whale.⁷³ This is opposed to 'the whale' in, e.g., 'The whale is called Moby-Dick'. In this case, the same phrase is employed to refer to an individual called 'Moby-Dick' which is a whale.

Stevens argues in his (2011) that Russell's notion of a denoting phrase is purely grammatical. 'The classification of phrases as denoting,' says Stevens, 'does not carry any assumption, then, regarding their semantics.'⁷⁴ 'Grammatical' is opposed by Stevens to 'semantical'. One page earlier, Stevens explains his reason for this interpretation. He says that 'the existence and non-existence of the apparent denotation is irrelevant to the classification of the corresponding phrase as a denoting phrase.'⁷⁵ Assuming provisionally that the theory of denoting concepts makes room for referential failures,⁷⁶ Stevens' point is actually true of both Russell's theories, the Theory of Denoting Concepts and his later Theory of Descriptions. Indeed, it is not essential for a denoting phrase to be one that denotes something. 'The current king of France' is as good a denoting phrase as 'the current president of the US'. But from this it does not follow that Russell's notion of a denoting phrase, whether in *Principles* or in 'On Denoting' and later, is *purely* grammatical!

I will argue that Russell's notion of a denoting phrase was primarily epistemological. It is couched within considerations about requirements that have to be met if one is to be able to understand a sentence.

73 See Strawson (1950), 320.

74 Stevens (2011), 11.

75 *Ibid.*, 10.

76 The next chapter is devoted to a discussion of this possibility.

Implicitly though, already in *Principles* Russell adheres to the distinction between knowledge by acquaintance and knowledge by description. This distinction is manifestly utilized in Russell's consideration about what makes possible for a finite mind to possess knowledge about infinite collections. Russell takes the task of explaining this possibility to be the 'logical purpose' of the theory of denoting. He argues as follows:

... the logical purpose which is served by the theory of denoting is, to enable proposition of finite complexity to deal with infinite classes of terms: this object is effected by all, any and every, and if it were not effected, every general proposition about an infinite class would have to be infinitely complex. Now, for my part, I see no possible way of deciding whether proposition of infinite complexity are possible or not; but this at least is clear, that all the proposition known to us (and, it would seem, all propositions that we can know) are of finite complexity. It is only by obtaining such propositions about infinite classes that we are enabled to deal with infinity; and it is a remarkable and fortunate fact that this method is successful.⁷⁷

In Chapter 6 of *Principles*, Russell puts his point as follows:

With regard to infinite classes, say the class of numbers, it is to be observed that the concept of *all numbers*, though not itself infinitely complex, yet denotes an infinitely complex object. This is the inmost secret of our power to deal with infinity. An infinitely complex concept, though there may be such, can certainly not be manipulated by the human intelligence; but infinite collections, owing to the notion of denoting, can be manipulated without introducing any concepts of infinite complexity.⁷⁸

For example, we cannot know that every natural number has an immediate successor unless we can grasp the content of this knowledge. This content which is a proposition cannot involve the whole infinitude of natural numbers. For otherwise, as Russell points

⁷⁷ *Principles*, §141.

⁷⁸ *Principles*, §72.

out, we (*qua* ‘the human intelligence’) would not be able to grasp it. We would be at loss to understand what ‘every natural number has an immediate successor’ means. But this is not the case.

In his argument, Russell deploys several epistemological doctrines. One concerns the nature of acquaintance. Russell maintains that acquaintance cannot be a multiple relation between mind and an infinite number of terms.⁷⁹ Acquaintance is a *dual* relation between a mind and a term. This relation is always associated with a particular mental act. Acquaintance with an infinitude of terms, had it been possible, would include a performance of an infinitude of mental acts.

Another Russell’s doctrine is based on a principle usually referred to as the ‘Fundamental Principle’. According to this principle,

(Fundamental Principle I) We cannot grasp a proposition unless we are acquainted with all its constituents.

According to Russell, understanding a sentence requires us to grasp the proposition expressed by the given sentence. This leads to the following linguistic version of the Fundamental principle:

(Fundamental Principle II) We cannot understand a sentence unless we are acquainted with all the constituents of the proposition expressed by that sentence.

Russell’s theory of acquaintance, Fundamental Principle I and the premise that our minds are finite yield the conclusion that we cannot grasp a proposition with infinitely many constituents. Embracing also the linguistic version of the Fundamental Principle, it follows that we cannot understand sentences which express propositions involving infinitely many constituents. We can, however, understand sentences *about* infinitely many terms such as ‘every natural number has an immediate successor’. The infinitude of terms which is the

⁷⁹ In his (1996, 130), Ricketts acknowledges that Frege’s theory of judgment involves an analogous restriction. Judgments which consist of an infinite number of conjuncts are impossible for Frege.

subject of our knowledge cannot constitute the proposition expressed. The question then arises, for Russell, of how reference to an infinite number of terms is established.

Knowledge whose subject is an infinite collection ‘is effected by all, any and every,’ says Russell in §141 of *Principles*. In this remark, he refers to his view of how phrases beginning with the determiners ‘all’, ‘any’ and ‘every’ function from the semantical point of view. *They are denoting*. In the proposition expressed by ‘Every natural number has an immediate successor’, the phrase ‘every natural number’ expresses a denoting concept which is itself one, but denotes an infinitude. Our knowledge is effected by grasping the proposition </every natural number/ has /an immediate successor/>. This proposition is finite, hence it is graspable by a finite mind. And since we can grasp this proposition, we are able to understand the sentence ‘every natural number has an immediate successor’.

The subject matter Russell is concerned with in his theory of denoting concepts can be described at several levels of abstraction. We start with the most specific case.

- The subject matter is knowledge about infinite collections.
- It is knowledge about something, be that an infinite collection or not, which is not involved in the content of our knowledge. In Russell’s jargon adopted at around 1911, this is how cases of *knowledge by description* are conceived.
- Third, it is the apprehension of a proposition p such that p purports to be about some a and a is not involved in p . As Hylton (1990) puts it, ‘[denoting, MS] is ... Russell’s way of acknowledging that there must be exceptions to the rule that the things which a proposition is about are among its constituents.’⁸⁰
- Fourth, the subject-matter is understanding a sentence s such that s is about some a and a is not involved the proposition expressed by s .

80 Hylton (1990), 207.

The last two points concern understanding, not knowledge. Understanding is more fundamental than knowledge. Before I know or err to know something, I must understand some affirmable content. At the fundamental level of understanding, the theory of denoting purports to explain what enables us to entertain a proposition which is about something that is not within the reach of our immediate knowledge, i.e., acquaintance.

Makin calls the phenomenon involved in the subject-matter of Russell's theory of denoting 'aboutness-shifting'.⁸¹ The general purpose of a theory of denoting is to explain thinking and use of language which is directed beyond the sphere of our acquaintance. Not only can we think about something which is in our *immediate* grasp, we can also, so to speak, point in a descriptive way to something outside the realm of our acquaintance. The linguistic vehicle of descriptive or indirect reference is a denoting phrase. In accordance to this observation, Russell's notion of a denoting phrase is defined as follows:

(D2) An expression *d* is a denoting phrase =def
a sentence in which *d* occurs can be understood without acquaintance with the term which *d* is supposed to pick out, unless *d* picks out some object *o* and the sentence contains a (logically proper) name whose bearer is *o*.

The qualification in (D2) concerns cases like 'the first African American president of the US is Barack Obama'. (As before, I suppose for simplicity that 'Barack Obama' is a logically proper name.) In a sentence like this, some term (Barack Obama) is referred to in both possible ways, directly and indirectly. The direct sort of reference requires that one who understands the sentence *is* acquainted with the object referred to. For this reason, 'the first African American president of the US is Barack Obama' cannot be understood without acquaintance with the term picked out by the phrase 'the first African American president of the US'. In such cases, we can say that the usual epistemological effect of indirect reference is cancelled.

Putting aside the cases in which the effect of indirect reference is canceled, we introduce the terminology popularized by Neale (1990). Following Gareth Evans, Neale

81 Makin (2011), 18.

takes sentences containing a denoting phrase to express so-called ‘object-independent’ propositions. Derivatively, we can call the sentences themselves ‘object-independent’. E.g., ‘the first African American president of the US is human’ is object-independent in the sense that one can understand this sentence without being acquainted with the denotation of ‘the first African American president of the US’, that is, with Barack Obama himself. This feature is captured by (D2).

The proposed definition provides us with a test of whether something is a denoting phrase or not. This test is applicable independently of the linguistic characterization of a denoting phrase given by (D1) above. For example, is ‘your father’ in ‘Your father is alive’ a denoting phrase? I can understand this sentence without ever being acquainted with your father. The phrase, therefore, is denoting, and it is irrelevant whether it involves a determiner or not. How about ‘the whale’ in ‘the whale is a mammal’? Here, the phrase is not denoting because I would not understand this sentence without being acquainted with what is picked out by ‘the whale’ (the zoological concept of a whale). Despite the fact that ‘the whale’ is grammatically a determiner noun phrase, it is not, in the given context, a denoting phrase.

If a phrase does not pass the test because it denotes a term which is, in the same sentence, referred to by a name, we still have a means to recognize the phrase as denoting without any recourse to (D1). If the name is replaced by a name of something which is not the denotation of the given phrase, the proposition expressed is object-independent in Neale’s sense. For example, in ‘the first African American president of the US is Barack Obama’ we replace ‘Barack Obama’ by, e.g., ‘Charlie Chaplin’. What we get is a sentence which expresses an object-independent proposition because we can understand this sentence without being acquainted with the denotation of ‘the first Afro-American president of the US’.

Surely, one can ponder whether a purely grammatical characterization of a denoting phrase is possible in a particular language. I suspect, however, that no existing natural language has such a uniform syntax as to satisfy this condition. Consider English. Recall the facts about English like one that ‘my only son’ is equivalent in an *intensional* sense to ‘the only son of mine’. Something like this is indispensable if we want to extrapolate the

Russell-style linguistic characterization of a denoting phrase to the effect that we cover the cases of denoting phrases without determiners. Notice that the required notion of equivalence is semantical, not grammatical, because it makes use of a notion of the proposition (more neutrally: the content expressed). In English, therefore, a purely grammatical definition of a denoting phrase is not possible.

Stevens, we can add in conclusion, over-interprets Russell when he takes him to be searching for a purely grammatical definition. Neither in *Principles*, nor in ‘On Denoting’ or other works, does Russell state that his linguistic characterizations of denoting phrases are anything more than useful guides.

2.2 Quantification, variables and denoting

The semantical theory of *Principles* consists of two main parts. The first is the theory of Russellian propositions. This aims to explain the semantic value of sentences without denoting phrases. The second is the Theory of Denoting Concepts [henceforth ‘TDC’] whose purpose is to account for the semantic value of denoting phrases and the sentences with such phrases. For the foundational logicist project of *Principles*, TDC carries more importance than the theory of Russellian propositions. Generality which is central to Russell’s logic cannot be expressed without denoting phrases. No proposition of logic, and by Russell’s logicist doctrine nor any one of pure mathematics, is a Russellian proposition. This comes with the fact that formulas of logic cannot contain other expressions than these: the implication sign (material and formal), variables, class-terms and the class-membership sign. Importantly, logical formulas cannot contain names. There is no place for names in the language of Russell’s logic and mathematics.

‘The whole theory of definition, of identity, of classes, of symbolism, and of the variable is wrapped up in the theory of denoting,’ says Russell in §54 of the *Principles*. Specially important, in this list, is the theory of the variable. Acknowledging how this theory is ‘wrapped up’ in TDC will help us to appreciate better the radical turn in Russell’s thought that eventually led him to his Theory of Descriptions, the celebrated successor of TDC.

Following Peano, Russell distinguishes two sorts of variable. *Apparent* variables are those that are bound by a quantifier, *real* are those that are not so bound. This is a correct thing to say, however, only as long as we are careful enough not to treat quantifiers as primitive logical constants. The real and apparent variables of *Principles* are, in the respective order, *analogous to, but not identical with*, free and bound variables of the modern quantification theory.

Real variables occur in so-called propositional functions. The notion of the propositional function is one of the most controversial in Russell's philosophical thinking. Propositional functions, perhaps surprisingly, cannot be identified with concepts (relations, universals). Quine is right when he remarks that propositional functions differ from concepts in that the former has a 'more nominalistic tone'.⁸² Indeed, not only do propositional functions differ from concepts, they are not even as real as them. While this is maybe correct, the rest of Quine's commentary on Russell's notion of the propositional function is misguided. We will return to this issue later. At this point it suffices to keep in mind that the early Russell's propositional functions are not, despite their nominalistic tone, open sentences (sentential expressions with one or more real variables).

The early Russell is clear that propositional functions are entities expressed by open sentences. In a sense, they seem to be fragments of propositions. In §86 of *Principles*, we read that ' Φx , the propositional function, is what is denoted by *the* proposition of the form Φ in which x occurs'. The italicized 'the' here indicates, in a clumsy way, that a denoting concept is mentioned. In our notation, Russell's statement is rendered in this way: /the proposition of the form Φ in which x occurs/ denotes the propositional function Φx . If ' Φx ' does not contain any real variable except ' x ' and ' x ' is replaced by some name, say, 'Socrates' (suppose both is the case), then the denotation becomes $\langle \Phi(\text{Socrates}) \rangle$. In general, among functions there are some which yield propositions as their values. Those are called by Russell 'propositional functions'.

Suppose, for simplicity, that ' Φx ' is ' x is bald'. Our proposition is, then, $\langle \text{Socrates is bald} \rangle$ and ' Φx ' is a shorthand for the function that x is bald, $\langle x \text{ is bald} \rangle$. $\langle x \text{ is bald} \rangle$ - denoted by /the proposition of the form Φ in which x occurs/ - seems to be part of

82 Quine (1966), 661.

<Socrates is bald>, yet it is not, according to Russell, the concept of baldness. Later I will discuss this point at length.

To appreciate the importance of Russell's notion of a variable, let us consider his logical system. This system is built upon the notion of the so-called 'formal implication', a notion which was also due to Peano. Formal implications are generally quantified conditionals. Nowadays we write ' $(x) (Fx \supset Gx)$ ' and we take such formulas to be analyzable in terms of a general quantifier and material implication. As Byrd notes, 'Russell did not accept this modern conception of formal implication. He viewed it as a genuine primitive, although he does offer an "analysis" of formal implication involving the notion of class, denoting and *any*.'⁸³ Here, a slight alteration Byrd's statement is in order. Formal implication is not, in *Principles*, a genuine primitive, but rather a sort of one which is not genuine. For the purposes of inferential practice, formal implication is treated by Russell as primitive, but, as Byrd says, he offers an analysis of this notion. Russell writes:

For the technical study of Symbolic Logic, it is convenient to take as a single indefinable the notion of a formal implication, i.e. of such propositions as 'x is a man implies x is a mortal, for all values of x'—propositions whose general type is: " $\Phi(x)$ implies $\Psi(x)$ for all values of x," where $\Phi(x)$, $\Psi(x)$, for all values of x are propositions. The analysis of this notion of formal implication belongs to the principles of the subject, but is not required for its formal development.⁸⁴

This is closely related to Russell's distinction between mathematical and philosophical definability.⁸⁵ The purpose of a formal system is to represent valid deductive relations between propositions. Any notation and its definitions can suffice for this task without being philosophically adequate (to judge upon such matters is a task belonging to what Russell's calls the 'theory of symbolism'). Thus, for example, Russell admits in §19 of

83 Byrd (1989), 345.

84 *Principles*, §12.

85 See *ibid.* §§19, 31; Cf. §63.

Principles, that from the philosophical point of view his definitions of disjunction and negation are inadequate. He admits that they do not give ‘the true philosophic analysis of the matter’. But insofar as they endow one’s formal system with the required expressive power, the definitions can be accepted as *mathematical*, that is, suitable for a formal development of the system of logic. Based on the same constraint of analysis, Russell treats formal implication as primitive.

Of course, the Russell of *Principles* is not interested in formal implications like ‘ (x) (x is human \square x is mortal)’, except for expository purposes. His focus is on those formal implications which belong to logic. These contain logical constants only. Russell’s often referred definition of logicism draws on the fact that the whole of logic is expressible by means of the formulas of this form. He writes:

PURE mathematics is the class of all propositions of the form “ p implies q ,” where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants.⁸⁶

For example, all axioms of Russell’s propositional calculus listed in §18 of *Principles* are formal implications. It is worth noting that we should resist reading Russell ‘ p ’, ‘ q ’ and ‘ s ’ in these axioms as schematic letters. Russell’s calculus of propositions (and the rest of his logic as well) contains bound variables, not schematic letters like the modern sentential calculi.⁸⁷

Importantly enough, the notion of the real variable, as well as the associated notion of the apparent variable, are not primitive. The definition of this notion is, for Russell, a task pertaining, in part at least, to TDC. This is even clearer from the exposition of his logicist doctrine in Russell’s ‘Mathematics and the Metaphysicians’ (1901/1917):

Pure mathematics consists entirely of assertions to the effect that, if such and such a proposition is true of *anything*, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true,

⁸⁶ *Principles*, §1.

⁸⁷ On this point, see Landini (1996), 561.

and not to mention what the anything is, of which it is supposed to be true. Both these points would belong to applied mathematics. We start, in pure mathematics, from certain rules of inference, by which we can infer that *if* one proposition is true, then so is some other proposition. These rules of inference constitute the major part of the principles of formal logic. We take any hypothesis that seems amusing, and deduce its consequences. *If* our hypothesis is about *anything*, then our deductions constitute mathematics.⁸⁸

The propositions of logic/pure mathematics are about *anything*. Here, Russell assimilates the meaning of a variable in logic to the natural language pronoun ‘anything’.⁸⁹ In his view, variables in logic are universally quantified and wholly unrestricted. Their domain is the domain of *all* terms. This view was soon to be undermined by the theory of types, however, it shows that Russell envisages variables to be denoting phrases of a sort. They mean something like ‘any term (of a certain type)’. This analysis was to some extent tentative and, by the time of *Principles*, Russell came with a more complicated picture of how the variables of logic function from the semantic point of view.

In Chapter 8 of *Principles*, Russell begins with an explanation of the nature of the variable in terms of what he calls ‘constancy of form’.⁹⁰ Take an arbitrary proposition. Then choose a term which occurs in that proposition as term. Vary this term across the range of all other terms. The propositions resulting from the variation are taken, by Russell, to form a class of propositions of constant form. ‘Taking *any* term, a certain member of any class of propositions of constant form will contain that term.’⁹¹ Take, e.g., Socrates. <Socrates was bald> contains Socrates and belongs to the class of all propositions of the form <*x* was bald>. <Socrates \square The Earth is round>, <Obama is bald

88 Russell (1901/1917), 75.

89 In a natural language, English in our case, we use the word “anything” as restricted to a certain domain, the restriction usually being implicit. Russell’s “anything” is unrestricted.

90 To be clear on the topic, Russell’s analysis discussed in this paragraph concerns the notion of real variable.

91 *Principles*, §86.

\square Socrates> are some of many propositions containing Socrates and belonging to the class of all propositions of the form $\langle x \square y \rangle$. ‘ x ’ and ‘ y ’ here are real variables and the symbols ‘ $\langle x$ was bald>’ and ‘ $\langle x \square y \rangle$ ’ express propositional functions. This is Russell’s point: whichever term t and whichever propositional function F you take, you can always find a proposition satisfying both these conditions:

- it contains t and
- it belongs to the class of the propositions of the form determined by F .

‘Thus x , the variable,’ Russell continues, ‘is what is denoted by *any term*, and Φx , the propositional function, is what is denoted by *the* proposition of the form Φ in which x occurs. [...] Thus in addition to propositional functions, the notions of *any* and of denoting are presupposed in the notion of the variable.’⁹²

The difference between real and apparent variables consists in the absence or presence of assertion.⁹³ $\langle x$ is /a term/> is a propositional function which is denoted by the nominal phrase ‘Anything’s being a term’ or (in Russell’s other way of putting it) ‘the proposition of the form x is /a term/,’ in short, this is: $\langle x$ is /a term/>. Russell calls the denoting concept which refers to $\langle x$ is /a term/> a propositional concept. We can denote this concept by means of ‘//Something’s/ being /a term//’. Such contents can be asserted. In this way, they are ambiguous between a denoting concept and a proposition. To use a simpler example, the propositional concept

/Socrates’ being bald/,

if asserted, turns into \langle Socrates is bald>. By the same token,

//Anything’s/ being /a term//,

92 Ibid.

93 This is not a mental act, but Russell’s ontological ‘indefinable element of assertion’ which we discussed in the previous chapter.

if asserted, turns into $\langle \text{Anything} / \text{is a term} \rangle$. This underlies the difference between ‘ (x) (x is a term)’ and ‘ x is a term’, i.e., the difference between the variable as an apparent variable and the variable as a real variable.

The real variable in ‘ x is a term’ turns into an apparent variable once we assert ‘ x is a term’. The modern reader may tend to add: ‘We must assert it *for all values of x* .’ But this qualification is already present in ‘ x is a term’ because unlike the modern free variable, Russell’s real variables are denoting phrases including the meaning of ‘any’ (or ‘some’ in other cases).

It should be noted that after publishing *Principles* Russell soon adopted a view that real variables (in this context, considered as linguistic devices) can form a closed formula. ‘ x is a term’, where ‘ x ’ is a real variable, became a closed wff of Russell’s logical system. In *Principles*, there is no indication, insofar as I know, of this view and it is likely that Russell adopted it from Frege whose works he studied in the period after submitting *Principles* to print.

Russell’s analysis of the variable in *Principles*, in fact, does not offer any grounds for distinguishing formulas such as ‘ x is a term’ from open sentences. Obviously, what is needed is a distinction between real (free) variables and schematic letters. Where ‘ α ’ is a schematic letter for any term, ‘ α is a term’ is an open formula expressing a propositional function. If adopted, this modern ingredient necessitates to depart from the account of propositional functions endorsed in *Principles*. ‘ α ’ is not a denoting phrase but a mere placeholder without any referential meaning! It is not necessary to go into greater details here. We should be aware that Russell’s view about the nature of generality and the way of capturing its aspects in his logical notations were rapidly developing at around the year 1903.

At first glance, the current exposition of Russell’s theory in *Principles* might seem to suffer from a confusion. On the one hand, free variables are said to be part of propositional functions and thus not to be symbolic in their nature. On the other, I have proposed a distinction between real and apparent variables in which variables are taken to be denoting phrases, hence symbols of a sort. Both ways of using the term ‘variable’ are in

fact correct. In the most adequate, all-embracing sense of this term, we take Russell's variables to manifest these three aspects:

- denoting phrase
- denoting concept
- denotation.

The free variable which is part of a propositional function is a variable considered in the third of these aspects. x in $\langle x \text{ is /a term/} \rangle$ is what Russell classifies as an 'object'.⁹⁴ This object is denoted by 'anything' and is, in Russell's slightly esoteric terminology, called 'ambiguous term' (or 'variable conjunction'). It is, roughly speaking, the moment of saying 'whichever term we choose from the class of all terms' reified and envisaged as a denoted unit. Russell's infamous mention of an ambiguous man at the outset of 'On Denoting' is reminiscent of this theory!⁹⁵ In the next section, I criticize this component of TDC.

2.3 Variables in formal implications

Russell's theory poses greater difficulties when we approach cases that are more complicated than ' x is a term'. Any formal implication is such a case. The complication comes when the variable has more than one instance. ' $(x) (x \text{ is human} \sqsupset x \text{ is mortal})$ ' does not mean the same as 'If something is human, something is mortal'. The correct reading is 'If something is human, it is mortal.' This shows, as Russell argues in §89 of *Principles*, that the whole propositional function $\langle Fx \sqsupset Gx \rangle$ is involved in a formal implication. This propositional function forms, as we have already seen, a class of propositions of constant form. In our case this is the class of the propositions of the form $\langle x \text{ is human} \sqsupset x \text{ is mortal} \rangle$. If we, then, assert ' $(x) (x \text{ is human} \sqsupset x \text{ is mortal})$ ', we state, according to Russell, that any proposition belonging to this class is true.

⁹⁴ See *Principles*, §58.

⁹⁵ *OD*, 379.

In general, every formal implication has this form: ‘Any proposition of the form Fx is true’. In this way ‘any is presupposed in mathematical formalism.’⁹⁶ The qualification of the ‘any’-phrase effected by the word ‘proposition’ here is in fact redundant because no non-propositional term can have the form expressed by a open sentence (Fx).⁹⁷ So, from now on we proceed by taking every formal implication to have this form: ‘Any term of the form Fx is true’.

This makes Russell’s logic and his related theory of (pure) mathematics reliant on TDC. In practice, however, Russell never used sentences of the form ‘any term of the form $\langle Fx \rangle$ is true’. Following Peano, he was using quantifiers subscripted to the material implication sign (e.g., x is human \square_x x is mortal). Or, shortly after publishing *Principles*, quantifiers prefixed to a formula. Notational practices notwithstanding, Russell’s view of formal implication cannot be bypassed. It is an essential part of Russell’s semantic views in *Principles* and all its consequences must be reckoned with seriously. In the rest of this section, I will discuss one of its possible consequences which threatens to make Russell’s conception of logic deeply confused.

By modern standards, any formal system is strictly separated from its semantics. This is a part of what the epithet ‘formal’ means nowadays. From this it follows, in particular, that the predicate ‘is true’ (or its opposite) will not occur in any theorem of the formal system. However, from Russell’s theory of formal implication, it seems, *prima facie*, to follow that every theorem contains this predicate. ‘is true’ in ‘any term of the form $\langle Fx \rangle$ is true’ is not deflatable, since ‘any term of the form $\langle Fx \rangle$ ’ is incapable of a truth-value and cannot be asserted. This is a kind of a truth-ascription which Ramsey recognizes in natural language as an ascription of truth to a proposition which is not explicitly given, but merely described.⁹⁸ In such cases, Ramsey recommends to effect the deflation of the truth-predicate as follows:

96 *Principles*, §89.

97 This comes with the very fact that the notion of a proposition is derived by means of the notion of a sentential structure.

98 See Ramsey (1927, 157).

In the second case in which the proposition is described and not given explicitly, we have perhaps more of a problem, for we get statements from which we cannot in ordinary language eliminate the words “true” and “false.” Thus if I say “he is always right” I mean that the propositions he asserts are always true, and there does not seem to be any way of expressing this without using the word “true.” But suppose we put it thus “For all p , if he asserts p , p is true.” then we see that the propositional function p is true is simply the same as p , as *e.g.* its value “Caesar was murdered is true,” is the same as “Caesar was murdered.” We have in English to add “is true” to give the sentence a verb, forgetting that “ p ” already contains a (variable) verb.⁹⁹

Let us see whether this trick can help the early Russell to avoid the untoward consequence of introducing ‘is true’ into the formal system of his logic. ‘any term of the form $\langle Fx \rangle$ is true’, according to Ramsey’s principle of translation, receives this reformulation:

(1) For all p , if p is of the form Fx , p .

In the logic of *Principles*, (1) is rendered as follows:

(1') (p) (p is of the form $Fx \supset p$).

This is, again, a formal implication. Ramsey’s principle of translation and Russell’s theory of formal implication appear in a vicious circle. (1') will be analyzed, in accordance to Russell’s theory, as follows:

(2) Any term of the form $\langle p \supset p \cdot \cdot p$ is of the form $Fx \supset p \rangle$ is true.

Applying Ramsey’s principle on (2), we get this:

(3) (q) (q is of the form $\langle p \supset p \cdot \cdot p$ is of the form $Fx \supset p \rangle \supset q$).

⁹⁹ Ibid., 158.

By Russell's measures, (3) is recognized as a formal implication. We are thereby compelled to introduce the predicate 'is true' again. In general, by applying Ramsey's trick over and over again we submit the original formula, 'any term of the form $\langle Fx \rangle$ is true', to a more and more complicated task of reformulation, but we never succeed in avoiding the truth-predicate.

Ramsey's principle does not help, yet the situation is not hopeless for Russell. In his parlance, Russell often uses the predicates 'is true' and 'is false', meaning by them absence and presence of negation respectively. When Russell says ' p is false', he means ' $\sim p$ '. When he says ' p is true', he means ' p '. It should be said, however, that the absence of negation does not necessarily amount to an assertion of a proposition. In general, if ' p ' is embedded in an extensional context like, e.g., ' $q \supset p$ ', the whole is asserted, while ' p ' is not.

Russell reads ' $q \supset p$ ' by saying 'if q is true, then p is true'. 'is true', in its both occurrences here, does not mean anything semantic. And it does not indicate an assertion either. The presence or absence of negation is a *purely syntactic* feature. This is what Russell means by 'is true' and 'is false' respectively. For our purposes, it remains to acknowledge how the Russell of *Principles* understands negation. According to the definition of negation in *Principles*, §19, ' $\sim p$ ' means the following: 'every proposition is implied by p '.

As we saw, 'is true' in 'any term of the form $\langle Fx \rangle$ is true' is not deflatable, since we cannot assert 'any proposition of the form $\langle Fx \rangle$ '. (Note that, here, what is said to be true is not embedded in an extensional context, so a requirement for an assertion *is* in order.) However, due to Russell's understanding of negation, we can render the contradictory of 'any term of the form $\langle Fx \rangle$ is true', that is, 'some term of the form $\langle Fx \rangle$ is false', in the following manner:

(4) 'Every proposition is implied by some term of the form $\langle Fx \rangle$.'

By negating (4), we can get an assertion of every proposition of the form $\langle Fx \rangle$. It is, therefore, double negation which solves the problem. ' $\sim(4)$ ' reads as follows:

(5) ‘That every proposition is implied by some term of the form $\langle Fx \rangle$, implies every proposition.’

The proposition expressed by (5) is this:

(5’) $\langle \langle \text{Every proposition} \rangle \text{ is implied by } \langle \text{some term of the form } \langle Fx \rangle \rangle \rangle \text{ implies } \langle \text{every proposition} \rangle$.

(5) expresses the same proposition as ‘it is false that some proposition of the form $\langle Fx \rangle$ is false’, and the proposition expressed by both is (5’). Since (5’) is not by its form a formal implication, the vicious circle we encountered above is avoided.

A similar solution to the problem can be achieved with the help of the substitutional theory of propositions developed by Russell in 1905. The formula starts with a comprehension of every proposition of a certain form in terms of a substitutional pattern. We first need an instance of a proposition of the form $\langle Fx \rangle$, say, $\langle Fa \rangle$. Then the comprehension is as follows: $(x) (\exists q) (Fa/a;x!q)$. This says: For every x , there is q such that q results from substituting x for every instance of a in Fa . The remaining ingredient is an assertion of q . This is effected by an assertion this formula:

(6) $(x) (\exists q) (Fa/a;x!q) \& q$.

(In Russell’s notation at the time of the *Principles*, the conjuncts are juxtaposed; I will use, throughout this work, ‘&’ instead, meaning the same as Russell’s juxtaposition.) ‘&’ is a defined symbol in the *Principles*. We have the following definition:

(D3) $p \& q = \text{Df. } (p \sqsubseteq p) : \sqsubseteq : q \sqsubseteq q . \sqsubseteq . (p \sqsubseteq (q \sqsubseteq r)) \sqsubseteq r$

The conditions ‘ $p \sqsubseteq p$ ’ and ‘ $q \sqsubseteq q$ ’ assure that p and q are propositions. Only those terms that are propositional imply themselves. Since ‘ $(x) (\exists q) (Fa/a;x!q)$ ’ and ‘ q ’ are well-formed

propositional expressions (terms) of Russell’s logic based on the substitutional theory, and every expression of this sort is an expression for a proposition, we can leave these conditions aside. Here, I omit for the sake of convenience the peculiarity of the substitutional theory that a well-formed propositional expression like ‘ $(x) (\Box q) (Fa/a;x!q)$ ’ expresses a class of singular propositions, rather than one general proposition which would be taken to have many singular instances. (6) is, then, expanded as follows:

$$(7) (r) (((x) (\Box q) (Fa/a;x!q) \Box (q \Box r)) \Box r)$$

As we can see (7) is a formal implication. Therefore, we are still without a working solution of the above problem with ‘is true’. The method of achieving such a solution is, however, available. It is the same as in the previous solution above—it consists in an employment of TDC instead of the apparatus of the quantification theory. The following is an alternative for (7):

(8) That any term resulting from a substitution in $\langle Fa \rangle$ of a for some term in every occurrence of a in $\langle Fa \rangle$ implies every term, implies every term.

This is the proposition expressed by (8):

(8’) $\langle \langle \text{any term} \rangle \text{ resulting from } \langle \text{a substitution in } \langle Fa \rangle \text{ of } a \text{ for } \langle \text{some term} \rangle \text{ in } \langle \text{every occurrence of } a \text{ in } \langle Fa \rangle \rangle \rangle \text{ implies } \langle \text{every term} \rangle \text{ implies } \langle \text{every term} \rangle$

(8’) is the proposition expressed by (8) as well as by (6). Since (8’) is not by its form a formal implication, the vicious circle encountered above is avoided.

Both solutions of the problem with ‘is true’ hang upon Russell’s analysis of generality in terms of denoting concepts. Russell’s logic in *Principles* is thus shown to stick to the modern standard of separating a formal system from its semantics, however, at the price of being fundamentally dependent on TDC. This means that every problem of TDC should be counted as a problem of Russell’s logic in *Principles* and, consequently, as a problem of Russell’s early logicism.

3 Russell analysis of denoting before ‘On Denoting’

3.1 The purview of Russell’s Theory of Denoting Concepts

Theory of Denoting Concepts [TDC] is to some extent similar to Frege’s mature semantics of *Sinn* and *Bedeutung*.¹⁰⁰ Like Frege’s theory, Russell’s differentiates in an expression two semantical levels. But unlike Frege’s theory which applies to every semantically complete expression, Russell’s applies only to some such expressions. Moreover, as we will see, Russell and Frege opine differently about which expressions count as semantically complete.¹⁰¹

For Frege, the semantic value of any semantically complete expression splits into two levels, where one of the values is called ‘sense’ (*Sinn*) and the other ‘meaning’ or ‘reference’ (*Bedeutung*). The sense is suggestively characterized by Frege as a mode of presentation (*Gegebenheitsweise*). The reference is what is presented.¹⁰² Russell’s theory draws on the same idea. His analogue for Frege’s distinction between sense and reference is the distinction between a denoting concept and the entity denoted (denotation). In the manuscripts in which Russell kept developing TDC after the completion of *Principles*, a different terminology was adopted. Denoting concepts were called ‘meanings’. In one such manuscript, ‘Points on Denoting’ (1903-4) [henceforth ‘PD’], we read that ‘meaning [= denoting concept, MS] has to do not with what a thing is, but with *the road by which it is reached*.’¹⁰³

Frege’s two-level semantics and TDC differ in their purviews. In ‘On Meaning and Denotation’ (1903-4), Russell is explicit for the first time about his view that some proper

100 Frege published his mature semantical views in ‘Funktion und Begriff’ (1891), ‘Über Begriff und Gegenstand’ (1892) and ‘Über Sinn und Bedeutung’ (1892).

101 To be semantically complete means for an expression that it is endowed with meaning independently from its placing in a context. In Russell’s parlance, such expressions are complete symbols as opposed to incomplete ones.

102 The distinction between sense and reference is not absolute because senses can be referred to. See Frege (1892/1960, 58).

103 CP4, 308. (my emphasis)

names are shorthand definite descriptions.¹⁰⁴ This addition to TDC (called the descriptive theory of names), it should be clear, does not compromise Russell’s fundamental semantic opposition between names and denoting phrases. Some proper names are shorthand descriptions, some are proper names in Russell’s strict sense;¹⁰⁵ TDC applies *only* to those names that are shorthand descriptions. Frege’s theory of names is descriptive through and through. For Frege, *every* name carries some descriptive content.¹⁰⁶ This is the first difference. The second is that Frege’s theory does not apply to the majority of determiner noun phrases. Frege does not treat, e.g., ‘all men’ as a semantically complete unit. His analysis of all determiner noun phrases other than definite descriptions is quantificational. For example, Frege takes ‘All men are mortal’ to express a proposition (*Gedanke*) which can also be expressed by ‘For all x , if x is a man, x is mortal’.¹⁰⁷ Russell disagrees on this point. In his opinion, the sentences in question are materially equivalent, but they do not express the same proposition.¹⁰⁸

Including other determiner noun phrases than definite descriptions in the purview of TDC led Russell to adopt a peculiar doctrine which does not have anything analogous to it in Frege’s mature semantics. For the sake of comparison, let us, again, start with Frege. For Frege, if a semantically complete expression is endowed with a reference, then the reference is always a single entity. Frege’s references are analogous to Russell’s

104 See *CP4*, 324.

105 Any proper name is a proper name in Russell’s strict sense if and only if that name stands for its bearer independently from any description of it.

106 This follows from Frege’s view that every (semantically complete) expression must contribute something to the sense of the sentence in which it occurs. The sense of the sentence, the *Gedanke*, then points to or presents a truth-value. Frege could not assume Russell’s notion of a genuine (logically proper) name, since such names would have nothing to contribute to the senses of the sentences in which they occur.

107 In his introduction of the so-called ‘condition-stroke’ in the first volume of *The Basic Laws of Arithmetic* (1893), Frege gives the following example:

$$\text{---}^a \left[\begin{array}{l} a^4 = 1 \\ a^2 = 1 \end{array} \right]$$

This says ‘in words: *if* the square of something is 1, *then* its fourth power also is 1; or: *all* square roots of 1 are fourth roots of 1.[fn. 20] Here we have the *subordination* of a concept under a concept, a *universal* affirmative proposition. (Frege 1893, 55)

108 See *Principles*, §73.

denotations. So, we can ask: Are Russell's denotations always single entities? Russell answers in the negative. Some denotations are, in his view, *combinations of terms*, where at least some such combinations are not single.

In the following passage Russell introduces the term 'object' which covers everything which can be denoted:

There is, connected with every predicate, a great variety of closely allied concepts [...] Starting, for example, with *human*, we have man, men, all men, every man, the human race, of which all except the first are twofold, a denoting concept and an object denoted; we have also, less closely analogous, the notions "a man" and "some man," which again denote objects* other than themselves.¹⁰⁹

The footnote marked in this passage by the asterisk reveals that objects do not need to be terms. We read the following:

I shall use the word *object* in a wider sense than *term*, to cover both singular and plural, and also cases of ambiguity, such as "a man." The fact that a word can be framed with a wider meaning than *term* raises grave logical difficulties.¹¹⁰

The category of objects is, in Russell's view, wider than that of terms. All terms are singular. In addition to terms, we also have objects called 'plurals' and 'cases of ambiguity'.

In what follows, we start with Russell's notion of a plural and then we move to discuss the cases of ambiguity. As we will see, objects of both these sorts present problems which Russell struggled to solve. This will substantiate Russell's mention of 'grave logical difficulties' in the quote above. In the rest of this chapter, I will rectify Peter Hylton's interpretation of TDC.

109 *Principles*, §58.

110 *Ibid.*, fn.

3.2 Russell's solution of the paradox of plurals

The plural is what Russell calls elsewhere 'set as many'.¹¹¹ This is opposed to 'set as one'. Plurals consist of terms combined by numerical conjunction. This sort of conjunction should not be confused with the propositional conjunction. The propositional conjunction is defined in terms of material implication. It is a complex relation usually considered as holding between propositions. Why usually? The distinction between the two conjunctions is not in that the later one applies to propositions only. Surprisingly perhaps, the propositional conjunction applies to any two terms, propositional or not. Russell's logic, one should be aware, was intended to be *maximally general*. Consequently, relations expressed by the logical connectives should apply to anything whatsoever. For instance, '<Obama is bald \square Socrates>' is a well-formed propositional term, although one which picks out a false proposition.¹¹²

As in many other cases, the early Russell exploits in the case of plurals a certain feature of natural language and reifies it, i.e., he posits its ontological counterpart. In the present case, it is the notion of a list. The possibility of listing things by means of 'and' reveals, in Russell's naïve realist view, the existence and structure of plurals. For example, 'Phobos and Deimos' is a well-formed expression listing all natural satellites of Mars. It is not a denoting phrase proper, yet it is not a name either. At any rate, it is, for Russell, a singular term which picks out the set (as many) of all natural satellites of Mars. An extensionally equivalent denoting phrase is, for example, the one we have just used, 'all natural satellites of Mars'.

The plural is a combination of terms which itself is not a term because it fails to satisfy the condition of singularity. This brings us to the 'grave logical difficulties' which Russell refers to in footnote * from §58 of *Principles* cited above. Throughout the chapters

111 See *Principles*, §74.

112 In §18 of *Principles*, Russell proposes a conditional definition of propositional conjunction which requires that the conjuncts are propositions. However, as Bird argues, this definition is at variance with Russell's treatment of implication, and, more importantly, it is at variance with Russell's formulation of the axiom Simplification. (See Bird 1989, 351) The formulation of Simplification shows that in practice Russell's propositional conjunction was unrestricted.

that precede Russell's introduction of TDC, Russell embraces the following principle about reference:

(P1) If something can be referred to, it can occur as term in a proposition.¹¹³

From Chapter 4 of *Principles*, we know that Russell held to this:

(P2) Something can occur as term in a proposition if and only if it is singular.¹¹⁴

Now, take an arbitrary plural *l*. Plurals are *not* singular. They are not single entities—that is why Russell speaks of 'sets as many', opposing thus plurals to 'sets as one'. From this supposition and an instance of the contrapositive of (P2), we can infer, by Modus Ponens, that

(1) *l* cannot occur as term in a proposition

The contrapositive of (P1) is as follows:

(P1c) If something cannot occur as term in a proposition, it cannot be referred to.

From (1) and an instance of (P1c), we arrive, by Modus Ponens, at this:

(1) *l* cannot be referred to.

This is self-refuting. If (1) is true, *l* is referred to and, by supposition, it is truly ascribed a property of being incapable of being referred to. But (1) cannot be true unless *l* is capable of being referred to. So, if (1) is true, it is false, and *vice versa*. This problem is already at

113 For what it means, for an entity, to occur in a proposition as term (or as subject), see Section 1.1 in the first chapter.

114 See *Principles*, §47.

the stage of (P1c). The antecedent of this principle cannot have a true instance, on pain of inconsistency.¹¹⁵

After a brief exposition in §74 of *Principles* of the paradox of plurals, Russell explains how this paradox should be resolved. He begins with an optimistic claim that he does not see ‘any way of eliciting a precise contradiction in this case [of plurals]’. In other words, the contradiction should be imprecise or, better still, just apparent—not real. Russell continues as follows:

In such a proposition as ‘*A* and *B* are two’, there is no logical subject: the assertion is not about *A*, not about *B*, nor about the whole composed of both, but strictly and only about *A* and *B*. Thus it would seem that assertions are not necessarily *about* single subjects, but may be about many subjects; and this removes the contradiction which arose, in the case of concepts, from the impossibility of making assertions about them unless they were turned into subjects. This impossibility being here absent, the contradiction which was to be feared does not arise.¹¹⁶

Russell admits that ‘*A* and *B* are two’ contains an assertion about something. ‘*A* and *B*’, therefore, picks out something. Russell first explains what this *something* (the intentional object) cannot be. He rules out two obvious possibilities: **(a)** The object which is truly said to be two is not $\{A, B\}$; this set is a single entity, hence it is one, not two. **(b)** Nor is it *A* and *B* *severally*; ‘*A* and *B* are two’ is not equivalent to ‘*A* is two and *B* is two’. Russell seeks to assume a possibility which lies between (a) and (b). On the one hand, ‘*A* and *B*’ must pick out *A* and *B* in some plural manner. On the other, *A* and *B* must be supplied for predication as a collection, not severally, not one after another. Russell concludes that the

115 In section 1.5.1, we encountered a similar issue. If relations (‘concepts’ in the early Russell’s jargon) are taken to be essentially relating, we end up holding to a self-refuting theory. Both problems, with essentially relating relations and with plurals, are instances of the same species. In general, any inconsistency of this general sort lies in an assumption that something is not a term. From an assumption that some *x* is not a term, it follows, under the premises of the early Russell’s philosophy, that *x* cannot be referred to. This is self-refuting.

116 *Principles*, §74.

logical subject of ‘*A* and *B* are two’ is a plural—a peculiar sort of entity which, especially as opposed to a set as one, is not singular.

The ontological part of Russell’s proposal maps in its content to the part which concerns the linguistic notions of reference and aboutness. In the proposition <*A* and *B* are two>, the concept involved in <... are two> holds of *A* and *B* by means of a *many-one* relation (copula or *relating-ness*).

Russell’s strategy is to deny (P2). He denies this: something can occur as term in a proposition if and only if it is singular. Consequently, he denies that whatever can be referred to and ascribed a property must be singular. In §127 of *Principles*, Russell asks whether the notion of *one* is presupposed every time we speak of a term. After a brief discussion, he reaffirms his resolution to the paradox offered earlier in the book: ‘[...] oneness belongs, in this view, to a certain type of logical subjects, but classes which are not one may yet have assertions made about them.’¹¹⁷

3.3 Distributive conjunction

In §59 of *Principles*, Russell proposes an intriguing explanation of which sorts of objects are denoted by different types of denoting phrases. He decides to discuss ‘the’ later in Chapter 5 and proceeds, in §59, to give an account of ‘all’-, ‘every’-, ‘any’-, ‘a’- and ‘some’-phrases.

We have already discussed ‘all’-phrases. These are supposed to denote numerical conjunctions. Only when the extension of a concept joined with *all* is a singleton, the object denoted is a single term. The denotation of ‘all natural satellites of the Earth’ is such. So, only when the extension is a set of two or more terms, Russell deploys his notion of the numerical conjunction. The characteristic feature of an ‘all’-phrase is that it concerns all terms involved in its denotation *collectively*. This is illustrated, in §59, by ‘Brown and Jones are two of Miss Smith’s suitors’. The predicate in this sentence does not apply to Brown and Jones severally. Neither of them is two of Miss Smith’s suitors. The predicate does not apply to the set (as one) consisting solely of Brown and Jones either. According to

117 Ibid., §127.

Russell, it applies to these two individuals, as it were, collectively. Russell's resolution to the paradox described in the previous section relies on this consideration.

The objects denoted by 'every'-phrases - again, except when the extension of the class-concept is a singleton - is explained in terms of *distributive* conjunction. This is illustrated by 'Brown and Jones are paying court to Miss Smith'. The predicate in this sentence concerns Brown and Jones distributively: each single of them is paying court to Miss Smith.

In order to semantically separate 'all' from 'every', Russell acknowledges two sorts of conjunctive combinations. This is at odds with the use of 'all' and 'every' in English. Notably, Russell admits this. He says that

I intend to distinguish between *a* and *some* in a way not warranted by language; the distinction between *all* and *every* is also straining of usage. Both are necessary to avoid circumlocution.¹¹⁸

Russell sees himself as rectifying the impreciseness of natural language in order to set up the semantical apparatus for his logic, of which the theory of denoting is a part. In 'On Denoting', however, he adopts the opposite course of analysis. We read:

Consider next the proposition 'all men are mortal'. This proposition is really hypothetical and states that *if* anything is a man, it is mortal. That is, it states that if *x* is a man, *x* is mortal, whatever *x* may be. ...

"C(all men)" means "'If *x* is human, then C(*x*) is true' is always true". ...

"C(every man)" will mean the same as "C(all men)".¹¹⁹

In his exposition of Theory of Descriptions, there is no semantic difference assumed between 'all men' and 'every man'. 'All men are mortal' and 'Every man is mortal', for instance, are different verbalizations of the same proposition. This proposition is of the form ' $(x) (Gx \supset Fx)$ '. Putting for now aside the specifics of Russell's Theory of Descriptions,

118 Ibid., §59 fn.

119 OD, 481.

what made Russell in 1905 to change his mind and treat ‘all’ and ‘every’ as giving the same semantical contribution?

The artificial semantic separation of ‘all’-phrases from ‘every’-phrases involved in TDC seems to be a consequence of Russell’s struggle to deal with statements in which we ascribe numbers. In his account of how ‘all’-phrases function from the semantical point of view, Russell uses as an example this sentence: ‘Brown and Jones are two of Miss Smith’s suitors’. The issue involved in Russell’s analysis of ‘all’ is encapsulated in this example. Its truth-conditions can be formulated in terms of *two* logically independent statements as follows:

‘Brown and Jones are two of Miss Smith’s suitors’ is true if and only if

- (i) Brown and Jones are two and
- (ii) Brown and Jones are suitors of Miss Smith.

Take an arbitrary concept *C* (a class concept in Russell’s terms) such that *C* is satisfied by Brown, *C* is satisfied by Jones, Brown \neq Jones and nothing else satisfies *C*. Now, we can formulate (i) and (ii) as follows:

- (i*) All *Cs* are two.
- (ii*) All *Cs* are suitors of Miss Smith.

In (ii*) the predication is clearly distributive. Each of *Cs* is a suitor of Miss Smith. Consequently, there is no reason to take ‘All *Cs* are suitors of Miss Smith’ and ‘Every *C* is a suitor of Miss Smith’ to express distinct propositions. We can say in the spirit of Russell’s 1905 analysis that these sentences are merely different verbalizations of the same proposition. As long as we are concerned with the predicate ‘are suitors of Miss Smith’ as combined with ‘All *Cs*’, we have no reason to treat ‘all *Cs*’ and ‘every *C*’ differently. Russell’s view that these phrases function differently thus boils down to the conditions (i) and (i*).

Similarly to Frege, Russell came to the conclusion that the number predicates like ‘are two’ are not first order predicates. The form of the proposition expressed by ‘*A* and *B* are two’ is different from the form of the proposition expressed, e.g., by ‘*A* and *B* are human’. About the nature of cardinal numbers Russell says, in §111 of *Principles*, that ‘mathematically, a number is nothing but a class of similar classes,’ where similarity is defined in terms of one-one correlation between classes. This seems to put Russell’s analysis of statements like ‘Brown and Jones are two’ and ‘All *Cs* are two’ under pressure.

In both the resolution to the paradox of plurals and his account of ‘all’-phrases Russell treats the number predicate as a first-order one. This is at variance with *Principles*’ theory of cardinal numbers. If a cardinal number is ‘nothing but a class of similar classes,’ what is it to ascribe a number predicate? In particular, what the semantic value of ‘All *Cs* are two’ should be?

If *Principles*’ theory of cardinal numbers is to be preserved, clearly we must treat the predicate ‘are two’ as expressing class membership. Let us call the class of two-membered classes (which is identified, by Russell, with number 2) as TWO. Then, ‘are two’ must be read as ‘belongs to TWO’. Obviously we must interpret the semantic contribution of ‘all *Cs*’ in accordance with this idea. The statement ‘All *Cs* are two’ expresses the proposition which is adequately formulated by ‘The class of all *Cs* belongs to TWO’. At this point, Russell’s notion of collective vs. distributive ascription of a property becomes obsolete. It does not hold that ‘are two’ is ascribed collectively to the members of the class of all *Cs*. The predicate ‘are two’ is to be replaced by ‘belongs to TWO’ and this predicate is ascribed to a class which we can treat without any issue as a single entity (set as one).

With Russell’s theory of cardinal numbers, the notion of set as many seems to lost its original motivation. Any plural can be treated as a set which is as single as any other term (Socrates etc.). Consequently, the paradox of plurals and the doctrines that Russell had to assume to resolve it are irrelevant. More importantly, the idiosyncratic semantic treatment of ‘all’-phrases which marks a peculiar difference between TDC and its successor, Theory of Descriptions disappears.

3.4 Variable conjunction

The nature of the objects denoted by ‘any’-phrases can be explained, according to Russell, by an analysis of this sentence:

(iii) If it was Brown or Jones you met, it was a very ardent lover.

The proposition expressed by (iii) ‘gives the kind of conjunction by which *any* is defined.’¹²⁰ The object the proposition (iii) explicates is called by Russell ‘variable conjunction’.¹²¹ Russell supposes that objects of this kind are denoted by all ‘any’-phrases.

Take any arbitrary concept *C*, where *C* is satisfied by Brown, *C* is satisfied by Jones, Brown \neq Jones and nothing else satisfies *C*. Then (iii) can be paired up with this:

(iv) If it was anyone of *Cs* you met, it was a very ardent lover.

The connective ‘or’ in ‘Brown or Jones’ picks out the relation by which Brown and Jones are combined in the object denoted by ‘anyone of *Cs*’. Notice that the concept *C* is part of the *intension* of ‘anyone of *Cs*’. It is part of /anyone of *Cs*/. The extension is just Brown and Jones connected by the relation of variable conjunction.¹²²

It might be confusing to characterize the relation the connective ‘or’ picks out as conjunction of . Why not disjunction? To this end, Russell says that the notion of the variable conjunction ‘seems half-way between a conjunction and disjunction.’¹²³ To explain what Russell means, we can resort to an example which is simpler than (iv). Consider this sentence:

120 *Principles*, §59.

121 Russell uses the term ‘variable conjunction’ ambiguously. In some contexts, ‘variable conjunction’ is just a shorthand for ‘the relation of variable conjunction’. In other contexts, Russell calls the whole object denoted by ‘any’-phrase by the term in question. This remark applies also to other associated terms like ‘numerical conjunction’.

122 Now it is apparent why Brown \neq Jones must have been supposed. If Brown = Jones, then we would have an individual instead of having a variable conjunction.

123 *Ibid.*

(v) Anyone of *Cs* is a very ardent lover.

Given our suppositions about the concept *C* and the supposition that Jones and Brown are not the same person, it follows that *they both* are very ardent lovers. This is the conjunctive aspect. The disjunctive aspect of the variable conjunction comes with the observation that ‘anyone of *Cs*’ means the same as ‘whichever one you choose from those that are *C*’.¹²⁴ Returning to (iv), we can rephrase this sentence in this way: ‘Choose anyone of those that are *C*. If you met him, you met a very ardent lover.’ Provided we know that Jones and Brown are the only members of the extension of *C*, we can add: Be it Jones or Brown, if you met him, you met a very ardent lover. Here, the disjunctive aspect is involved.

3.5 Russell’s analysis of ‘a’ and the ambiguous man problem

The phrases beginning with ‘a’ denote, according to Russell, variable disjunctions. Russell proposes the following sentence to illustrate this sort of objects:

If it was one of Miss Smith’s suitors, it must have been Brown or Jones.

The proposition expressed by this sentence involves the object denoted by ‘a suitor of Miss Smith’. If you met a suitor of Miss Smith, ‘it is not true that it must have been Brown, nor yet that it must have been Jones.’¹²⁵ Denoting by means of ‘a suitor of Miss Smith’ is *ambiguous* between picking out Brown and picking out Jones. It is not, however, the relation of denoting where the ambiguity lies. Nor is it /a suitor of Miss Smith/. Russell’s notion of the variable disjunction rests in reifying the ambiguity in question *as the denotation*. This is confirmed in §57 of *Principles*, fn. †. Russell proposes two ways of parsing ‘Socrates is a man’. In one of these parsings, ‘a man’ is treated as a denoting

124 For this see point (3) in §61 of *Principles*.

125 *Ibid.*, §59.

phrase and ‘Socrates is a man’ is taken to express an assertion of identity between Socrates and ‘an ambiguous individual’.¹²⁶ I will show that this view is rather implausible.

Let us begin with that part of Russell’s theory which is plausible. ‘Socrates is a man’ is paired with a disjunction of the formulas of the form ‘Socrates = x ’. This disjunction is such that

- x is human,
- each instance of ‘Socrates = x ’ is such that x is distinct from each substitution for x in any other instance of ‘Socrates = x ’,
- the number of the disjuncts is the same as the number of all men (that is, the disjunctive formula exhausts the set of all men).

If one of the substituends of x is Socrates, that is, ‘Socrates = Socrates’ occurs among the disjuncts, then ‘Socrates is a man’ is true. This shows that Russell’s theory of ‘a’-phrases provides us with a way of expressing the truth-conditions of the sentences containing such phrases. However, there is more to Russell’s theory of ‘a’-phrases.

The long disjunction which we arrive at in an inquiry into the truth-conditions of ‘Socrates is a man’ was taken by Russell to map the structure of the object denoted by ‘a man’. This object is a variable disjunction, a combination of all men in a certain disjunctive way. Let us call this combination ‘O’. If ‘a man’ denotes O, it is necessary to take ‘Socrates is a man’ to express identity between O and Socrates. This presents a serious issue for Russell.

Provided that ‘Socrates is a man’ is a true sentence, Russell’s theory leads us to identify Socrates with O. This follows from the fact that by asserting ‘Socrates is a man’ we assert the relation of identity as holding between Socrates and the denotation of ‘a

126 This is the ambiguous man infamously mentioned in the first paragraph of ‘On Denoting’—importantly enough, despite the fact that there was no room for him after Russell’s theory descriptions took over TDC. Cf. Russell’s mature statement of how ‘a’ is treated in his theory of descriptions: ‘The identity in “Socrates is a man” is identity between an object named (accepting “Socrates” as a name, subject to qualifications explained later) and an object ambiguously described. An object ambiguously described will “exist” when at least one such proposition is true, i.e. when there is at least one true proposition of the form x is a so-and-so,” where “ x ” is a name.’ (*IMP*, 172) There is nothing ambiguous about the entity picked out by ‘a man’. This entity is Socrates who is named by ‘Socrates’ and ambiguously described by ‘a man’.

man'. Since O is the denotation, we assert identity between Socrates and O. Quite clearly, Socrates *cannot be* identical with a complex object in which every man is involved as a proper part. Had Socrates been such an object, it would contain every man. He would, in consequence, contain himself as a proper part. This is absurd as long as Socrates is the whole of Socrates. To conclude the argument, Russell's semantic theory of 'a'-phrases renders 'Socrates is a man' a false sentence. Since the sentence is plainly true, Russell's theory is plainly false.

It would seem natural to admit that the denotation of 'a man' in our example is Socrates himself. The ambiguity involved in one's use of 'a man' must consist in *the way how the concept /a man/ denotes*. (See the fn. on the previous page) It denotes Socrates by means of a characterization which can belong to many individuals.

It was, I suspect, issues of this sort which let Russell to pose at the end of Chapter V of *Principles* the following questions:

Is there one way of denoting six different kinds of objects, or are the ways of denoting different? And in the latter case, is the object denoted the same in all six cases, or does the object differ as well as the way of denoting it?¹²⁷

Russell did not find satisfactory answers. But, as we have demonstrated, in the case an 'a'-phrase, it is very implausible to posit the disjunctive aspect involved in the functioning of such phrases in the denotation. If a disjunctive way of combination is just the manner of how /a man/ denotes Socrates, there is no such combination but only a specific relation of denoting. The combination as an entity must be merely an abstraction derived from an observation of how /a man/ denotes.

The 'ambiguous individual' problem can be formulated for 'every'-, 'any'-, 'some'-phrases¹²⁸ as well (sticking to the common usage of 'all'-phrases, these phrases are also part of the problem—see the discussion of 'every' above in this chapter). Let us consider,

¹²⁷ *Principles*, §59.

¹²⁸ For the sake of brevity, I am leaving out Russell's analysis of 'some'. Our goal in this chapter is not to expose Russell's theory in its entirety. This would be, in any case, an impossible task, as Russell's own account is full of gaps and unresolved tensions. Rather, our goal is to explicate the theory enough to acknowledge its problematic parts.

for illustration, an ‘any’-phrase. ‘Any man’ in ‘any men is mortal’ does not pick out any particular man. The phrase means ‘whichever man you choose’, leaving open which one you choose at which occasion. So, is there an object which could be described, in analogy to the ambiguous man, as a ‘whichever man’? I assume Russell tends to say ‘yes’. This whichever man is all men combined by variable conjunction. But - and here is the linchpin of my criticism - this combination far from being mortal. It is an abstract individual which neither lives nor dies, nor it begins to exist in space time and ceases to so exist at some point. The particular men, Socrates among them, live and die, hence they are mortal.¹²⁹

3.6 Plurals again: Rectifying Hylton’s interpretation

In §62 of *Principles*, Russell remarks that, except definite descriptions, denoting phrases denote ‘very paradoxical objects’. Interpreting this remark, Hylton (1990) acknowledges a point closely related to the paradox of plurals discussed at length in Section 3.2 above. He acknowledges Russell’s idea that plurals fail to possess termhood, i.e., are not single entities. This is a valid point, but the rest of Hylton’s account of Russell’s conundrum concerning the nature of combinations is not right, as Hylton misrepresents the category of plurals. He has it that *all five* types of combinations are plurals and proposes, in turn, that the aspect of plurality possessed by each of them is that which makes them ‘very paradoxical’. He says:

What makes these objects ‘very paradoxical’ is not simply their peculiarity. These objects are essentially plural; each one is not a term but a combination of terms. It is this fact, that these objects are not terms, that causes **the paradox**: ‘I shall use the world *object* in wider sense than *term*, to cover both singular and plural. ... The fact

129 Cf. *IMP*; On page 173, we read: ‘[...] when we have enumerated all the men in the world, there is nothing left of which we can say “This is a man, and not only so, but it is the ‘a man,’ the quintessential entity that is just an indefinite man without being anybody in particular.” It is of course quite clear that whatever there is in the world is definite: if it is a man it is one definite man and not any other.’ This proves that Russell was in his retrospective aware of the issue.

that a word can be framed with a wider meaning than *term* raises grave logical problems' (*Principles*, 58n.)'¹³⁰

As we will see, the textual evidence shows that Russell struggled to accommodate all types of combinations into his ontology grounded in the all-embracing category of terms. But this does not imply that all combinations were plurals for him. In fact, only the denotations of 'all'-phrases, i.e., sets as many, were. This is simply how the word 'plural' is deployed by Russell in *Principles*. So, contra Hylton, Russell does not take the other types of combinations to be plurals.

The inadequacy of Hylton's account is not merely terminological. The paradox of plurals we discussed above is a problem specific to the denotations of 'all'-phrases and concerns their absence of singularity as a result of Russell's struggle to explain the semantics of the cases like 'All Cs are two'. Taking 'are two' in its face value as a first-order predicate, Russell resorts to thinking that 'All Cs' denotes a set as many, as opposed to a set as one. These general consequences follow:

- (a) Provided whatever is denoted is (has being), there are entities in Russell's ontology which are not singular (combinations).
- (b) There is a peculiar way of predication in which a concept is asserted of the denotation collectively.
- (c) There is a peculiar sort of how a concept applies to its terms in a proposition when the proposition is the semantical value of a sentence in which the concept is asserted of some (distinct) terms collectively.

The paradox of plurals serves, according to Russell in *Principles*, to show that (b) needs to be embraced. (c) and (a) concern ontology and follow once (b) is embraced. Hylton spots the fact that plurals are not singular, he misses, however, that this comes as part of Russell's resolution to the paradox of plurals. This is important: In the case of the other phrases, Russell's way of arriving at the conclusion that their denotations lack singularity comes from a very different consideration.

130 Hylton (1990), 208.

By conflating ‘all’-phrases with the other four types of the denoting phrases, Hylton makes a false impression that there was one problem covering all denoting phrases (the definite descriptions excluded) and its core lies in some simple fact that plurality entails lack of singularity. This is why Hylton’s stretching the term ‘plural’ over all five types of denoting phrases goes beyond a mere terminological inaccuracy.

Let us examine whether the paradox of plurals applies, e.g., to the denotation of an ‘any’-phrase, provided the denotation has more than one member. Consider, e.g., ‘Any person could do this’. Here the deed which any person is said to be capable of doing is not ascribed to its subjects collectively! If the sentence is true, I could do this, you could do this, Iggy Pop could do this etc. Russell’s introduction of plurals is motivated by the cases like “Brown and Jones are two”, where the predicate is, in his view, applied to its subjects collectively.¹³¹ Neither Brown, nor Jones are two, but both of them in a peculiar sense are.

Plurals (sets as many) are objects that we, according to Russell, denote by ‘all’-phrases and by ‘all’-phrases only. The paradox of plurals applies to the denotations of ‘all’-phrases only. Hylton’s account is oblivious to this fact. More importantly, the following textual evidence proves him incorrect:

It is to be observed that these five combinations yield neither terms, nor concepts, but strictly and only combinations of terms. The first yields many terms, while the others yield something absolutely peculiar, which is neither one nor many.¹³²

When Russell says that ‘the first [type of combination] yields many’, he means the type belonging to ‘all’-phrases. The others types, like the type belonging to ‘any’-phrases, yield ‘something absolutely peculiar, which is neither one or many.’ If something is not many, it means that it is not a plural.

Putting aside plurals, why the rest of the combinations cannot be terms? Russell says that these combinations are not many (plurals), but ‘something absolutely peculiar’.

¹³¹ See Section 3.2 above.

¹³² *Principles*, §59.

He also admits that they are not one (singular). Like plurals they also fail to satisfy the condition of singularity. Clearly this must be based on grounds that differ from the case of plurals. I maintain that at this point a generalization of the ambiguous individual problem takes its part in the overall failure of Russell's Theory of Denoting Concepts. Let us return to the example 'Socrates is a man'.

The sentence expresses, in Russell's view, the proposition <Socrates = /a man/>. Based on what Russell says in his analysis of 'a'-phrases, /a man/ is supposed to denote a combination of all men. Now, if this combination is a solid entity, it is a complex of all men interrelated by a certain relation, namely by the relation which Russell calls 'variable disjunction'. But, again, it is quite clear that Socrates cannot be identical to some such entity. He is identical with himself. In response to this issue, Russell adopts a nominalistic course with regard of combinations, trying to refrain from positing them in his ontology. This is evidenced by his claim that 'the combinations are combinations of terms, effected without the use of relations.'¹³³ Variable disjunction which is supposed to tie all men as to yield the denotation of 'a man' is not really a relation. Consequently, the combination of all men effected by variable disjunction is not really an entity. In turn, it is not really the denotation of /a man/. This is how Russell tries to make Socrates into the denotation of /a man/. That this was Russell's preferred view is confirmed at the outset of Chapter 5 of *Principles*. Russell says the following:

If I say 'I met a man,' the proposition is not about *a man*: this is a concept which does not walk the streets but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public-house and a drunken wife.¹³⁴

Surely, the actual man Russell is talking about here is not the combination of all men effected by variable disjunction. The combination lives 'in the shadowy limbo of the logic-books' no less than /a man/ does.

133 Ibid.

134 Ibid., §56.

This course of reasoning, of course, leaves the combinations belonging to ‘every’, ‘any’, ‘a’ and ‘some’ without an actual place in both Russell’s ontology and in his semantic analysis of denoting phrases. On the one hand, they cannot serve the purpose for which they were introduced if they fail to be denotations and if they fail to be terms. On the other hand, for obvious reasons explained above they cannot be the denotations of the denoting phrases in question, which leads Russell to weaken their ontological status by taking them as mere abstractions as opposed to entities in the standard sense of his ontology. This nominalistic turn in Russell’s understanding what combinations are rises an important question. If they are mere abstractions, of which aspect of denoting they are abstractions?

This question is unanswered in *Principles*. As Russell never returned to his notion of combinations in the subsequent years, TDC must be considered as an undeveloped theory suffering from serious tensions and gaps. To sum up the whole failure of the theory, combinations are entities which due to their plurality or ambiguity fail to be terms, which contradicts Russell’s doctrine that *everything which has being is a term* (the doctrine called in the secondary literature ‘termism’). As for plurals, Russell was able to explain, however insufficiently, *predication / the unity of proposition* in the cases where something is *ascribed to / holds of a plural*. As for the rest of the combinations, Russell failed to make them work in his semantic outlook of language in any reasonable way and resorted to puzzling nominalistic remarks.

In the preceding chapter, we saw that Russell’s notion of generality which was in the heart of his theory of logic and mathematics depended in a substantial way on TDC. It is not, therefore, incorrect to say that the early Russell’s logicism was an unsuccessful project as long as TDC was left in a state of an undeveloped and highly problematic semantical theory. After Russell completed *Principles*, it took him more than three years to adopt the Fregean, quantificational approach to ‘all’, ‘every’, ‘any’, ‘a’ and ‘some’. In his Theory of Descriptions, not only that he adopted this approach with regard to the phrases beginning by these determiners, he also extended the Fregean approach to definite descriptions.

4 Contradictory entities

4.1 The question of contradictory entities

Quine's 1966 paper, titled 'Russell's Ontological Development', popularized an interpretation according to which the ontology of *Principles* is very similar to Meinong's *Gegenstandstheorie*. Under the influence of Quine, the early Russell has often been attributed an ontological commitment to such entities as the present king of France. As Quine's view of the matter continues, only by 1905 when Russell invented the Theory of Descriptions was he able to liberate himself from the undesired exuberance of the Meinongian realm of being. 'In *Principles of Mathematics*, 1903,' says Quine, Russell's ontology was unrestrained. Every word referred to something. [...] beyond existence, there were the rest of the entities: "numbers, the Homeric gods, relations, chimeras, and four-dimensional spaces" [quoted from *Principles*, §427].¹³⁵ This is continued as follows:

[...] gods and chimeras are as real for Russell as numbers. Now this is an intolerably indiscriminate ontology. For, take impossible numbers: prime numbers divisible by 6. It must in some sense be false that there are such; and this must be false in some sense in which it is true that there are prime numbers. In this sense are there chimeras? Are chimeras then as firm as the good prime numbers and firmer than the primes divisible by 6?

Russell may have meant to admit certain chimeras (the possible ones) to the realm of being, and still exclude the primes divisible by 6 as impossibles. Or he may, like Meinong, have intended a place even for impossible objects. I do not see that in *Principles of Mathematics* Russell faced that question.¹³⁶

In this passage, Quine raises two questions. First, should possibilia like a chimera which does not exist, but is included in the furniture of reality (has some form of being), be

¹³⁵ Quine (1966), 658.

¹³⁶ Ibid.

treated as being ontologically on a par with entities such as (good) prime numbers? Second, did the Russell of *Principles* believe that there are, in some sense, entities which are contradictory? In this chapter, I deal with the second question.

According to Makin, Quine believed that in *Principles* Russell posited contradictory entities.¹³⁷ This is unfair, since in this matter Quine professes agnosticism. Quine says he ‘could not see that in *Principles of Mathematics* Russell faced that question.’ So, when Makin says, ‘I find nothing in Russell’s writings to support it [= the view that Russell did posit contradictory entities, MS],’¹³⁸ he makes no point against Quine at all. Makin’s statement is compatible with Quine’s agnosticism. I will argue that there are several reasons, some of them rather obvious, showing that within the framework of *Principles* an admission of contradictory entities should not be possible.

4.2 The problem of explosion

The first thing to consider is the question what would an admission of contradictory objects do to Russell’s logicist project. We will see that it would make it impossible. The logic of *Principles* is classical in the sense of including what is called the principle of explosion. Russell states that ‘false propositions imply all propositions.’¹³⁹ For any formula, therefore, it holds that, if that formula is contradictory, it implies every proposition, that is,

$$(1) (q) (q \sqsupset q \cdot \sqsupset \cdot (p) (p \& \sim p \cdot \sqsupset \cdot q)),$$

where, according to the following definition introduced in §16 of the *Principles*, the antecedent ‘ $q \sqsupset q$ ’ means ‘ q is a proposition’:

137 Makin (2000), p. 55n.

138 Ibid.

139 *Principles*, §16.

(D1) x is a proposition =Df. $x \sqcap x$, where x is any term.

If (1) holds, every entity which is such that $Fx \ \& \ \sim Fx$ will have every property. That is, schematically it will be $G(x, x_1, x_2 \dots x_n)$ for every G , where ‘ G ’ is a predicate with the arity n and $x_1, x_2 \dots x_n$ are terms to which x has the relation G if the arity of G is higher than 1. For instance, where an arbitrary entity a is round and not round, we have: a is round & a is not round. This is a contradiction, so by an instance of (1), any formula of the form ‘ $G(a, x_1, x_2 \dots x_n)$ ’ follows. E.g., it follows that a is an even prime. a is, therefore, a member of the class of all even primes. Due to the generality of (1), every contradictory object ends up being a member of this class. So, the class of all even primes which we naturally suppose to have just one member, the integer 2, includes as its members all contradictory objects. Since there is, presumably, an infinite number of contradictory entities, the class of all even primes has an infinite number of members.

Generally, contradictory objects invade every class, making thus the cardinality of every class infinite. This, in turn, makes Russell’s logicist reconstruction of the pure mathematics impossible. Russell defines cardinal numbers as classes of similar classes, where two classes are similar if and only if there is one-one correspondence between them (see the discussion in §109-111 of *Principles*). Thus, e.g., the integer 1 is the class of all singletons. In this view, to say, for example, that the number of all natural satellites of the Earth is 1 means to say that the class of all natural satellites of the Earth is a member of the class of all singletons. Thus, in general, cardinal number is a relational property of classes; it is the membership of a class to the class of all classes similar to the given class.¹⁴⁰ The explosion resulting from an admission of contradictory objects leads to a consequence that there are no singletons (and, in general, no classes having a finite number of members). From this, it seems that the class of all singletons (the integer 1) is empty, but this, in fact, cannot be the case, since empty classes are impossible either. *Every* class contains all contradictory objects. This yields a contradiction: the class of all singletons is empty and it is not empty.

140 We have already acknowledged Russell’s theory of cardinal numbers when we discussed his analysis of ‘all’-phrases in section 3.3.

If one imposes type restrictions on the class-membership relation, it is possible to avoid a consequence that contradictory ur-elements like the round square can be significantly said to be members of the class of all singletons, however, contradictory classes like a class which is empty and not empty will still be counted as members of the class of all singletons. The contradiction indicated above, therefore, cannot be avoided by the simple type theory as applied to the theory of classes.

The combination of the principle of explosion and an admission of contradictory objects yields a logical framework which cannot underpin a consistent theory of cardinal numbers and, consequently, it cannot underpin a consistent rendering of all what Russell's conceives as the pure mathematics. Presumably, Russell would have to adopt some sort of para-consistent logic, rejecting thus the principle of explosion. But Russell's logic is clearly classical. So, it is likewise clear that the Meinogianism about contradictory entities was *not* an option for the Russell of *Principles*.

4.3 The Law of Contradiction

In *Principles*, section *A* of Chapter 2, Russell introduces his calculus of propositions. In §18, he puts forward the axioms of the calculus and the rule of inference. He adds that the law of contradiction is derivable from the axioms. In the *Principles*, the law of contradiction receives the following formulation:

$$(q) (q \supset q : \supset : (p) (p \supset p : \supset : (q \& (r) (r \supset r . \supset . q \supset r) . \supset . p))$$

This formulation of the law of contradiction draws on the early Russell's definition negation. The definition reads as follows:

$$(D2) \quad \sim x = \text{Df. } (r) (r \supset r . \supset . x \supset r), \text{ where } x \text{ is any term.}$$

By (D2), the above formulation of the law can be simplified in this way:

(LoC) $(q \supset q) \supset \sim(q \ \& \ \sim q)$

In Russell's jargon: for any term q , if q is a proposition, then ' q is true and q is false' is false.

Suppose, for the sake of argument, that the round square is an inhabitant of the early Russell's realm of being. The contradictoriness of the round square is derivable by means of the conditional 'If x is a square, x is not round' (or, alternatively, 'if x is round, x is not a square'). Call the round square ' a '. Then a is round because the round square is round. But a is a square also, and since 'if something is a square, it is not round' holds, it follows that a is not round. So, with the round square endowed with being, we can derive both ' a is round' and ' a is not round'. Since ' a is not round' and ' $\sim(a \text{ is round})$ ' are materially equivalent, it follows that $\sim(a \text{ is round})$. Hence, we arrive at this contradiction:

$a \text{ is round} \ \& \ \sim(a \text{ is round})$.

This is at odds with (LoC). Since ' a is round' is a proposition, ' a is round' implies itself. ' $(a \text{ is round} \supset a \text{ is round}) \supset \sim(a \text{ is round} \ \& \ \sim(a \text{ is round}))$ ' results from (LoC) by the rule of Universal Instantiation. By Modus Ponens, we infer that $\sim(a \text{ is round} \ \& \ \sim(a \text{ is round}))$. So, $a \text{ is round} \ \& \ \sim(a \text{ is round}) \ \& \ \sim(a \text{ is round} \ \& \ \sim(a \text{ is round}))$. Had Russell posited contradictory objects, his logic (under Russell's intended interpretation) would be plagued with as many contradictions of this sort as is the number of the posited contradictory objects.

As long as Russell insisted that (LoC) is part of his logical system, we cannot, on his behalf, drop this law. With the contradictory objects, the situation is rather opposite. Russell does not say anywhere in the *Principles* that contradictory objects have being. Since he does not say anywhere in the book the opposite either, the question whether contradictory entities have being or not seems to be open for him. The present argument shows that, had Russell faced this question, he would have to renounce contradictory objects because of his reliance on the law of contradiction.

4.4 The early Russell's notion of the null class

The last argument I wish to propose, in order to show that Russell could not subscribe himself to a view that contradictory entities have being, concerns Russell's notion of the null class. In *Principles*, Russell holds to a distinction between a class and the concepts whose extension the class is (he calls such concepts 'class-concepts'). In this way, he adopts a strictly extensional view of classes. The identity criteria of a class are given solely with its members. The associated class-concept (the intension) is, then, irrelevant to the identity criteria of the class. This turned out troublesome when the class is null (empty) or has just one member. Most likely, it was this peculiar problem which led Russell to confess in the the preface of the *Principles* that he failed 'to perceive any concept fulfilling the conditions requisite for the notion of a *class*.'¹⁴¹

Concerning classes having just one member, singleton classes, Russell acknowledges in §484 of *Principles* that Frege distinguishes a singleton class from its only member. But in which respect does this difference lie if classes are conceived in a strictly extensional manner? The difference between a and $\{a\}$ is marked by the fact that a belongs to $\{a\}$, whereas $\{a\}$ does not belong to itself. This, however, is a matter of intension. We describe the class $\{a\}$ as one whose *only* member is a and derive, accordingly, that it is not a member of itself. From Russell's extensional point of view, 'a class having only one term is to be identified ... with that one term.'¹⁴²

A similar issue concerns the notion of the null class. Russell explains it in the following passage:

[W]ith the strictly extensional view of classes [...], a class which has no terms fails to be anything at all: what is merely and solely a collection of terms cannot subsist when all the terms are removed. Thus we either find a different interpretation of classes, or else we find a method of dispensing with the null-class.¹⁴³

141 *Principles*, vi.

142 *Ibid.*, §69.

143 *Ibid.*, §73.

Russell suggests the later course of adjustment. He suggests that the null class ought to be identified by stipulation with ‘the class of null class-concepts or of all null propositional functions.’ (Ibid.) In this way, the null class is not empty. However, in the context of the *Principles*, it is difficult to take this suggestion seriously, for in the more substantial parts of the book Russell takes the null class to subsist (have being). Russell’s discussion of the notion of the cardinal number in §109 is an example.

In §109, Russell seeks after an adequate definition of a statement that two classes have the same number. He first offers the following definition:

(i) [T]hey [two classes, MS] have the same number when their terms can be correlated one to one, so that any one term of either corresponds to one and only one term of the other.’ (§109)

Almost immediately the definition is objected as follows:

But in order to provide for the case of two classes which have no terms, it is necessary to modify slightly the above account of what is meant by saying that two classes have the same number. For if there are no terms, the terms cannot be correlated one to one. (Ibid.)

Let us first consider whether what Russell means by his definition is the Hume principle. This principle amounts to the following definition:

(HP) The classes A and B have the same number =Df. $(\exists R)(R \text{ is a 1-1 relation } \& .(x)(x \in A \rightarrow (\exists y)(y \in B \& xRy)) \& (y)(y \in B \rightarrow (\exists x)(x \in A \& xRy)))$.

Strangely enough, if this is what Russell means by (i), the worry about the cases of two classes which have no terms would not arise. If A and B are null, then *they do have* the same number in the sense of (HP). Suppose that ‘ A ’ and ‘ B ’ in (HP) are class terms both denoting the class which has no members. The antecedent in the second conjunct of (HP)

' $x \in A$ ' is false for every value of x , rendering thus the whole conditional ' $(x)(x \in A \supset ((\exists y)(y \in B \ \& \ xRy)))$ ' true. The same applies to the third conjunct of (HP). In other words, there is some R which is a one-one relation such that it correlates in the sense of (HP) the members of the null-class to the members of the null class.

This observation is analogous to the following part of Russell's objection against Hugh McColl in 'Existential Import of Propositions' (1905). We read:

Thus, if ' $XA = X$ ' means 'every X is an A ', [fn. 2] then ' $OA = O$ ' means 'every member of the class which has no members is an A ', or 'for every value of x , " x is a member of the class which has no members" implies " x is an A ". This hypothetical is true for all values of x , because its hypothesis is false for all values of x , and a hypothetical with a false hypothesis is true.¹⁴⁴

If A and B in (HP) are null, the two hypotheticals occurring in the given instance of (HP) are true in the manner described in this quote.

There are two possibilities concerning statement (1) from §109 of *Principles*. Either by this statement Russell proposes (HP) and his subsequent objection is misguided, or by that statement he does not propose (HP), but rather (HP) plus a requirement that the classes A and B must contain at least one member (or, rather, a proposition which is equivalent to the conjunction of (HP) and this requirement). I will argue that in all likelihood the latter is the case.

In order to read (1) correctly, we need to acknowledge that (1) is worded tentatively as a statement *about* the terms of the classes A and B . The correct formal rendering of (1) would include an existential assumption that A and B contain some terms, and this will be followed by a claim that these terms are correlated one to one. Where the existential assumption comes from? It is very likely that Russell took (1) to express a proposition involving a denoting concept /the terms of A and B /. The whole proposition is expressed as follows:

(1') </the terms of A and B / are one-one correlated by /some relation/>.

¹⁴⁴ Russell (1905/1973), 101.

If /the terms of A and B / fails to pick up anything, the whole proposition is false. So, there is no confusion about (1) and the objection against it is valid.

In his objection to (1), Russell is concerned with a case in which two different class concepts are empty in their extension. For instance, the extensions of the concept of a Martian and that of a natural satellite of Mercury are empty. This should ground the truth of a statement that the class of all Martians and that of all natural satellites of Mercury have the same number which is zero. Such a case, however, cannot be accounted for by (1'), since there are no terms which could be one-one correlated by some relation.¹⁴⁵

Russell's amended definition of a statement that two classes have the same number reads as follows:

Two classes have the same number, when, and only when, there is a one-one relation whose domain includes the one class, and which is such that the class of correlates of the terms of the one class is identical with the other class.¹⁴⁶

'From this it appears,' Russell continues, 'that two classes having no terms have always the same number of terms; for if we take any one-one relation whatever, its domain includes the null-class, and the class of correlates of the null-class is again the null-class.' (Ibid.) Formally, Russell definition is as follows:

(ii) The classes A and B have the same number =Df. $(\exists R) (R \text{ is a 1-1 relation} \ \& \ A \exists \text{ the domain of } R \ \& \ \{x: (\exists y) (yRx \ \& \ y \in A)\} = B.$

Why the second definition works when A and B are null? It works because there is the 1-1 relation R such that: the null class is a subclass of the domain of R ; $\{x: (\exists y) (yRx \ \& \ y \in A)\} = \text{the null class}$ and $B = \text{the null class}$, therefore $\{x: (\exists y) (yRx \ \& \ y \in A)\} = B$. (2) is, in fact,

145 Note that this presupposes that the extension of the two concepts is the null class in the sense of the class containing no terms at all.

146 Ibid., §109.

a formal equivalent of (HP). In §109 of *Principles*, Russell is laboring towards the Hume principle.

The domain of every one-one relation (of which there are presumably many) includes, i.e., has as its sub-class, the null-class. The class of the correlates of the null-class is a sub-class of the range of any one-one relation. This is, again, the null class. Extensionally, two empty classes are identical, that is, we have two empty class-concepts sharing the null-class as their extension, and they satisfy the amended definition. The question now arises whether the whole consideration in §109 would make sense in the backdrop of Russell's rejection in §73 of the null class conceived as a class having no terms.

Firstly, the problem which leads to Russell's redefinition of the fact that two classes have the same number *makes sense only* if what he means by 'the null class' is the class which has no members! With the class of all null class-concepts (or all null propositional functions), or in general with any class which has some members, no problem established on the fact that we lack terms for one-one correlation can arise.

Second, if we stipulate that the null class is the class of all null class-concepts, any two classes which are null would satisfy either of Russell's two proposed definitions of the fact that two classes have the same number. But, ask yourself, what that number would be? Would that be zero? Hardly so. There is, presumably, an infinite number of null class-concepts. So should we, then, accept that the number of all Martians and that of natural satellites of Mercury is the same and that it is an infinite number? That would be absurd.

Another substantial use of the concept of the null class as the class with no terms takes place in Chapter 8 of *Principles*. The relevant passage reads as follows:

If there is one term x which can be taken away from u to leave a similar class u' , it is easily proved that if any other term y is taken away instead of x we also get a class similar to u . When it is possible to take away one term from u and leave a class u' similar to u , we say that u is an *infinite* class. When this is not possible, we say that u is a *finite* class. From these definitions it follows that the null-class is finite, since no term can be taken from it.¹⁴⁷

147 Ibid., §117.

If the null class is the class of all null class-concepts (or all null propositional functions), then (a) the null class is an infinite class as long as there is an infinite number of null class-concepts and (b) *it is possible* to take a term from it. Since, in the cited passage, Russell denies (a) and (b), it is not the revised notion of the null class from §73 which is employed in §117.

Russell's proposal in §73 that the null class will not be treated as empty was most likely a later addition to the bulk of *Principles*. As this addition was not reconciled with Russell's substantial doctrines belonging to his theory of logic and mathematics, its status was very tentative. The centrality of logic and mathematics to *Principles* makes necessary that we ascribe Russell that notion of the null class which is actually deployed in his logic and mathematics. How to solve the problem of what the null class is under a strictly extensional view of class remained open for Russell. It remained unsolved in *Principles*. The solution of this whole conundrum, of course, came with Russell's later development of the no-classes theory according to which there are no classes at all.

4.41 The null class and contradictory entities

Russell's exposition of his calculus of classes in section B of Chapter 2 of *Principles* is another substantial part of the book which reckons with the null class as the class containing no terms. Henceforth, when I speak of the null class I mean the null class in this sense. My objective is to show that the null class could not be reckoned with if Russell maintained that contradictory entities have being.

§23 of the *Principles* introduces the Cantorian comprehension principle for classes. We read that 'a class may be defined as all the terms satisfying some propositional function.' Russell adds that some limitation of the principle might be needed in order to avoid (the class-theoretic version of) the paradox named after him, but this is a question beyond our current interest, so I assume no limitation of the comprehension. Formally, the Cantorian comprehension schema reads as follows:

$$(CP) \quad (EM) (x \in M \equiv_x Ax),$$

where A is free for x . The class M is comprehended, in this scheme, by the propositional function $\langle Ax \rangle$. (CP), the axiom of extensionality and the principle ‘if x belongs to the class of terms satisfying a proposition function Φx , then Φx is true’ amount to the basis of Russell’s calculus of classes.

Russell’s further suggests a definition an existent class. ‘A class is said to exist when it has at least one term. A formal definition is as follows: a is an existent class when and only when any proposition is true provided “ x is an a ” always implies it whatever value we may give to x .’¹⁴⁸ Here, we need to remind ourselves of the definition (D2) which was introduced above:

$$\sim x = \text{Df. } (r) (r \in r \cdot \supset x \in r), \text{ where } x \text{ is any term.}$$

To be a proposition is to imply itself and the negation of a proposition is defined as the property of implying every proposition. The latter comes with the fact that, if a proposition is false, it implies every proposition.¹⁴⁹ The reader of *Principles* should be aware that ‘is true’ and ‘is false’ are usually used by Russell to indicate assertion and negation respectively. ‘is true’ and ‘is false’ in the semantic sense do not appear in Russell’s formal system. Thus ‘ p is false’ is to be rendered in accordance with (D2) as ‘ $(r \in r) \supset r (p \in r)$ ’. Now it is easy to capture formally the enigmatic statement ‘any proposition is true provided “ x is an a ” always implies it whatever value we may give to x ’. We get this definition:

$$(D3) \quad a \text{ is an existent class} = \text{Df. } (p) (p \in p \cdot \supset (x) ((x \text{ is an } a) \supset p) \cdot \supset p)$$

By (D1) and (D2), the definiens is equivalent to this:

148 Ibid., §25.

149 See *ibid.*, §16.

$$\sim(x) \sim(x \text{ is an } a)$$

With the existential quantifier defined in the usual way, the definiens is also equivalent to this:

$$(\Box x) (x \text{ is an } a)$$

A class which is null is defined, accordingly, as the class a such that $(x) \sim(x \text{ is an } a)$. Russell says: ‘A propositional function is said to be null when it is false for all values of x ; and the class of all x ’s satisfying the function is called the null-class, being in fact a class of no terms. Either the function or the class, following Peano, I shall denote Λ .’¹⁵⁰

Notice that ‘ Λ ’ is not a name, but a shorthand for a denoting phrase. At the end of §25, Russell suggests several ways of describing the null class, but for our purposes it suffices to stick with one: ‘the class of x ’s satisfying any propositional function Φx which is false for all values of x ’. To bring Russell’s definition closer to the modern standards, I avoid the quantification over propositional functions, using instead a schematic symbol for an open formula. Thus we get this definition:

$$(D4) \Lambda = \text{Df. } \{x: (\sim(\Box y)(Ay) \ \& \ Ax)\},$$

where A is any truth-functional context free for x . If ‘ $\sim(\Box y)(Ay)$ ’ is true, nothing satisfies ‘ Ax ’, so ‘ Ax ’ is false whichever value we choose for x . If ‘ Ax ’ is true for some value of x , then ‘ $\sim(\Box y)(Ay)$ ’ will be false. Therefore, nothing satisfies the instances of ‘ $\sim(\Box y)(Ay) \ \& \ Ax$ ’.

It is important to realize that (D4) is a mere blueprint for generating the class terms which denote the null class. That is, due to our use of the schematic letter ‘ A ’, it is an improper definition. Take ‘ Ax ’ to be ‘ x is a man’. Then we get the following name: $\{x: \sim(\Box y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$. Regardless of whether x is a man or not, it is false that there is no one who is a man. Thence the class denoted is empty. For a change, take ‘ Ax ’ to

150 Ibid., §25.

be ' $x \neq x$ '. We get this: $\{x: \sim(\forall y)(y \neq y) \ \& \ x \neq x\}$. $\sim(\forall y)(y \neq y)$ is true. For that reason, no x can satisfy the other conjunct, so, again, the class denoted is empty. In general, whichever open formula we assign to ' Ax ' in (D4), we get the null class. Of course, ' Λ ' cannot have more than *one* definiens. We must, therefore, choose one of the possible names of the null class as the definiens of ' Λ '.

Let us proceed to a comprehension of the null class. For this purpose, any class term denoting the null class will do. We choose ' $\{x: \sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$ '. Accordingly, ' Λ ' means by stipulation ' $\{x: \sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$ '. The related comprehension of the null class is this:

$$(2) (\forall M) (x) (x \in M \equiv \sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man})$$

If the null class is comprehended, it follows that ' $\{x: \sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$ ' is a non-empty denoting phrase. The null class is the denotation. In Russell's terms, the propositional function $\langle \sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man} \rangle$ is false for all values of x . In the framework of *Principles*, every propositional function is correlative to a concept, the difference between them being in the fact that the propositional function retains the relating relation in its assertional form and contains one or more real variables as subject(s). Thus, $\langle \text{Socrates is wise} \rangle$ receives this variety of analyses:

- a subject/subjects and a concept, e.g., {Socrates, wisdom}.
- a subject/subjects and an assertion, e.g. {Socrates, ... is wise}.
- a subject/subjects and a propositional function, where the subject(s) is/are conceived as a value/values of the real variable(s) contained in the propositional function, e.g. {Socrates, $\langle x \text{ is wise} \rangle$ }.

If a propositional function is false for all values of x , the correlative concept is empty. In particular, the concept correlative to $\langle \sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man} \rangle$ is empty. From this concept, we can derive a denoting phrase which is empty. For example, 'an x such that $\sim(\forall y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ' is such a phrase. Then, the following equivalence holds:

(2) ‘an x such that $\sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ’ does not denote anything if and only if ‘ $\{x: \sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$ ’ denotes the class with no members.

(We also have this general schema: ‘an x such that Ax ’ denotes if and only if ‘ $\{x: Ax\}$ ’ has a member, where A is free for x .)

By the comprehension of the null class above, (1), the right side of (2) is true, so the left side of (2) is likewise true. Therefore,

(3) ‘an x such that $\sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ’ is empty.

Suppose for a reductio that Russell was a Meinongian in the sense of assuming that contradictory objects are included among what there is. I will argue that this is at odds with the possibility of comprehending the null class.

The open sentence ‘ $\sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ’ is contradictory, though not in the simple sense of having the form ‘ $x \text{ is } F \ \& \ x \text{ is not } F$ ’. It is a contradiction in the semantic sense of being false in every interpretation. If Russell was willing to admit that there are contradictory objects, he would have to treat the denoting phrase ‘an x such that $\sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ’ to have a denotation. If this phrase has a denotation, it is either a term such that $\sim(\exists y)(y \text{ is a man})$ and it is a man, or a combination made out of such terms (a combination of the sort called ‘variable disjunction’). In either case, the ontology of contradictory objects ensures that ‘an x such that $\sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ’ is a non-empty phrase. This has an immediate drawback of blocking the comprehension of the null class suggested above.

If ‘an x such that $\sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}$ ’ is not empty, then, by (2), ‘ $\{x: \sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$ ’ does not denote the null class. To be sure, this class term is still having a denotation, but this denotation is not the null class. The comprehension (1),

$(\exists M) (x \in M \equiv \sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}),$

would be a comprehension of the class which collects all x such that $\sim(\exists y)(y \text{ is a man}) \ \& \ x$ is a man, and, due to our Meinongian supposition about Russell, this class would contain one or more contradictory objects. Therefore, $\{x: \sim(\exists y)(y \text{ is a man}) \ \& \ x \text{ is a man}\}$ is by (D3) a class which is existent.

An analogous token of reasoning holds for any instance of the schematic definition (D4) $(\Lambda = \text{Df. } \{x: (\sim(\exists y)(Ay) \ \& \ Ax)\})$. The impact the Meinongian supposition is, however, more general. (D4) does not provide the only way of defining the null class. In general, the null class can only be comprehended by means of a contradictory open sentence (or, in Russell's framework, 'propositional function'). The commonest way of defining the null class is to use ' $x \neq x$ '. In this way, we get the following definition and its related comprehension:

$$\text{(D4')} \ \Lambda = \text{Df. } \{x: x \neq x\}$$

$$(1') \ (\exists M) (x) (x \in M \equiv x \neq x)$$

The Meinongian supposition, again, ensures that (1') comprehends a class with one or more members. Accordingly, it ensures that (D4') is not a definition of the null class. The core of the problem lies in the considered sort of Meinongianism which entails the principle that contradictory open sentences are always satisfied by some entities—entities that are contradictory.

Three reasons derived from the logic and the related theory of mathematics endorsed in *Principles* were proposed in this chapter to demonstrate that Russell could not include contradictory entities among what there is. First, Russell could not propose his theory of cardinal numbers had he combined the principle of explosion with contradictory entities. Second, had Russell posited contradictory entities, he would have to renounce the Law of Contradiction. Third, Russell's logic and theory of mathematics depend on taking the null class to be a class with no members; it would be impossible, for Russell, to reckon with this notion, had he admitted contradictory entities. Quine in his (1966) paper claims that the early Russell's ontology is unrestrained. 'Every word referred to something,' says

Quine.¹⁵¹ As we pointed out, he remained agnostic about whether this principle applies to descriptions of contradictory entities. We are now justified to insist that it could not apply to such descriptions. Yet, as we will see in the next two chapters, that there were semantic reasons which were dragging Russell towards the opposite end.

151 Quine (1966), 658.

5 Did the early Russell take the present king of France to be?

5.1 Quine and his critics

There is a solid amount of evidence showing that Russell could not maintain that there are contradictory entities. Contradictory entities should, therefore, present one demarkation line between the early Russell and Meinong.¹⁵² But contradictory entities are usually not at issue in the literature about Russell. Russell scholars have shown much more interest in the question whether the early Russell embraced possibilia.

Let us first settle what it means for an entity to be a possibilium. Possibilia are coherent, hence possible. They can be actual, but as a matter of fact they happen not to be (they do not obtain in time). As they are not actual, they do not exist. The golden mountain and the present king of France are typical examples.

In the previous chapter, we saw that Quine asks this question: Concerning Russell's ontology in *Principles*, should possibilia be treated as being ontologically on a par with such entities as (good) prime numbers? Raising this question presupposes that in *Principles* Russell *did* embrace possibilia. So, according to Quine, the early Russell believed that entities like the golden mountain or the present king of France are in some sense. This interpretation has been recently opposed by several Russell scholars. Today it is widely accepted that Russell's position in *Principles* was not as definite as Quine presented it in his 1966 paper. Some scholars, notably Stevens (2011), have even ventured to claim that the ontology of *Principles* was as free of possibilia as Russell's ontology after adopting Theory of Descriptions was.

The issue of the dispute has been set up by Quine's influential account of how Russell reformed his views towards ontological austerity. 'The reform was no simple change of heart; it hinged on his discovery of a means of dispensing with the unwelcome

¹⁵² Later it will be emphasized that, for Meinong, contradictory entities are beyond being. By Meinong's Independence Principle, the so-being of an entity is independent from whether the entity has being or not. This allowed Meinong to maintain that a contradictory entity is something mental acts can be directed to despite the fact the entity fails to possess any form of being. Any comparison of Russell with Meinong must, in the end, mention that Russell never adhered to Meinong's Independence Principle and the related view of intentionality.

objects. The device was Russell's theory of singular descriptions [...].¹⁵³ The unwelcome objects are, or at least include, possibilia. If only with Theory of Descriptions at hand could Russell expel possibilia out of the inventory of reality, then either TDC could not supply this effect, or, more radically, it had the opposite effect, i.e., TDC was itself responsible for Russell's alleged admission in *Principles* of the unwelcome objects. Accordingly, we need to ask this question:

(q1) Did TDC commit Russell to include possibilia among what there is?

The dispute between Quine and the Russell scholars is concerned with ontology insofar as its conclusions follow from a *semantic* consideration. Since TDC does not exhaust the early Russell's semantics, it is necessary to generalize (q1) in the following way:

(q2) Were there, in *Principles*, any semantic grounds that committed Russell to include possibilia among what there is?

Taking this more general question into account has an advantage of covering Russell's early semantical thought as a whole. I maintain that, if there are semantical reasons, whether they are based on TDC or not, that led Russell to embrace possibilia, exposing these reasons is highly relevant to the debate over Quine's influential account.

It is common, in the literature, to use the term 'Meinongianism', but rarely its users sufficiently clarify what they have in mind. This label is closely related to Quine's use of the epithets 'unwanted' and 'fictitious', and also to his phrase 'the exuberance of Meinong's realm of being'.¹⁵⁴ Invoking Meinong in a discussion of the early Russell's views in this simplistic way can be misleading. For the early Russell, all entities (terms) fall under the category of being. Everything has being. For Meinong, this is not the case! Possibilia and impossible (contradictory) objects are, according to Meinong, beyond being

153 Quine (1966), 659.

154 See *Ibid.*, 658.

(*Ausserseiende*).¹⁵⁵ Meinong's theory is, in this respect, more of a descriptive phenomenology than of an ontology of the early Russell's style. For this reason, it is crucial to use terms like 'Meinongianism' and 'Meinongian entities' with caution.

Let us first see what Meinongianism is not. Depending on one's notion of existence, not every view that there are non-existents is Meinongian. For Russell, only individuals can exist.¹⁵⁶ Relations, for example, cannot exist. They are said by Russell to subsist or to have being. Embracing subsistent entities does not make Russell's ontology Meinongian. As Stevens points out, the extent to which one's ontology can be termed 'Meinongian' must be 'a matter of *which kinds* of objects are taken to have being and on what grounds, rather than simply a matter of acknowledging being as a an ontological category in addition to existence.'¹⁵⁷ Stevens offers the following account of Meinongianism:

[...] I will use the term 'Meinongianism' to denote a certain kind of ontological position which is primarily motivated by semantic considerations: I will use the term to denote any ontological position which takes the existence of an object to follow immediately from the use of a term to refer to that object so as to express a proposition about it, and which treats the question whether a term is a referring term to be wholly answered by observation of its grammatical behaviour.¹⁵⁸

In Stevens' view, the notion of Meinongianism is specific to a certain way of explaining reference. If the following two conditions are met, one's ontology can, in accordance to Stevens' use of the term, be termed 'Meinongian':

- The existence (but 'being' is a more appropriate term here) of an entity follows immediately from the use of a term to refer to that entity.
- The fact that a term is referring follows solely from its grammatical behavior..

155 For a detailed discussion of Meinong's notion of *Aussersein* of the pure object, see, e.g., Dale (2011), chapter 4.

156 *EIP*, 98.

157 Stevens (2011), 50.

158 *Ibid*.

In what follows, I use the term ‘Meinongianism’ in a wider sense represented by the generalization involved in the question (q2). *Any* semantic reason which leads to an admission of entities like the golden mountain will be taken to lead to a Meinongian ontology in my more comprehensive sense.

4.2 The puzzle of negative existentials: Quinean reading

What can be said in Quine’s defense? Quine’s interpretation cannot be renounced as a wild speculation because there actually are passages in Russell’s writings which seem to undoubtedly support it. One such passage occurs in Russell’s intellectual autobiography published in 1959. Russell writes:

Another important distinction between names and descriptions is that a name cannot occur significantly in a proposition unless there is something that it names, whereas a description is not subject to this limitation. Meinong, for whose work I had had a great respect, had failed to note this difference. He pointed out that one can make statements in which the logical subject is ‘the golden mountain’ although no golden mountain exists. He argued, if you say that the golden mountain does not exist, it is obvious that there is something that you are saying does not exist - namely, the golden mountain; therefore the golden mountain must subsist in some shadowy Platonic world of being, for otherwise your statement that the golden mountain does not exist would have no meaning. I confess that, until I hit upon the theory of descriptions, this argument seemed to me convincing.¹⁵⁹

The problem invoked in this passage is traditionally called the puzzle of negative (singular) existentials. How can we, the question of the puzzle goes, truly deny existence of something? If such a denial is significant (has its purported meaning), there must exist

¹⁵⁹ Russell (1959), 64.

some x of which we truly state it does not exist. But if the denial is true, x does not exist. So, x both does and does not exist.¹⁶⁰

Russell claims in his (1959) to reproduce Meinong's take on the puzzle and confesses that, until he hit upon Theory of Descriptions, he had been in agreement with Meinong. Let us, at this moment, abstract from the question related to Meinong. What Russell says about his position before discovering Theory of Descriptions is quite clear. He allegedly held to a doctrine instantiated by this claim: 'The golden mountain does not exist' is not meaningful unless the golden mountain has some form of being. Russell speaks of subsistence and suggests that subsistence is a weaker sort of being. The difference between existence and subsistence presents a solution to the puzzle. We can truly deny the existence of the golden mountain without running into a contradiction because the fact that 'the golden mountain' is significant requires the subsistence of the golden mountain and subsistence can live without existence. Whether this was Meinong's take on the puzzle or not, our present question concerns Russell: Was this the early Russell's take on the puzzle? Did he adhere to the doctrine which endows possibilia with subsistence?

The passage from Russell (1959) seems to reproduce an argument sketched in §427 of *Principles*. In §427, just after distinguishing between existence and being, Russell sets out to criticize the so-called 'existential theory of judgment' (a theory that every judgment is existential). In the course of his criticism, Russell remarks that the distinction between existence and being (which the existential theory of judgment rejects) 'is essential, if we are ever to deny the existence of anything.'¹⁶¹ Subsequently, he expands on this point as follows:

160 I appeal to the reader to consider this statement of the puzzle carefully. Later in this chapter, I expose that critics of Quine establish their point on misunderstanding the puzzle. They miss that it is concerned with aboutness. The golden mountain must be in some sense because, in order to state the truth 'The golden mountain does not exist', we need something of which we truly deny existence, or alternatively: about which we truly say it does not exist.

161 *Principles*, §427.

For what does not exist must be something, or it would be meaningless to deny its existence; and hence we need the concept of being, as that which belongs even to the non-existent.¹⁶²

Talking here of a non-existent, i.e., an entity which does not exist but is, nevertheless, endowed with being, does Russell talk about possibilia like the golden mountain? From the Quinean perspective the answer is 'yes'. The passage from §427 of *Principles* contains a sketch of the Meinongian argument referred to in the 1959 retrospective. From the Quinean perspective, then, the Russell of *Principles* was committed to a view that possibilia are among what there is.

5.3 The puzzle of negative existentials: An alternative reading

Graham Stevens, one of the current critics of Quine's interpretation, proposed an alternative take on Russell's retrospective. Russell, according to him, misinterprets his thought before the discovery of Theory of Descriptions. 'Both Quine and Russell are wrong to think that the semantic theory of the *Principles* is committed to a Meinongian ontology.'¹⁶³

First, we need to detail the crux of Russell's alleged misinterpretation. Stevens has it that the objection presented in Russell's retrospective points to a failure, in a semantic theory of singular terms, to distinguish *names* and *descriptions*. In Stevens' view, Russell complains that Meinong failed to realize that descriptions, as opposed to (logically proper) names, do not contribute their referents (denotations) to the meaning of the sentences in which they occur. What about Theory of Denoting Concepts [TDC]?

According to TDC, a denoting concept is contributed, not the denotation. As Stevens points out, from the fact that '*Fa*' is meaningful, where '*a*' is a genuine proper name, one can infer that *a* has being.¹⁶⁴ However, this line of reasoning, Stevens argues, is

¹⁶² Ibid.

¹⁶³ Stevens (2011), 52.

¹⁶⁴ Ibid., 55.

not available if 'a' is a description and TDC, rather than a naive semantics associated by Russell with Meinong's name, is endorsed. Stevens argues as follows:

In the proposition that the present King of France is bald, the present king of France is not to be found, only the denoting concept /the present king of France/ is present in his place. But, in this case, the argument previously taken to ensure the being of *a* will only, when transposed to this case, ensure the being of the *concept* /the present King of France/ [...]¹⁶⁵

By analogy, the meaningfulness of 'The golden mountain does not exist' presupposes that the denoting concept expressed by 'the golden mountain' has being, not the being of the purported denotation, the golden mountain itself. So, according to Stevens, Russell is unfaithful to himself when he suggests that before his discovery of Theory of Descriptions he adhered to a Meinongian view which necessitates the being of such entities as the golden mountain.

The same interpretation of Russell's retrospective is assumed in Bostock (2013). Bostock also emphasizes that a denoting concept suffices to make a description meaningful. Descriptions 'would retain this meaning even if they failed to denote anything. Moreover, a proposition that is expressed by means of a denoting phrase contains only the meaning of this phrase as a constituent, and not the object(s) that it denotes.'¹⁶⁶ 'The golden mountain' fails to denote and contributes a denoting concept into the proposition expressed by 'The golden mountain does not exist'.

If Stevens and Bostock are right, we have to assume that the early Russell must have confined his argument in §427 to *just* those statements of the form 'x does not exist' which have a genuine proper name in the place of 'x'. Since 'the golden mountain' is not a genuine proper name, the argument could not be used to support a view that the golden mountain has being. So, from the standpoint of these critics of Quine, it is concluded that Russell (1959) *does not* reproduce the argument from §427 and must be taken to

¹⁶⁵ Ibid., 56.

¹⁶⁶ Bostock (2012), 33.

misinterpret the position he held at the time of *Principles*. In the following, I will challenge this interpretation.

5.4 In defense of Quine

The above cited passage from §427 is closely related to another passage which can be found earlier in *Principles*, namely in §71. This other passage strongly suggest that, *contra* Stevens and Bostock, in §427 Russell does include in his discussion those statements of the form ‘*x* does not exist’ which have a definite description in the place of ‘*x*’! The relevant part of §71 reads:

A and *B* may be any conceivable entities, any possible objects of thought, they may be points or numbers or true or false propositions or events or people, in short anything that can be counted. A teaspoon and the number 3, or a chimaera and a four-dimensional space, are certainly two. Thus no restriction whatever is to be placed on *A* and *B*, except that neither is to be many. It should be observed that *A* and *B* need not exist, but must, like anything that can be mentioned, have Being. The distinction of Being and existence is important, and is well illustrated by the process of counting. What can be counted must be something, and must certainly be, though it need by no means be possessed of the further privilege of existence. Thus what we demand of the terms of our collection is merely that each should be an entity. (§71)

Russell’s schematic letters ‘*A*’ and ‘*B*’ are replaced three times by an indefinite description and once by a definite description. Notice that if the denoting concept expressed by ‘a chimera’ is empty, it could not be the case that a chimera and a four-dimensional space are two. So, since they are two, there must be at least one chimera which is a member of the extension of /a chimera/ and which together with a four-dimensional space creates a couple.

Both arguments, that from §71 and that from §427, are proposed to show that the distinction between existence and being is indispensable. The former derives this conclusion from an observation about ‘the process of counting’ while the latter from an observation about true denials of existence. As we will see, both arguments are based on the same principle.

In its primary sense, counting is, for Russell, stating about something that it is one. In §47 of *Principles*, Russell explains that countability is an attribute of everything. Every term is one. Is a chimera one? Russell’s answer in *Principles* seems to be ‘yes’. His argument can be reproduced as follows: Since ‘a chimera is one’ is meaningful, we can use it to make a statement; to use it to make a statement means to assert *of something* that it is one; if there is nothing of which we can assert oneness when we state that a chimera is one, ‘a chimera is one’ is meaningless; therefore, there is, in some sense (being or subsistence), some chimera of which we assert oneness when saying that a chimera is one. The crucial principle applied in this piece of reasoning can be informally put as follows:

(IR) Intentionality Rule: Every sentence is meaningful if and only if there is an x such that one can use that sentence to make a statement about that x , i.e., one can use that sentence to assert something of x or deny something of x .

The argument from §71 concerns counting in the sense of saying of a couple of entities that they are two. The same consideration applies. ‘A chimera and a four-dimensional space are two’ is meaningful, therefore there must be some two entities of which we are saying that that they are two when we state the sentence.

Let us now return to the passage from Russell (1959). In Russell’s view, Meinong argued that ‘if you say that the golden mountain does not exist, it is obvious that there is something that you are saying does not exist - namely, the golden mountain [...]’¹⁶⁷ This is clearly an application of (IR)! This is overlooked by Stevens and Bostock. If you assert ‘the golden mountain does not exist’, there must be something, according to (IR), *of which* you can say that it does not exist when you state ‘the golden mountain does not exist’. The

¹⁶⁷ Based on what was said above, Meinong did not argue in this way. We are still operating under the assumption that Russell could be referring to an argument which he himself had endorsed before he adopted Theory of Descriptions in 1905.

golden mountain, therefore, has being in some sense. This is the Meinongian argument referred to in the retrospective.

The associated argument of §427 of the *Principles* is more difficult to interpret because it is too sketchy. Russell says merely that ‘what does not exist must be something, or it would be meaningless to deny its existence.’ What does he mean by ‘meaningless’ here?

One option is that ‘meaningless’ means that the grammatical subject of a sentence lacks a semantic value (it has nothing to stand for). This interpretive assumption was adopted by Bostock and Stevens in order to establish their criticism of the Quinean reading which links §427 to the retrospective in Russell (1959). To explain Bostock’s and Stevens’ reading, we took ‘the golden mountain does not exist’ as an example. We pointed out that TDC supplies ‘the golden mountain’ with a denoting concept. A genuine proper name must refer in order to be meaningful, not a description. Descriptions retain their meanings even if they fail to denote something. It suffices that they express denoting concepts. Based on this consideration, Stevens and Bostock confine Russell’s argument presented in §427 to the statements of the form ‘*x* does not exist’, where ‘*x*’ is a name, only.

It is now clear that Stevens’ and Bostock’s reading is questionable. Since the passage from §71 presents a piece of reasoning which is strikingly close to that of §427, we should treat the former as a key to a proper reading of the latter. Consequently, ‘meaningless’ in §427 does not concern the question of whether the grammatical subject of a sentence has a semantic value or not at all. I suggest, against Stevens’ and Bostock’s reading, that ‘meaningless’ in §427 means a violation of (IR).

Whether ‘*x*’ in a sentence of the form ‘*x* does not exist’ is a definite description or name, that sentence is, according to (IR), meaningful if and only if there is something of which we assert that it does not exist when we use that sentence to make a statement. In particular, ‘the golden mountain does not exist’ is meaningful if and only if ‘the golden mountain’ supplies us with a term of which we can assert that it does not exist when we state that the golden mountain does not exist. Obviously, the denoting concept expressed by ‘the golden mountain’ will not do. A statement that the golden mountain does not exist is no more about a denoting concept than, e.g., a statement that the current chancellor of Germany does not appreciate Donald Trump’s views is.

Stevens and Bostock are right when they emphasize that TDC provided the early Russell with a semantic differentiation between names and descriptions. This differentiation, however it might mark a superiority over the naive semantics associated by Russell with Meinong, is of a little help in preventing the early Russell's ontology from Meinongianism (in a wider sense defined above).

5.5 The early Russell's Meinongian argument reconstructed

We can now construe the schema of the Meinongian argument informally and very briefly suggested in §71 and §427 of *Principles* and referred to in Russell's 1959 retrospective. ' A ' is any singular term, a name or description. We start with an instance of (IR) and a sentence schema which captures the desideratum that every sentence of the form ' A does not exist' is meaningful:

- (1) If ' A does not exist' is meaningful, then $(\Box y)$ (we deny existence of y if we assert ' A does not exist') [an instance of (IR)]
- (2) ' A does not exist' is meaningful.
- (3) $(\Box y)$ (we deny existence of y if we assert ' A does not exist') [(1), (2), Modus Ponens]

The argument utilizes the following principle concerning aboutness (intentionality) and reference:

- (4) (y) ((we deny existence of y if we assert ' A does not exist') if and only if $y = A$).

The argument continues in this way:

- (5) We deny existence of a if we assert ' A does not exist'. [(3), The Rule of Existential Instantiation]

- (6) (we deny existence of a if we assert $\lceil A \text{ does not exist} \rceil$) if and only if $a = A$.
 [(4), The Rule of Universal Instantiation]
- (7) If we deny existence of a if we assert $\lceil A \text{ does not exist} \rceil$, then $a = A$. [(6),
 Simplification]
- (8) $a = A$. [(5), (6), Modus Ponens]
- (9) $(\exists y) (y = A)$. [(8), The Rule of Existential Generalization]

As a consequence, from the fact that a sentence of the form $\lceil A \text{ does not exist} \rceil$ is meaningful, we can infer that something is referred to by ' A ', that is, there is something named if ' A ' is a name, or described if ' A ' is a definite description. Let us call this argument 'M1'. Now, suppose, the following is the case:

- (10) A does not exist.

This takes us to Russell's considerations about the indispensability of the category of being. According to (9), there is (in the sense of an existential quantifier) an entity which is referred to by ' A ', but, according to (10), that entity is not an existent. Since there are true negative existentials, the domain of all terms must be wider than the domain of existents. Russell adopts, throughout *Principles*, the term 'being' or 'subsistence' as a label which covers the ontological status of *all* terms. In the present context, we can formalize this doctrine in the following way:

- (11) If $(\exists y) (y = A) \ \& \ A$ does not exist, then A subsists.

From (9) and (10), we know that the antecedent of (11) is true, so we can infer, by Modus Ponens, that

- (12) A subsists.

5.6 What Russell's 1959 retrospective says?

TDC is certainly better than the Meinong-style naive semantics in that it differentiates semantically descriptions from names. However, both TDC and the naive semantics are ineffective in avoiding M1 and the related Meinongianism. The validity of M1 depends crucially on (IR). We must read Russell's 1959 retrospective in accordance with these findings. Russell writes:

Another important distinction between names and descriptions is that a name cannot occur significantly in a proposition unless there is something that it names, whereas a description is not subject to this limitation. Meinong, for whose work I had had a great respect, had failed to note this difference.¹⁶⁸

This passage announces a presentation of M1, more precisely, the instance of M1 which concerns the meaningfulness of 'The golden mountain does not exist'. The reading of the passage, then, must reflect this peculiar context. For a name, to occur significantly in a sentence is, among other things, to provide that sentence with an entity *of which we assert something if we use that sentence to make an assertion*. Russell insists that 'a description is not subject to this limitation.' Why? Russell does not refer to TDC here, but clearly to Theory of Descriptions! The function of a description, according to Theory of Descriptions, is not to pick out the logical subject of a sentence. The description is as an incomplete symbol dissolved into an existential quantifier and what Russell calls a propositional function. Let us, again, take 'The golden mountain does not exist' as an example. According to Theory of Descriptions, this sentence expresses a proposition whose structure is captured, e.g., in this formula:

$$\sim(\exists x) (y) (y \text{ is a mountain} \ \& \ y \text{ is golden} \ \equiv \ y = x)$$

In Russell's usual idiom, this formula, when it is asserted, says that '(y) (y is a mountain & y is golden \equiv y = x)' is false for every value of x. The function '(y) (y is a mountain & y is

¹⁶⁸ Russell (1959), 64.

golden $\equiv y = x$)' is, accordingly, the logical subject and ' $\sim(\exists x)$ ' represents what is asserted about the subject.

Putting aside the issue what the function ' (y) (y is a mountain & y is golden $\equiv y = x$)' for the Russell of this or that period is or what it stands for, we can see now how Theory of Descriptions avoids the argument M1 and the related Meinongianism. Unlike names, descriptions do not supply the sentences in which they occur with their purported referents (denotations) as the logical subjects of those sentences. The logical subject of a sentence containing a description is a function. In our example above, it is a function of which it is said that it is false of every value of x . Accordingly, from the fact that 'The golden mountain does not exist' is meaningful can be validly inferred that the function ' (y) (y is a mountain & y is golden $\equiv y = x$)' must have being; we cannot, however, infer from that fact that the golden mountain has being.

Importantly enough, Theory of Descriptions is not incompatible with (IR). With Theory of Descriptions assumed, the aboutness feature of a sentence remained, for Russell, a precondition of the sentential significance. Russell did not avoid M1 by rejecting (IR)! The avoidance was effected by Russell's eliminative analysis of the logical form of the sentences containing descriptions. 'The golden mountain does not exist' is *only seemingly* of the same form as 'Socrates is wise' (provided 'Socrates' is a genuine name).¹⁶⁹ The phrase 'the golden mountain', as opposed to 'Socrates', does *not* serve to pick out an entity which functions as logical subject of the sentences having 'the golden mountain' in grammatical subject. *This* is the difference between names and descriptions which Russell (1959) says Meinong and his former self failed to acknowledge. TDC was a failure in this very peculiar sense.

5.7 Concluding remarks

Quine, we conclude, was not wrong in claiming that only with Theory of Descriptions could Russell liberate himself from the exuberance of the realm of possibilia. However, as we will see in the next chapter, Stevens, Bostock and some others are partially

¹⁶⁹ This topic will occupy us in a greater detail in Chapter 7.

right in claiming that the early Russell was working under an assumption that there are no possibilities in the realm of being when he was developing his TDC. I assume, therefore, that Russell was experimenting in *Principles* with conflicting semantic views, in all likelihood without being aware of the extent to which they were in tension.

Let us now I return to Stevens' account of Meinongianism. Stevens calls an ontology 'Meinongian' if it satisfies the following two criteria:

- The being of an entity follows immediately from the use of a term to refer to that entity.
- The fact that a term is referring follows solely from its grammatical behavior.

This is hopelessly narrow. The source of Russell's Meinongianism consists in a certain understanding of aboutness, not reference. The being of an entity, say *a*, is warranted by the following three premises:

A (well-formed) sentence containing a term which purports to refer to *a* is not about anything if *a* does not have being.

A sentence which is not about anything is meaningless.

A sentence containing a term which purports to refer to *a* is not meaningless.¹⁷⁰

Neither of Stevens' two criteria makes sufficiently clear that the notion of aboutness can be involved in the roots of Meinongianism. Stevens' definition of Meinongianism is, in our present context, acceptable only if what he means by 'reference' and 'grammatical behavior of a term' involves the function of a term to supply the sentence in which it occurs with a logical subject; we can speak of the 'aboutness function'. Given Stevens' misinterpretation of Russell, we are entitled to assume that this function was not in his focus at all.

In section 5.1, we decided to treat an ontology as Meinongian (about possibilities) if and only if there is *any* semantic reason which commits one to include possibilities among

¹⁷⁰ Here, the word 'term' is used in all its occurrences in the modern sense of a linguistic unit, not in the early Russell's ontological sense.

what there is. Whether it is a reason which concerns reference or something else is irrelevant. Returning to (q1), since (IR) is not part of TDC, it should be said that TDC itself did not commit Russell to include possibilia among what there is. Accordingly, (q1) should be answered in the negative. But since there was *some* semantic reason which committed Russell to include possibilia among what there is, we have to say 'yes' in response to (q2). The early Russell's semantics makes necessary that he takes entities like the present king of France to be. With the positive answer to (q2), we conclude, in the same breath, that Russell's ontology in *Principles* was Meinongian in the wider sense of this term assumed in section 5.1.

6 The tensions surrounding the Theory of Denoting Concepts

6.1 An overview of Russell's conflicting views

I suggested, in the previous chapter, that the most accurate way to describe the early Russell's semantics is to take Russell to be experimenting in *Principles* with conflicting semantic views, in all likelihood without being aware of the extent to which they were in tension. In this chapter, the following three conflicts in the early Russell's semantical thought will be discussed:

- The first conflict is based on our findings in the previous chapter about the early Russell's ontological commitment to possibillia. Holding to (IR) alongside advocating the Theory of Denoting Concepts [TDC] made it impossible for Russell to dispense with possibilia. Yet, at some point, he wanted TDC to account for genuine referential failures.
- We have discussed what the ontological consequences of the fact that true denials of existence are possible would be under the joint rule of (IR) and TDC. What about true denials of being? For the early Russell, contradictory objects could not have being. His semantical thought, therefore must have allowed for true denials of being. Sadly, that was not possible as long as both (IR) and TDC were endorsed by the early Russell.
- In Chapter 1, we saw that the theory of Russellian propositions endows false propositions with being. We will discover that this leads to a specific sort of Meinongianism, which populates Russell's realm of being with possibilia. As mentioned above, Russell decided, at some point, that TDC should account for genuine referential failures. That was impossible as long as false propositions were supposed to possess being.

In what follows, I discuss these conflicts in this respective order. Starting with the first conflict in the list above will provide us with an opportunity to get a better sense of where scholars such as G. Stevens and S. Bostock find their reasons to oppose Quine's account of Russell's ontological development.

6.2 Genuine referential failures: An unfulfilled ambition

Russell says in *Principles* that 'it is necessary to realize, in the first place, that a concept may denote although it does not denote anything.'¹⁷¹ The first occurrence of 'denote' in this statement clearly does not mean the same as the second. I assume that Russell means to say that we need to realize that a denoting concept may *purport to* denote something but fail to do so. In what follows in §73, *Principles*, Russell proposes two examples of a concept that 'may denote although it does not denote anything', one expressed by the word 'chimeras', the other by 'even prime numbers other than two 2'. In Russell's typology discussed in Chapter 3, these are 'all'-phrases and the concepts expressed by them are

/all chimeras/

and

/all even prime numbers other than 2/

respectively.

Russell is clear that 'the concepts in question do not denote anything.'¹⁷² Later in §73, *Principles*, these cases are generalized into a definition of denoting concepts which do not denote anything. It reads as follows:

All denoting concepts, as we saw, are derived from class-concepts: and *a* is a class-concept when "*x* is an *a*" is a propositional function. The denoting concepts

¹⁷¹ *Principles*, §73.

¹⁷² *Ibid.*

associated with a will not denote anything when and only when “ x is an a ” is false for all values of x . This is a complete definition of a denoting concept which does not denote anything; and in this case we shall say that a is a null class-concept, and that “all a ’s” is a null concept of a class.¹⁷³

This is closely related to Russell’s definitions of an existent class and a null class from Chapter 2 of *Principles*.¹⁷⁴ The definition of an existent class (proposed in one of its notational variants) reads as follows: the class a is existent if and only if, by definition, $(\exists x) (x \text{ is an } a)$. Accordingly, the class a is null if and only if $\sim(\exists x) (x \text{ is an } a)$. Let us signify denoting concepts by the schema $/\Gamma \text{det}^1 + \Gamma a^1/$, where ‘det’ is a determiner and ‘ a ’ is an expression for a class concept. Russell’s definition of an empty denoting concept then goes as follows:

(D5) $/\Gamma \text{det}^1 + \Gamma a^1/$ is null =Df. $\sim(\exists x) (x \text{ is an } a)$

We should be clear about the fact that the definition of an empty denoting concept ensures that it has an instance no more than a definition of God ensures that God exists. This is why it is crucial that Russell states about the concept of a chimera and the concept of an even prime other than 2 that they are empty. This shows that, according to Russell, (D5) does have an instance. The first of the concepts is a concept of a possible entity which happens not to exist, the second is presumably a contradictory concept which thus belongs to the same group as */the round square/*.

It is beyond doubt that, at some point, Russell intended TDC to account for genuine referential failures. Explanations such as the following one should have been possible: When I assert ‘The present king of France is bald’, I fail in an attempt to refer to something, since */the present king of France/* is by (D5) a null denoting concept. How about the truth-value in such cases?

According to Frege (later joined by G. Ryle and P. Strawson), a failure to refer to something is associated with a truth-value gap. ‘The present king of France is bald’ is

173 Ibid.

174 Both definitions are discussed in detail in Section 4.5.

neither true, nor false (for Strawson, this applies to a statement of the sentence). However, for Russell, this does not seem to be the case. In the second paragraph of §73, *Principles*, he proposes a preliminary definition of a null denoting concept. This definition suggests that sentences such as ‘The present king of France is bald’ were considered by Russell as false, not truth-valueless. He writes:

It is necessary to realize, in the first place, that a concept may denote although it does not denote anything. This occurs when there are propositions in which the said concept occurs, and which are not about the said concept, but all such propositions are false. (*Principles*, §73)

This definition is immediately rejected because there are propositions which contain an empty (null) denoting concept, are not about that concept, and yet they are true, not false. Russell’s example is the proposition expressed by ‘Chimeras are animals’ or that expressed by ‘even primes other than 2 are numbers’. The definition is deemed flawed and replaced by (D5), but it is not entirely useless. It reveals that Russell intends to associate *some* cases of a referential failure with falsehood, not with the lack of a truth-value. Quite trivially, the cases of a referential failure associated with falsehood are those *which are not associated with truth*. ‘The present king of France is bald’ is such a case. This accords with Russell’s claim in *OD* that ‘The present king of France is bald’ is plainly false.¹⁷⁵

The intended capacity of TDC to account for genuine referential failures distinguishes TDC from the naive semantical theory attributed in *OD* to Meinong. In an often cited passage, Russell writes:

This [Meinong’s, MS] theory regards any grammatically correct denoting phrase as standing for an *object*. Thus “the present King of France,” “the round square,” etc., are supposed to be genuine objects. It is admitted that such objects do not *subsist*, but nevertheless they are supposed to be objects.¹⁷⁶

175 *OD*, 484.

176 *Ibid.*, 482-3.

According to the discussed proposal in §73 of *Principles*, every well-formed denoting phrase, expresses a denoting concept, where *only some of the concepts expressed stand for objects*, i.e., denote something. The phrases ‘the present king of France’ and ‘the round square’ presumably do not stand for objects, i.e., they purport to denote something, but fail to do so; these are null denoting concepts.

How should we integrate these findings into a more comprehensive interpretation of the early Russell’s ontological (and semantical) development? Stevens and Bostock took the evidence gathered from §73 of *Principles* to be the lead principle of their interpretation while undermining, or simply ignoring, those passages which speak in support of their competitor, Quine’s interpretation. In Chapter 5, we saw that their attempt is not compelling. The evidence gathered from §73, I contend, shows that Russell decided, at some point in the process of writing *Principles*, that TDC should account for referential failures. He was hoping that TDC could stand this test. Unfortunately, this amounts to an ambition only. Thus I take Stevens and Bostock to mistake an ambition to construe a successful theory for such a theory itself.

6.3 True denials of being

As we discovered in Chapter 5, Russell’s 1959 retrospective and two passages from *Principles* show that the Russell of *Principles* was committed to a principle we decided to call the ‘Intentionality Rule’. To remind ourselves of this principle, it reads as follows:

(IR) Every sentence is meaningful if and only if there is an x such that one can use that sentence to make a statement about that x , i.e., one can use that sentence to assert something of x or deny something of x .

In Section 5.5, we saw how (IR) is involved in the early Russell’s resolution of the puzzle of negative existentials. In what follows, I extend Russell’s use of (IR) to show that there is a cognate puzzle which concerns true denials of being (as we saw, Russell distinguishes existence and being/subsistence). From what we established in Chapter 4 concerning contradictory objects, it is absolutely certain that, for Russell, such denials must have been

possible. It must have been possible, e.g., to deny truly that the round circle has being. The puzzle that is cognate to the puzzle of negative existentials shows that, for the early Russell, it was impossible to coherently allow for true denials of being.

Let us stick to the round circle [trc in short] as an example. Now, in the early Russell's view, 'trc does not have being' must be a true sentence (as long as it is a well-formed sentence, which the early Russell does not refute). For if it is not, his theory of cardinal numbers, his logic based on the principle of explosion and Law of Contradiction and, finally, the notion of the null class deployed in *Principles*—all of this must be discarded. So, we assume, on Russell's behalf, that 'trc does not have being' is true. If this is correct, then the sentence is meaningful. So,

- (1) If 'trc does not have being' is meaningful.

The argument continues as follows:

- (2) If 'trc does not have being' is meaningful, then $(\neg y)$ (we deny being of y if we assert 'trc does not have being'). [an instance of (IR)]
- (3) $(\neg y)$ (we deny being of y if we assert 'trc does not have being'). [(2), (1), Modus Ponens]

The argument utilizes, at this point, the following principle concerning aboutness (intentionality) and reference:

- (4) $(y) ((\text{we deny being of } y \text{ if we assert 'trc does not have being'}) \text{ if and only if } y = \text{trc}).$

We continue in this way:

- (5) We deny being of a if we assert 'trc does not have being'. [(3), The Rule of Existential Instantiation]

- (6) (we deny being of a if we assert ‘trc does not have being’) if and only if $a = \text{trc}$. [(4), the Rule of Universal Instantiation]
- (7) If we deny being of a if we assert ‘trc does not have being’, then $a = \text{trc}$. [(6), Simplification]
- (8) $a = \text{trc}$. [(5), (6), Modus Ponens]
- (9) $(\exists y)(y = \text{trc})$. [(8), The Rule of Existential Generalization]

Call this argument ‘M2’. M2 is analogous to M1 from the previous chapter. The latter was followed by Russell’s introduction of the category of being/subsistence, which helped him to avoid a contradiction. What is not in the sense of existence must be, according to Russell, in some other sense: in the sense of being or subsistence. In the case M2, this move is not available, at least not for Russell. It is, however, possible that something similar to this move was involved in Meinong’s introduction of the phenomenological category of extra-being (*Aussersein*).

Russell could embrace contradictory entities only at the expense of destroying of his conception of logic, mathematics and philosophy. He could not embrace them. Insofar as the descriptions of contradictory objects are well formed descriptions, it is possible to deny the being of a contradictory object. In other words, such denials seem to be perfectly meaningful. Russell never said or implied they are not. Being perfectly meaningful, can such a denial be true? M2 demonstrates that ‘no’ must be the right answer here. Russell’s whole intellectual enterprise appeared at stake.

6.4 Propositions as Meinongian entities

In Chapter 1, we saw that, following G. E. Moore, Russell committed himself to a view that both true and false propositions have being.¹⁷⁷ This view, it was shown in the last

¹⁷⁷ An ontological commitment to false propositions is implied by Moore’s statement, in his ‘The Nature of Judgment’ (1899), on p. 189, of a primitivist theory of truth. He writes: ‘A proposition is constituted by any number of concepts, together with a specific relation between them; and according to the nature of this relation the proposition may be either true or false. What kind of relation makes a proposition true, what false, cannot be further defined, but must be immediately recognized.’

section of Chapter 1, collapses once it is held alongside another doctrine endorsed by Russell in *Principles*. This is the doctrine which takes propositions such as these two:

<*a* differs from *b*>

and

<*difference* relates *a* and *b*>,

to be materially equivalent. The collapse unfolds as follows: No matter if the first proposition in the pair above is true or false, it certainly must be a unity; since its being a unity makes the second proposition true and the propositions are materially equivalent, the first must be true as well; every proposition, therefore, must be true after all. The collapse brings about a further untoward consequence for the early Russell that emerges in the context of his semantics of denoting phrases.

Suppose that *a* does not differ from *b*, i.e., $a = b$. Say, *a* and *b* are roses; and as we suppose that they are the same, they simply have the same color, occupy the same region of space and time and so on. Had they been different, they would be two different roses, perhaps different in color etc. Now, in the name of ontological austerity which we associate, in the context of Russell's advocacy of TDC discussed in Section 6.2, with the possibility of genuine referential failures, we want to claim this: Provided $a = b$, there is no such thing among what there is as the difference between *a* and *b*; that is, the phrase 'the difference between *a* and *b*' should fail denote anything actual *in a sense that the extension of the concept is empty*. We want to claim, on Russell's behalf, that TDC allows for the case of a genuine referential failure here. With the early Russell's theory of propositions endorsed, is this possible?

We need remind ourselves about Russell's category of things. Things are opposed to concepts in the early Russell's ontology. While concepts can hold of something (in the sense of an application of a property or relation: *redness* holds of this rose; *c* and *d* stand in the relation of being next to each other) as well as occur as terms (as subjects) in a proposition, things can do only the latter.¹⁷⁸ Propositions are among things. Although

¹⁷⁸ See Sections 1.1 and 1.5.1.

Russell does not, in *Principles*, explain how an analysis of things such as a table or a rose should look like, he most probably held to some such view as that endorsed by Moore in his early papers that ordinary things are clusters of propositions.¹⁷⁹ On this view, propositions are the elements out of which the world of ordinary objects is constructed.

Let us now we return to our example, $\langle a \text{ differs from } b \rangle$. According to the early Russell's ontology, this proposition is a thing and as such it belongs to the realm of being regardless of its truth-value. Excluding $\langle a \text{ differs from } b \rangle$ from the inventory of reality would lead to the unacceptable consequence that ' $a \text{ differs from } b$ ' is a meaningless token of linguistic symbols. If $\langle a \text{ differs from } b \rangle$ has being, it is a unity, and, if it is a unity, it must be true that the relation of difference relates a and b . Russell admits that. And if he does, then how could the difference between a and b fail to be? Is he not forced to ditch the supposition that $a = b$?

Perhaps, it might be objected that $\langle a \text{ differs from } b \rangle$ was for the Russell of *Principles* something *essentially different* from the denotation of 'the difference between a and b '. And, as the objections goes, this denotation might fail to be despite the fact that $\langle a \text{ differs from } b \rangle$ has being. In the rest of my argument, I oppose this course of thinking. There is no evidence for any such distinction in the early Russell's works in the first place. In fact, the way the early Russell treated nominalizations of sentences supports a view that there was no such distinction for him.

The nominalization of a sentence is a grammatical form that belongs to the basic elements of the language of Russell's logic. Let's take an arbitrary proposition, p , then p can be either asserted or not asserted. If we assert it, we can use a sentence expressing it as a premise. This is how a proposition enters our inferential practices. What does it mean when the proposition is not asserted? One obvious option is twofold, we either refuse to assert a sentence expressing p and the same with its contradictory, or we assert the contradictory. There is, however, a third option. The proposition is unasserted whenever we assert a wff ' Ap ', where A is any extensional context, e.g., $q \sqsupset p$. In some such cases, p is implied, but that does not mean that p is asserted in whatever implies it unless this is

179 See, e.g., Thomas Baldwin's interpretation, in his (1990), Chapter II, of the early Moore's atomistic ontology of concepts.

identical with p (every proposition implies itself). In general, in any wff ' Ap ', where A is any extensional context, p is unasserted.¹⁸⁰ If I state, e.g., that 'Socrates is bald' and someone else states that the proposition expressed by this sentence implies another proposition (or itself), we both express <Socrates is bald>, but while I express this proposition as asserted, the other person expresses it as unasserted and, in fact, as part of a larger unit which is asserted.

For example, <Socrates is bald> is not asserted in a statement that it implies itself. It is referred to, denoted. A proposition of the form $\langle x \supset x \rangle$ is asserted here whose relating relation is material implication. In the proposition which is asserted in our example this relation holds of the terms which occur as terms in it. And there are only two possible ways of expressing such terms in the expression of our proposition: by names or by denoting phrases; choosing normally the latter, we write:

(i) $\langle \text{Socrates is bald} \rangle \supset \langle \text{Socrates is bald} \rangle$,

where ' $\langle \text{Socrates is bald} \rangle$ ' is a definite description picking out the proposition expressed by 'Socrates is bald'. The primary way of reading ' $\langle \text{Socrates is bald} \rangle$ ' does not rely on a description beginning with 'the proposition that', but it consists in nominalizing the sentence expressing the proposition denoted. Russell's primary way of reading (i) is as follows:

(i)' Socrates' baldness implies Socrates' baldness

or

(i)'' Socrates' being bald implies Socrates' being bald.

Accordingly, whenever a proposition p occurs in an extensional context, Ax , we *must* nominalize the sentence which is used to express p . Only with the nominalization at hand

¹⁸⁰ An assertion of the contradictory of a sentence expressing p , in fact, falls within this category if we mean that a negated content is asserted, for $\sim p$, where p is a proposition, is an extensional context. The other option is, to be precise, a denial of p .

are we able to express the wff whose meaning is $\langle Ap \rangle$.¹⁸¹ This shows how the difference between a sentence and its nominalization is used in the language of Russell's logic.

In this essay, we avoid nominalizing brackets in the formal expositions of the arguments. That is, we write 'Socrates is bald \square Socrates is bald' instead of ' \langle Socrates is bald $\rangle \square \langle$ Socrates is bald \rangle '. This practice makes the expositions less difficult to read. We should, however, be aware that the nominalization of a sentence is presupposed in every case where a proposition occurs as a term in some other proposition which we express. The expression of this larger unit, in which the proposition is embedded, is not nominalized if and only if it amounts to a line of an argument.

The role of nominalization in Russell's language of logic shows clearly that nominalizations of sentences stand for propositions. To be more precise, nominalizations of sentences *denote* propositions. In any occurrence of denoting a proposition, these three layers are involved:

- the nominalization of a sentence (a denoting phrase),
- a denoting concept expressed by the nominalization (Russell calls such concepts 'propositional concepts')
- the proposition denoted.

Thus the denotation of 'the difference between a and b ', if there is any, must be identical to the proposition expressed by ' a differs from b '.

Metaphysical nuances aside, there seems to be a straightforward linguistic intuition at play which is akin to the intuition that the pairs of sentences like {' a differs from b ', '*difference* relates a and b '} are pairs of materially equivalent units. (Concerning the latter we spoke, in the concluding section of Chapter 1, about a brute linguistic fact.) That intuition can be formulated in this way:

(PD) s if and only if there is such a thing as $\langle s \rangle$,

181 Thus, " $\langle \langle \rangle \rangle$ " and " $\langle \rangle \rangle$ " indicates the form of a nominalization of the sentence written in between such brackets.

where ' $\langle s \rangle$ ' represents the nominalization of s (the nominalization of a sentence in the usual sense, considered outside of the context of Russell's ontology of propositions). We can call this principle the 'Principle of Denoting' [PD] Thus, as long as (PD) is a true principle, the following:

a differs from b iff there is such a thing as the difference between a and b

is a true instance of it. So, whether the entity denoted by 'the difference between a and b ' is the same as the proposition that a differs from b or it is something else, the link between these entities is such that, if the second of them is a true proposition, the first is included in the inventory of reality and *vice versa*. It follows that the reality of false propositions, had it been possible in the early Russell's ontology, would necessitate that we assume that the proposition expressed by ' s ' and the putative denotation of the nominalization of ' s ' are distinct entities. Why? Suppose ' a differs from b ' is false. Then by an instance of (PD) it follows that there is no such thing as the difference between a and b . Since our false sentence is meaningful, there is such thing as $\langle a$ differs from $b \rangle$. Therefore, the difference between a and $b \neq \langle a$ differs from $b \rangle$. The reality of false propositions however, as we concluded in Chapter 1, is impossible if the early Russell's ontology is endorsed.

The early Russell's theory of propositions (and their unity) renders every proposition true. Russellian propositions in general thus end up being necessarily true. Assuming that (PD) is a principle we cannot get around, every nominalization of a sentence is rendered as denoting. In this way, the early Russell's theory of propositions precludes the ontological austerity Russell was manifestly hoping to achieve by providing TDC with a capacity to account for genuine referential failures. Assuming further that the nominalization of a sentence is identical to the proposition expressed by that sentence, we are entitled to label certain propositions 'Meinongian'. If ' a differs from b ' should be false, the early Russell's theory of propositions

- renders this sentence true;

- renders the proposition expressed by it a Meinongian object which prevents every referential use of its nominalization, ‘the difference between a and b ’, to be a referential failure.

So, every proposition which should (by a supposition) be false appears, on the early Russell’s views, true and Meinongian.

Although descriptions like ‘the present King of France’ are not nominalizations of sentences (or, at least, not in the straightforward way ‘the difference between a and b ’ is), the current result concerning nominalizations of sentences extends unto them in a peculiar way. We can join every description which is not a nominalization with the predicate ‘has being’. So, we have, for instance, this:

The present King of France has being.

Since the proposition expressed is a unity, it must be true and ‘the being of the present King of France’ must, according our findings above, have an instance. It follows that there is a such a thing as the present King of France.

With every proposition being true, every version of reality alternative to the actual one must be the case. We can say that every possibilium must be, but in fact, on such a disastrous view, there is no difference between an actual entity and a related possibilium. For instance, there is no difference in actuality between the current France and an alternative version of it. Being forced to add contradictory objects to this already bloated and surreal vision of reality, we end up facing all the problems discussed in chapter 4: No arithmetics can be construed, the Law of Contradiction gets horribly infringed and the notion of null class turns out to be impossible.

7 Theory of Descriptions

7.1 The apparent form vs. the real form of a sentence

Theory of Descriptions [TD], as Russell puts it in ‘On Denoting’ [OD], ‘gives a reduction of all propositions in which denoting phrases occur to forms in which no such phrases occur.’¹⁸² In this important statement, Russell does not speak of propositions in the ontological sense introduced in Chapter 1. He does not speak of mind-independent complex individuals, but of linguistic items: declarative sentences. If such a sentence contains a description, TD is required in order to effect *a reduction of the given sentence to a form (another declarative sentence) in which no descriptions occur*. TD thus serves as a means of analyzing descriptions away whenever they occur in a sentence.

To set forth Russell’s theory in a precise way, we need to acknowledge properly that TD is a semantic theory whose subject matter is not descriptions alone, but descriptions in a certain context: sentences containing descriptions. This point is far from being merely rhetoric. It goes with the main principle of Russell’s theory. Let us call this principle the ‘Contextualization Principle’. In *OD*, we read:

This is the principle of the theory of denoting I wish to advocate
[(**Contextualization Principle**), MS]: that denoting phrases never have any
meaning in themselves, but that every proposition in whose verbal expression they
occur has a meaning.¹⁸³

Later in the paper, Russell says in the same vein that, from the semantic point of view, ‘a denoting phrase is essentially *part* of a sentence, and does not, like most single words, have any significance on its own account.’¹⁸⁴

182 *OD*, 482. The word ‘description’ is used as interchangeable with ‘denoting phrase’.

183 *Ibid.*

184 *Ibid.*, 488.

Why is a description ‘essentially *part* of a sentence’? This comes as a consequence of taking the semantic contribution of a description *to be holistic with regard to the sentence in which the description occurs*.¹⁸⁵ In other words, where a description is concerned, as opposed to a logically proper name,¹⁸⁶ we are dealing with a symbol that determines *the form* of a sentence in which it occurs, *not a constituent* of it (in Russell’s technical sense of constituency).¹⁸⁷ The semantic nature of descriptions is concealed by the grammar of an ordinary language, for the grammatical structure of a sentence containing a description fails to incorporate the peculiar way the description contributes to the whole meaning. Thus there are, in every semantic analysis of a sentence containing a description, two formal elements involved and, to put it in the early Wittgenstein’s words, ‘Russell’s merit is to have shown that the apparent logical form of the proposition need not be its real form.’¹⁸⁸ The purpose of TD is to reveal the *real* form of a sentence containing a description by assigning that sentence with an alternative rendering that embodies its real form in its explicit, *surface-level* structure. This alternative rendering is claimed by Russell to reveal the structure of the proposition expressed.

7.2 The core part of Theory of Descriptions

TD, as Russell sets it forth in *OD*, amounts to a set of eliminative rules according to which any semantic analysis of a sentence, if that sentence contains one description or more, has to proceed. In what follows, we adopt the order of Russell’s exposition by beginning with that portion of the theory’s subject matter which leaves ordinary definite descriptions for a later analysis. The exposition of the eliminative rules begins in the paper as follows:

185 In the last section of this chapter, I explicate a reason why we should establish, on Russell’s behalf, a talk about the semantical contribution of a description in spite of the fact that descriptions are deemed incomplete symbols.

186 See the outset of Chapter III of PM.

187 We will see in Section 7.5 that the contribution of a description can be represented by an open sentence.

188 Wittgenstein (1921), 4.0031.

I take the notion of the *variable* as fundamental; I use “ $C(x)$ ” to mean a proposition [fn. 2] in which x is a constituent, where x , the variable, is essentially and wholly undetermined. Then we can consider two notions “ $C(x)$ is always true” and “ $C(x)$ is sometimes true”. [fn. 3] Then *everything* and *nothing* and *something* (which are the most primitive of denoting phrase) are to be interpreted as follows:

$C(\text{everything})$ means “ $C(x)$ is always true”;

$C(\text{nothing})$ means “‘ $C(x)$ is false’ is always true”;

$C(\text{something})$ means “it is false that ‘ $C(x)$ is false’ is always true”.

Here the notion “ $C(x)$ is always true” is taken as ultimate and undefinable, and the others are defined by means of it. [fn. 4]¹⁸⁹

At first glance, ‘ $C(x)$ is always true’ looks cryptic, but this is just another way of expressing ‘ $(x) C(x)$ ’, where C is usually a predicate.¹⁹⁰ ‘ $C(x)$ is false’ is Russell’s awkward way of putting ‘ $\sim C(x)$ ’. As we noted at the outset of Chapter 3, ‘is false’ and ‘is true’ were used by Russell to signify a presence of negation (as applied within the scope indicated) and an absence of negation respectively.

Russell assumes the apparatus of the propositional logic (his equivalent of the standard modern sentential logic) and introduces the notion of an individual variable and the general quantifier as undefinables. The semantic analysis of ‘everything’, ‘nothing’ and ‘something’ is captured by the triad of rules that can be expressed using the modern logical notation as follows:

$C(\text{everything})$	means	$(x) C(x)$
$C(\text{nothing})$	means	$(x) \sim C(x)$
$C(\text{something})$	means	$\sim(x) \sim C(x)$

189 *OD*, 480.

190 In any properly general statement of the theory, $C(x)$ is any truth-functional context free for x . For brevity, we stick to Russell’s examples which replace C by a predicate, e.g., by ‘is human’. In Russell’s application of TD, $C(x)$ is allowed to represent intensional contexts such as ‘George IV. wished to know whether Scott was x ’ as well; in such cases however, the scope must be decided in order to semantically disambiguate the sentence analysed. See Landini (2006), pp. 194-200.

With the existential quantifier defined as usual, the last formula can be written in this familiar way: $(\exists x) C(x)$.

The real form of a sentence containing a description is quantificational. Each of ‘everything’, ‘nothing’ and ‘something’ signifies a specific quantifier within the scope of ‘ $C(x)$ ’ is embedded, provided C is free for x and x is the variable of the quantifier.

The simplicity of ‘everything’, ‘nothing’ and ‘something’ rests in the fact that these are *pure* quantifiers. In our ordinary, every day use of English we barely encounter these words as pure quantifiers, as we restrict implicitly (and more or less vaguely) our everyday applications of predicates to various sets of entities. When I state that everything makes me angry, as long as I intend to make a believably true statement, I am concerned, in my application of the predicate ‘makes me angry’, with the things that matter to me to a great extent, i.e., the things I eagerly list to anyone who dares to angry me by pointing out some things that should please me. So, technically the eliminative rules provided so far apply to a rather rare and artificial use of the words listed above. Importantly enough, empirical facts about our use of descriptions (or, more precisely, sentences containing them) like this one are beyond the scope of Russell’s theory. At most, such facts are relevant when we are concerned with a fully comprehensive application of TD to a particular language (English).

When it comes to carrying out the task of setting forth the theory, it makes perfect sense to begin with the cases in which a predicate is applied, within a *general* statement signaled by “everything”, “nothing” or “something”, onto an *unrestricted* domain, regardless of how unusual this is in our everyday linguistic practices. This approach adopted by Russell in *OD* makes perfect sense as far as it reveals Russell’s eliminative analysis in its core: it shows how the Contextualization Principle, ‘the principle of the theory of denoting I [= Russell, MS] wish to advocate,’ is implemented. From the semantical point of view, descriptions are quantifiers!

Surprising as it might sound, Russell’s semantics of sentences containing the unrestricted use ‘everything’, ‘nothing’ and ‘something’ exhausts TD in its core part. The rest of *OD* elaborates on two aspects of the core part: **(1)** a detail to be specified concerning an application of TD to a natural language (English, in our case) and **(2)** its validity as an idea that helps to provide compelling answers to certain philosophical

problems and its validity as a scientific theory *simpliciter*. We will proceed in accordance with this plan:

(1) An account of how the restriction of a predication domain is incorporated into TD conceived in its application to English

- qualitative restriction
- quantitative restriction

(2) A defense of TD as a valid theory

The last section of this chapter is devoted to (2). The scope of the discussion will be narrowed down so as to continue our explanation of how TD introduced, for Russell, a viable possibility of dispensing with questionable entities such as the present king of France and disastrous entities such as the round circle.

7.3 Qualitative restriction

The strategy Russell adopts in *OD* in order to set forth his new semantical theory is very close to what we have found in chapter 5 of *Principles*. Again, we are supplied with six description types differentiated from each other according to the determiner used: ‘all’, ‘every’, ‘any’, ‘a’, ‘some’ and ‘the’.¹⁹¹ Of course, this division stands as a useful simplification, i.e., an idealization whose accuracy and exhaustiveness with respect to our actual, every day use of English is limited.¹⁹² ‘The formal language and its semantical theory do not provide a naturalistic description of our language in its actual working [...]’, says Leonard Linsky and he continues: ‘In providing an idealized model, abstraction is made from *certain* features of actual language, such as ambiguity and vagueness, that are

191 See Chapter 3 of this essay.

192 For instance, the determiner ‘most’ as used, e.g., in ‘Most expats in Prague earn more than average’, is omitted in Russell’s account. Inquiring into different uses of one and the same determiner are also beyond the scope of Russell’s main interest of giving an account of TD.

not regarded as *essential* to the working of language [...].¹⁹³ What Linsky (discussing Frege and Russell) means by ‘the working of language’ is of course quite specific, and generally any particular abstraction made as part of one’s language-theoretic enterprise is subject to possible refutations depending on how we take the language to work. This is in fact admitted by Linsky. It is clear, nonetheless, that the fact itself that *some* abstractions must be made is beyond any objection and no theory should be criticized for doing so. Russell’s strategy in *OD* is one of finding balance between the following three goals: **(a)** The account of the theory must be rich enough to reveal in full its expressive power. **(b)** The account of the theory should be as minimal as possible in order to prevent conflation and confusions as much as possible. **(c)** The extent to which the account of the theory follows our use of English dictates its degree of plausibility and comprehensiveness for the reader as a competent user of English; therefore, any desirable account of the theory - construed in an application to an idealized version of English - should not depart from our actual use of English excessively.

‘The Theory of Descriptions has enormous expressive power,’ remarks Neale adding immediately that Russell’s theory can elegantly deal with what he calls ‘relativized descriptions’ such as ‘each girl’s father’ and provide sentences like ‘Every man loves the woman who raised him’ with accurate semantical values.¹⁹⁴ On the one hand, Russell’s account of TD should capture that ‘enormous expressive power’. This is required in (a). On the other hand, (a) should be met without lengthy debates on linguistic questions like those concerning how various kinds of descriptions are construed and used in this or that natural language, English in our case. This can be put aside to a great extent, and it should be. Generally, clean-cut stipulations are in order.

As we include (on top of ‘everything’, ‘nothing’ and ‘something’) those noun phrases that are construed by joining one of Russell’s five determiners with a concept word (what he called in *Principles* words standing for class concepts), the eliminative rules of TD are *extrapolated* so as to include the following rules:

193 Linsky (1983), xxxiv-xxxv. (my emphasis).

194 Neale (1990), 35.

“C(all men)” means “‘If x is human, then $C(x)$ is false’ is always true”.

“C(no men)” means “‘If x is human, then $C(x)$ is false’ is always false”.

“C(some men)” will mean the same as “C (a man),” fn. 2 and

“C(a man)” means “It is false that ‘ $C(x)$ and x is human’ is always false”.

“C(every man)” will mean the same as “C(all men)”.¹⁹⁵

Russell is thus finally free from the confused considerations that led him in *Principles* to separate ‘all’-phrases from ‘every’-phrases and ‘a’-phrases from ‘some’-phrases. Although it is admitted that there are differences in use between the phrases of these types, these are deemed irrelevant. To remind ourselves, Russell’s separation in *Principles* of ‘all’-phrases from ‘every’-phrases rests on a dubious assumption that the number predicates such as ‘are two’ function semantically as first-order predicates; needless to say, this is surprising (if not appalling) if we consider that, in *Principles*, Russell puts forward the orthodox logicist construction of cardinal numbers as sets of sets.¹⁹⁶ With *OD* published, this inconsistency seems to be a history to be forgotten.

As emphasized above, Russell can reasonably abstract from facts such as the one that our use of a ‘some’-phrase presupposes, in some cases, that the associated predication is restricted to more than one entity.¹⁹⁷ Again, facts such as this one are highly relevant to those who take up the specific task of developing, with the help of TD, *complete* semantics of a particular language (natural language semanticists)—a task that is beyond Russell’s *philosophical* interests.

Due to Russell’s methodological decision to abstract from the differences between our usage of ‘all’-phrases and ‘every’-phrases, and our usage of ‘some’-phrases and ‘a’-phrases, the five rules listed above are reduced to this compact list of three:

$C(\text{all } a / \text{every } a)$ means $(x) (x \text{ is } A \supset C(x))$

$C(\text{no } a)$ means $(x) (x \text{ is } A \supset \sim C(x))$

¹⁹⁵ *OD*, 481.

¹⁹⁶ For a detailed account of Russell’s construction of cardinal numbers in *Principles*, see Landini (1998), Chapter 1.

¹⁹⁷ See Russell’s remark on this point in *IMP*, 171.

$$\begin{aligned}
C(a \text{ } a / \text{ some } a) & \quad \text{means } \sim(x) (x \text{ is } A \supset \sim C(x)) \\
& \quad (\forall x) \sim(x \text{ is } A \supset \sim C(x)) \\
& \quad (\forall x) (x \text{ is } A \ \& \ x \ C(x))
\end{aligned}$$

The inclusion of a concept word schematized as ‘*a*’ in a description (where ‘is *A*’, in our notation, is a related predicate joined with a copula) is linked in a systematic way to the fact that whenever we assert any of ‘*C*(all *a* / every *a*)’, ‘*C*(no *a*)’ and ‘*C*(*a* *a* / some *a*)’, we make a general claim represented by the general quantifier, negated in the third case, about *C* under the restriction on the domain determined by *A*; thus in asserting that every *a* is *C*, we are saying about *C* that it is satisfied in all cases in which *A* is satisfied; in asserting that some *a* is *C*, we are saying about *C* that it is not satisfied in all cases in which *A* is satisfied, and so on.

The new rules map in fact onto the same region of meaning which is mapped by the triad of rules discussed in the previous section. So, what is the difference? It lies in the fact that the present rules comprehend a method of restricting *qualitatively* the extension of the domain to the members of which a predicate is applied. This is an essential step towards deploying TD successfully in describing the semantics of English. This qualitative restriction marked in English (and many other languages) by the use of a concept-word does not, however, belong to the core of TD.

7.4 Logical fictions: A refutation of Gödel’s interpretation

In *Introduction to Mathematical Philosophy* [IMP] (1919), Russell opens his semantical account of ‘the’-phrases, i.e., *definite* descriptions, (both in the singular and plural) by the following, often-cited passage:

[W]e shall consider the word ‘the’ in the singular, and in the next chapter we shall consider the word ‘the’ in the plural. It may be thought excessive to devote two chapters to one word, but to the philosophical mathematician it is a word of very great importance: like Browning’s grammarian with the enclitic δε, I would give

the doctrine of this word if I were ‘dead from the waist down’ and not merely in prison.¹⁹⁸

To a philosophical mathematician, says Russell, ‘the’ is a word of very great importance. Russell is here referring to his logicist doctrine here. The reformed, mature version of it inaugurated by his joint publication, in 1910, of *Principia Mathematica*, Vol. 1, [PM] with N. A. Whitehead endowed the quantificational treatment of ‘the’-phrases with a unique dimension of philosophical importance. The logicist doctrine of *PM* is established based on an idea that our cognition operates with so-called ‘logical fictions’—this is tied up with the quantificational treatment of definite descriptions, but before we take up this topic in a greater detail, we need to acknowledge that the *Principia*’s logicism elaborates on a presumption that logic is strictly general. As Landini emphasizes, the formal language of Russell’s logic developed in *PM* does not contain any individual constant.¹⁹⁹ Russell himself puts this point as follows:

[I]t is part of the definition of logic [...] that all propositions [of logic, MS] are completely general, *i.e.* they all consist of the assertion that some propositional function containing no constant terms is always true.²⁰⁰

Containing ‘no constant terms’, logic is purely formal (in a very different sense, however, from what modern mathematical logicians²⁰¹ mean when they say that the language of *a* logic is formal). As Russell and Whitehead contend that all knowledge of unapplied mathematics is capable of being formulated entirely by means of the language of their logic, the following question naturally arises: provided logic is purely formal, how can we by its means, and by its means only, establish any discourse about or knowledge of mathematical entities such as numbers, sets, spaces etc.?

198 *IMP*, 167.

199 Landini (2010/2011), 217.

200 *IMP*, 159.

201 Indeed, Russell’s ‘philosophical mathematician’ is a whole different breed from the modern mathematical logician.

Russell's and Whitehead's solution does not lie in TD *qua* a semantical theory whose explanative power rests in a reductive analysis. It lies in TD when it is considered in the *opposite* direction (synthesis), i.e., not from descriptions to quantifiers, but from quantifiers to descriptions.

By applying the idea of the reductive semantical analysis of TD backwards, so to speak, Russell and Whitehead developed a notion of synthesis. Already in *Principles*, Russell acknowledged that often we have knowledge about something, where the subject of our knowledge is beyond the reach of our acquaintance. For the early Russell, this was, e.g., a distant historical figure or an infinitude of natural numbers.²⁰² In such cases, our knowledge is descriptive and its content general. And the intentional character of it, i.e., the fact that the case of descriptive knowledge is *about a certain thing / things that it is / they are* such and such, is grounded, in a way, in its general nature. For instance, we know about Napoleon Bonaparte that he was a soldier, but as we have never been acquainted with him, the aboutness feature of our epistemic state about him is, roughly speaking, general in its essence. Of course, the early Russell envisaged the generality of a piece of knowledge such as this one in a way specific to his then theory of denoting. However, the insight involved, i.e., that our knowledge about something can be - and due to the limited reach of our acquaintance often is - *descriptive-general* in its nature, lies as for its validity beyond disputes between all alternative semantical analyses of descriptions. Russell's idea of a logical fiction came about as a way of expanding on that insight, in the particular context of giving an account of logical/mathematical knowledge. If we know about a set that it is empty, our knowledge in such a case is also general. The difference is, however, that Napoleon (if he really existed, which is what we know with some high degree of probability) is an entity, while, in Russell's and Whitehead's view, no set is.²⁰³ Sets are logical fictions. We cannot name them in Russell's strict sense of the term 'name'. We

202 See Chapter 2; Thanks to the no-classes theory, the Russell of *Principia* does not take sets to be, so neither does he take numbers or their infinitude to be. In *Principles*, Russell lacks the no-classes theory, we must take him to hold, at that time, to a realist position about classes, and by the same token about numbers as well.

203 This refers to Russell's position around 1910. Later on, Russell appropriated his eliminativist method typical of his *Principia* logicism to an application in empirical sciences such as physics (material objects) and psychology (minds).

cannot be acquainted with them.²⁰⁴ Sets, numbers and other entities of non-applied mathematics are certainly not entirely fictitious in the way Cinderella is. They are constructions to the effect that, although they are legitimate subjects of our knowledge, our knowledge about them is general and purely formal its essence, and as such it is absent of an ontological commitment to them.

A note deserves to be made about a temptation of a peculiar sort to misrepresent Russell. Let us begin with stating the foundation of Russell's eliminative method. His logical atomism (viewed as a research programme rather than a metaphysical theory) is best characterized as being established on the heuristic maxim, which is elucidated in the following excerpt from Russell's 1918 lectures:

When some set of supposed entities has neat logical properties, it turns out, in a great many instances, that the supposed entities can be replaced by purely logical structures composed of entities which have not such neat properties. In that case, in interpreting a body of propositions hitherto believed to be about the supposed entities, we can substitute the logical structures without altering any of the detail of the body of propositions in question. This is an economy, because entities with neat logical properties are always inferred, and if the propositions in which they occur can be interpreted without making this inference, the ground for the inference fails, and our body of propositions is secured against the need of a doubtful step. The principle may be stated in the form: "Wherever possible, substitute constructions out of known entities for inferences to unknown entities."²⁰⁵

For Russell, this embodies Occam's razor. Now, notice carefully that 'entities with neat logical properties *are always inferred*, [...];' (my emphasis) for instance, natural numbers

²⁰⁴ We cannot be acquainted with Napoleon Bonaparte as well. In his case, however, the impossibility rests upon empirical reasons. Had someone invented a time machine, a possibility to get acquainted with the French ingenious leader would emerge. To put it in a more technical language, any statement that someone cannot be acquainted with a set (number, space etc.) is, in Russell's and Whitehead's view, analytically true. This is vital to my argument in the second part of this section, which presents a refutation of Gödel's view of Russell's notion of a logical fiction.

²⁰⁵ *LA*, 130.

as determined by Peano's set of five axioms are entities with neat logical properties. What is actually inferred are statements that articulate an existential import: from a premise, e.g., that 2 is the smallest prime number, we infer that there is an x such that x is the smallest prime number, i.e., that there is an abstract individual called '2'. Russell's method consists in a *refutation* of such inferences by replacing the statements apparently about numbers by statements that are not about any individuals at all. The concept of the number as an entity, however, is not entirely disregarded. It is, at the end of the day, a theory of mathematics what is proposed. The number as an entity is constructed. The result of the construction is something we might call a 'pseudo-entity,' or more simply, in Russell's own words, a logical fiction. As we will immediately see, this point is prone to be badly misunderstood.

In his (1946), Gödel acknowledges Russell's method when he says that 'when he [Russell, MS] started on a concrete problem, the objects to be analyzed (e.g., the classes or propositions) soon for the most part turned into "logical fictions".'²⁰⁶ And throughout the first several pages of his paper, Gödel seems to realize that Russell's logical fictions were meant to *refute* ontological commitments. Unfortunately, he is not consequential in that respect as evidenced by the continuation of his statement quoted earlier; we read: 'Though perhaps this need not necessarily mean [...] that these things do not exist, but only that we have no direct perception of them.'²⁰⁷

What does 'these things' in Gödel's statement denote? Clearly, the denotation is the objects to be analyzed (e.g., the classes or propositions), but, in general, all logical fictions—the 'supposed objects,' in Russell's jargon of his 1918 lectures. What Gödel states, then, is that Russell need not necessarily mean that logical fictions do not exist, but only that we have no direct perception of them. That's staggering. That logical fictions do not exist is *true by definition!* It is, therefore, perfectly necessary that they do not exist and any talk about whether we can have a direct perception (acquaintance) with them or not is not only beside the point, but wholly contradictory.

Perhaps we should take another look at what Gödel tells us and consider whether his differentiation between the 'objects to be analyzed' and 'logical fictions' is merely verbal, or not. Let us suppose it is not. This line of reading Gödel suggests that he takes

206 Gödel (1946), 127.

207 Ibid.

Russell to first acknowledge certain objects and then to develop a way of talking about them which is free from an ontological commitment to them; they are *replaced* by logical constructions. This view makes Russell's logical constructions into subjects of a correspondence theory of truth. **(1)** The objects to be analyzed (such as classes etc.) are acknowledged in some pre-theoretical manner. **(2)** Our talk about them is then turned into a talk about logical fictions. **(3)** A possibility is admitted that the initial pre-theoretical acknowledgement of the objects, while not in itself strong enough to be capable of providing us with ontological commitments, could have been an acknowledgement of *actual* entities, where whether this was so or not we don't know, and possibly cannot ever know.

The second and more charitable reading at best represents Gödel's attempt to bring Russell's eliminativism closer to a possibility of being reconciled with mathematical platonism, but it by no means represents Russell's position itself. First of all, the heuristic method of Russell's logical atomism is *linguistic through and through*. It does not serve to analyze objects, but sentences that are only *prima facie* about certain objects. These are paraphrased so as to reveal their semantical values,²⁰⁸ by means of which it is, in the same breath, revealed that their aboutness feature is *apparent*: and so are the objects about which they apparently are, hence the *supposed* objects. Second, logical fictions/constructions cannot be subject to a correspondence theory of truth, as they are part of what is essentially a *façon de parler*. Only the formulas of the language of *Principia* are subject to a theory of truth, and these are, as we have emphasized above, purely formal in that they contain logical constants and bound variables only.²⁰⁹

²⁰⁸ We should speak of '*trans-semantical*' values of the sentences analysed insofar as Russell's eliminative analysis aims to replace pre-theoretical (semantical) contents by theoretical contents construed by means of logic, within the field of logic alone (logicism) as well as within a framework of extra-logical scientific theory such as physics.

²⁰⁹ In a similar vein, Landini refutes Ayer's interpretation that is based on a view that Russell's (and Whitehead's) logical constructions should be understood in terms of reductive identity. Taking classes as an example, Landini concludes his argument as follows: 'Ayer is mistaken. *Principia* enables one to emulate the results of a simple type-theory of classes without embracing an ontology of classes. This is not a reductive identity (i.e. the identification of classes with attributes of a certain sort). Classes do go the way of the gorgon sisters.' Landini (2010/2011), 116.

7.4 The word ‘the’ in the singular

Let us now transition from the importance of the semantics of ‘the’-phrases for Russell as a proponent of a version of the foundational-logicist approach to mathematics to the semantics of ‘the’ itself. In *OD*, we read:

“Scott was the author of Waverley" (i.e. "Scott was identical with the author of Waverley") becomes "One and only one entity wrote Waverley, and Scott was identical with that one" ; or, reverting to the wholly explicit form: "It is not always false of x that x wrote Waverley, that it is always true of y that if y wrote Waverley y is identical with x , and that Scott is identical with x ".²¹⁰

Let us replace ‘ $x = \text{Scott}$ ’ by a schematic symbol, ‘ $C(x)$ ’, where x is free for C , the predicate ‘wrote Waverley’ by ‘ D ’ and ‘author of Waverley’ by ‘ d ’. Then, we have:

$C(\text{the } d)$ means $(\exists x) (D(x) \ \& \ (y) (D(y) \supset y = x) \ \& \ C(x))$

Considering this equivalent formula: ‘ $\sim(x) ((D(x) \ \& \ (y) (D(y) \supset y = x)) \supset \sim C(x))$ ’, we can acknowledge that Russell’s semantical theory of ‘the’-phrases is a modified semantics of ‘a’/‘some’-phrases. In addition to the *qualitative* restriction indicated by the concept word ‘ d ’ which follows ‘the’ in the description, there is a *quantitative* restriction requiring ‘ $D(x)$ ’ to be satisfied by exactly *one* entity.

The proposed division between two restrictions, one qualitative, the other quantitative, can be made explicit by means of formulating Russell’s semantical analysis of definite descriptions in the way that was adopted by Russell in his writings on the subject after *OD*. ‘The author of Waverley was Scotch’ is Russell’s example to be analyzed in *IMP*. The sentence is associated with this list:

- (1) At least one person wrote Waverley.

²¹⁰ *OD*, 488.

- (2) At most one person wrote Waverley.
 (3) Whoever wrote Waverley was Scotch.²¹¹

‘All these three,’ Russell explains, ‘are implied by “The author of Waverley was Scotch.”’ Conversely, the three together (but no two of them) imply that the author of Waverley was Scotch.’²¹² As G. E. Moore pointed out, the verb to imply must have been used, in the passage cited, to state a fact about entailment, not a mere fact that something is implied in the sense of material implication.²¹³ The connection of mutual entailment between ‘The author of Waverley was Scotch’ and the joint conjunction of (1), (2) and (3), however, is *not* a point belonging to TD. The connection Russell is interested in as long as his intention is to set out in *IMP* a portion of the theory originally presented in *OD* (that is, as long as he is a semanticist) must be *semantic*. Such a connection is pointed out by Russell when he says that the joint conjunction of (1), (2) and (3) ‘may be taken as defining what is *meant* by “The author of Waverley was Scotch.”’²¹⁴

The joint conjunction of (1), (2) and (3) amounts, according to Russell, to a correct definition of the meaning of ‘The author of Waverley was Scotch.’ (1) and (2) are both addressing the question *how many* individuals wrote Waverley. We also have the *quality* of being an author of Waverley. (1) and (2) jointly state of that quality that it is satisfied by exactly one entity. A *quantity* is stated here.

This uniqueness condition can easily be replaced by the condition of satisfying the quantity of a couple, triple or any other quantity. It can also be replaced by a combination of several different conditions of this sort. Suppose a detective investigates a murder of which he is persuaded it was committed either by two individuals or by three. His claim

211 Our usage of (3), as G. E. Moore argues in his (1946), is such that in an assertion of (3) the speaker presupposes that there was someone who wrote Waverley, i.e. the truth of (3) presupposes the truth of (1). Russell does not use (3) in this standard way. Moore suggest to reformulate (3) as to match Russell’s intention as follows: ‘There never was a person who wrote Waverley and was not Scotch.’ See G. E. Moore (1946), 180.

212 *IMP*, 177.

213 Moore (1946), 181-2 and 187. Against Moore’s objection, it can be said that ‘ $(F)(G) ((\exists x)((y)(Fy \equiv y=x) \& Gx) \supset (\exists x)(Fx) \& \sim(x,y)(Fx \& Fy \& x \neq y) \& (x)(Fx \supset Gx))$ ’ is a theorem of the formal theory which is used in the semantic analysis of TD (the logic of *PM*). Clearly, the main connective of the theorem is material implication.

214 *IMP*, 177. (my emphasis)

that the murderers must have received a military training would, upon TD, receive the following ‘Russellian’ analysis:

$$(\exists x) (\exists y) (M(x) \& M(y) \& y \neq x \& (\exists z) (M(z) \supset ((z = x) \vee (z = y)))) \& T(x) \& T(y) \vee$$

$$(\exists x) (\exists y) (\exists z) [M(x) \& M(y) \& M(z) \& y \neq x \& y \neq z \& x \neq z \& (\exists w) (M(w) \supset ((w = x) \vee (w = y) \vee w = z))] \& T(x) \& T(y) \& T(z),$$

where the property of being an individual who committed the murder in question is represented by ‘ M ’ and the property of having received a military training by ‘ T ’.

Due to the expressive power of the second-order predicate logic with identity, the possibilities are countless. An infinitude of various ways to restrict the application of a concept numerically are available. We can also replace the condition of uniqueness by the condition of emptiness, which yields a contradictory formula. This possibility is, of course, entirely abstract and cannot be found in the actual use of descriptions in our every day linguistic practices.

7.5 The semantic contribution of a description

Let us reiterate the main principle of TD in a particular way adopted by Russell and Whitehead in the third chapter of the first volume of *Principia Mathematica*. We can schematize ‘Barrack Obama is bald’ and ‘The first African American president of the US is bald’ in the respective order as follows:

- (i) baldness(Barrack Obama)
- (ii) baldness(the first African American president of the US).

Taking ‘Barrack Obama’ as a genuine proper name (in Russell’s technical sense), the semantical contribution of the subject expressions in (i) and (ii) differ *in kind*. The contribution of ‘Barrack Obama’ to the meaning of (i) is its bearer, the person called ‘Barrack Obama’. By this, Russell means to imply that the bearer of ‘Barrack Obama’ is a

constituent of the proposition expressed by (i).²¹⁵ Besides bearing the name ‘Barrack Obama’, Barrack Obama is also the entity satisfying the description in (ii); in Russell’s parlance, he is the denotation of that description. However, the description does not contribute its denotation to the proposition expressed by (ii). So, what does it contribute to the proposition?

By the Contextualization Principle, the description in (ii) is deemed ‘incomplete’ and an eliminative rule is applied to (ii) in order to determine the proposition expressed. This poses a question: in which sense can we, in our interpretation of Russell, talk about the semantical contribution of a description? The phrase ‘semantical contribution’ seems to suggest something constitutive with regard to the meaning, i.e., an entity to be contributed to the proposition expressed. The application of TD, however, seems to have its point in taking a description not to contribute any entity to the meaning of the sentence in which it occurs. So, after all, shouldn’t we, in our interpretation, refrain from any talk about the semantical contribution of a description?

My answer is ‘no’. Surely, descriptions do not contribute their denotations to the propositions expressed. That does not mean, however, that no entities are contributed by them! To put this point differently, the Russellian semantical characterization of a definite description is not exhausted by claiming that the description needs to be analyzed away.

So, again, what is the contribution of a description to the meaning of the sentence in which it occurs? For expository purposes, let us separate the grammatical subject from the grammatical predicate in (i) and (ii). Since the grammatical predicate is identical in the two sentences, we get just three elements:

(i-s) Barrack Obama

(ii-s) the first African American president of the US

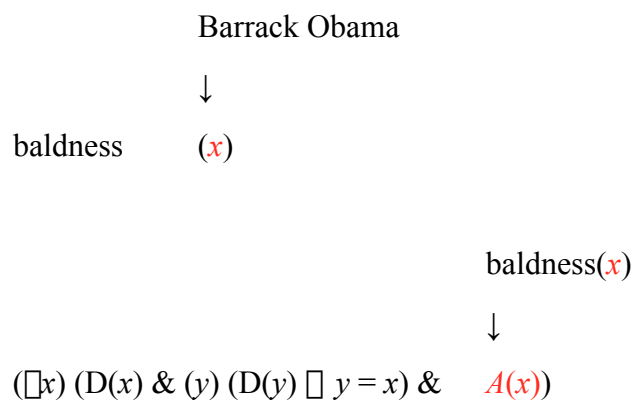
(p) baldness(*x*)

215 In 1910, Russell adopted, in his ‘On the Nature of Truth and Falsehood’, a view that not only descriptions, but also propositions are incomplete symbols. This view appears also in the first volume of *Principia* and we can find it tentatively suggested in Section 3 of the paper titled ‘On the Nature of Truth’ (1906). This is Multiple-Relation Theory of Judgment. TD surely presupposes some notion of a proposition, but not necessarily any of Russell’s versions of the Multiple-Relation theory. In the present, we therefore neglect this specific take on the notion of a proposition.

Now, the point of TD can then be then formulated in this way: There is an asymmetry in the way how the grammatical subject and grammatical predicate contribute to the meaning of the whole sentences; while (i-s) contributes a constituent to the proposition expressed, which is embedded in what is contributed by (p), (ii-s) contributes to the proposition expressed a constituent in which what is contributed by (p) is embedded. This is can be demonstrated once we reveal that the real structure of (ii) is quantificational. If we provide (ii) with a proper Russellian analysis and replace the predicate constant ‘*P*’ by a schematic letter, say ‘*A*’, we get:

$$(ii)' (\exists x) (D(x) \ \& \ (y) (D(y) \supset y = x) \ \& \ A(x)),$$

where ‘*D*’ represents the property of being an African American & holding the presidency of the US & having no African American predecessor in holding the presidency, and where *A* is free for *x*. The proposition expressed is, in both cases, determined by a sentential expression that results from completing (p), i.e., by means of combining (p) with some supplement. The difference between the two completions is a difference in kind. We schematize them as follows:



The predicate professes incompleteness in the sense that it needs something other than itself to form a sentence. That can be effected in two ways: in the first case above, a name is substituted into the argument place of baldness(*x*); in the second, the predicate itself is an argument substituted into an argument place of a function-expression. The first case yields (i), the second yields (ii). We can speak of

particularizing completion

vs.

generalizing completion.

Considering the variable ‘ x ’ in ‘baldness(x)’, we can say that, in the first case, the variable is determined by substitution, while in the second it is bound by a quantifier—in our rendering of (ii), this is ‘ \exists ’.

When Russell characterizes descriptions as incomplete, saying that they are devoid of meaning unless considered in the context of a whole sentence, he does not preclude the possibility that every description provides a sentence in which it occurs with a determinate semantical contribution. In *IMP* he takes ‘I met a unicorn’ as an example and says:

[I]t is only what we may call the concept that enters into the proposition. In the case of “unicorn,” for example, there is only the concept: there is not also, somewhere among the shades, something unreal which may be called “a unicorn.” Therefore, since it is significant (though false) to say “I met a unicorn,” it is clear that this proposition, rightly analysed, does not contain a constituent “a unicorn,” though it does contain the concept “unicorn.”²¹⁶

The concept (indicated by the word) ‘unicorn’, not an entity which may be called ‘a unicorn’, enters into the proposition expressed by ‘I met a unicorn’. As I read it, the subject matter of the sentence is the concept rather than a thing that belongs to the extension of it. Where the property of being met by me is G and the property of being a unicorn is F (this is Russell’s concept ‘unicorn’), ‘I met a unicorn’ receives, in accordance with TD, the following quantificational rendering:

$(\exists x) (G(x) \ \& \ F(x)).$

216 *IMP*, 168.

The semantical contribution of ‘a unicorn’ is determined by the function $(\exists x) (A(x) \& F(x))$, where ‘ A ’ stands for any truth-functional context free for x . So, the concept ‘unicorn’ is the subject matter of ‘I met a unicorn’ in this way: To state ‘I met a unicorn’ is, from the semantical point of view, to state that there is something that jointly satisfies $F(x)$ and $A(x)$, where $A(x)$ is determined as $G(x)$ (i.e., ‘I met x ’). This takes us right back to the discussion of Section 5.6.

7.6 The Intentionality Rule

In the fifth chapter, we reconstructed the puzzle of negative existentials as based on a piece of inference we called ‘M1’. M1 (or an equivalent of it) forced Russell to adopt a distinction between existence and being (subsistence) and, in consequence, to allow entities like the present king of France or the golden mountain to be in some sense. In the sixth chapter we saw that an analogous piece of inference, called ‘M2’, threatened to populate the early Russell’s ontology with contradictory objects. In Section 5.6, we briefly examined the strategy of avoiding M1 which became available when Russell adopted TD. Thanks to TD, Russell could readily adhere to the principle we called the ‘Intentionality Rule’, while avoiding the consequence of embracing possibilia. This is something he could not achieve while he was defending his Frege-style semantics of denoting concepts! The Intentionality Rule reads as follows:

(IR): Every sentence is meaningful if and only if there is an x such that one can use that sentence to make a statement about that x , i.e., one can use that sentence to assert something of x or deny something of x .

With TD endorsed, Russell still adheres to this principle, and he applies it to sentences that do not contain descriptions as well to those that do. When we return to $(\exists x) (G(x) \& F(x))$ as a formula embodying the *real* form of ‘I met a unicorn’, then in stating this sentence, we state, from the *semantical* point of view, that $G(x) \& F(x)$ yields a true sentence for at least one value of x . In a concluding remark, I would like to say two things about this aspect of Russell’s mature semantics.

First, the phrase ‘from the semantical point of view’ is crucial, since without making use of it, Russell’s interpretation of what it means to state a sentence with a description might seem highly artificial, if not outright ludicrous. One might object that, whenever a description is used in the usual way, we don’t seem to talk about concepts or functions; it does not seem, when ‘I met a unicorn’ is used in the usual way to make a statement, that the speaker asserts of $G(x) \ \& \ F(x)$ that it is true for at least one value of x or, to use an equivalent idiom, that he/she asserts of the concepts G and F that they are jointly satisfied by something. Against this, it suffices to say that Russell’s semantics, and more importantly also his application of it in the field of mathematics and other branches of science such as psychology or physics, does not aspire to affirm all of our pre-theoretic conceptions. It might well be argued that the progress of science often consists in showing that our pre-theoretical conceptions are wrong—that they appear on a thorough analysis contrary to the facts or to be incoherent. And by this I do not necessarily refer to groundbreaking discoveries such as Einstein’s theory of relativity. Consider something more mundane, e.g., a phonetic study of language. We often find surprising facts about how we actually use language in speech, some of them entirely new to a layman, some contrary to a layman’s pre-theoretical conception of language use.

Second, if we return to M1, we find that (IR) is implemented in M1 in a way that is not compatible with TD. This is the rendering of (IR) I used in my reconstruction of the puzzle of negative existentials:

- (1) If ‘ $\ulcorner A$ does not exist \urcorner ’ is meaningful, then $(\exists y)$ (we deny existence of y if we assert ‘ $\ulcorner A$ does not exist \urcorner ’) [an instance of (IR)],

where ‘ A ’ is a singular term, a name or description. Let us replace ‘ A ’ with ‘the present king of France’. Applying TD on ‘the present king of France does not exist’, (1) receives the following analysis

- (1)’ If ‘ $\sim(\exists x) (F(x) \ \& \ (y) (F(y) \ \square \ y = x))$ ’ is meaningful, then $(\exists y)$ (we deny existence of y if we assert ‘ $(\exists x) (F(x) \ \& \ (y) (F(y) \ \square \ y = x))$ ’),

where F is the property of being a present king of France. (1) is faulty. If we assert ‘ $(\exists x)(F(x) \ \& \ (y)(F(y) \supset y = x))$ ’, what we do not deny existence of some entity; we state that for every value of x either $F(x)$ is false, or $(y)(F(y) \supset y = x)$ is. As long as TD is endorsed, existence (in the sense in which we say that the present king of France and the golden mountain do not exist) *is not a first-order predicate*.²¹⁷ Any adoption of (IR) alongside TD must operate under this condition.

7.7 ‘On Denoting’ against Meinong(ianism)

In the last portion of this section, we will briefly look at two arguments Russell proposed in *OD* against Alexius Meinong. Both of them are presented very early in the paper, both are today regarded as being unjust to Meinong and applicable to the early Russell’s position represented by *Principles* rather than to Meinong’s. As we will see, there is some substance to this view. It is not our task, in this essay, to do justice to Meinong, since this would make the essay fairly much longer. Nonetheless, especially in connection with the first argument which, in my view, accords to the version of the puzzle of negative existentials presented in Chapter 6, I will suggest a direction in which a defense of Russell against Meinong could orient itself.

7.71 Argument I

Russell’s first mention of Meinong in *OD* follows immediately after the eliminative rules of Theory of Descriptions are set forth. A naive semantical theory of descriptions is attributed to Meinong and it is criticized in a passage whose content is rather cryptic and condensed; the passage reads:

This [Meinong’s] theory regards any grammatically correct denoting phrase as standing for an *object*. Thus “the present King of France,” “the round square,” etc.,

²¹⁷ See Landini (2002), 197.

are supposed to be genuine objects. It is admitted that such objects do not *subsist*, but nevertheless they are supposed to be objects. This is in itself a difficult view; but the chief objection is that such objects, admittedly, are apt to infringe the law of contradiction. It is contended, for example, that the existent present King of France exists, and also does not exist; that the round square is round, and also not round; etc. But this is intolerable; and if any theory can be found to avoid this result, it is surely to be preferred.²¹⁸

At first glance, it seems that Russell confuses descriptions with their denotations. However, when he says, on Meinong's behalf, that 'the present king of France' and 'the round square' are supposed to be genuine objects, he actually means to say this: the denotation of 'the present king of France' / 'the round square' is supposed to be a genuine object.²¹⁹ In Chapter 1, we discussed Russell's unusual use of grammatical categories in an ontological sense at length. The seeming sloppiness in the passage cited is just another case of this use.

In Russell's view, the denotations of 'the present King of France' and 'the round square' 'are apt to infringe the law of contradiction'. As Suter (1967) puts it, Russell means to say that Meinong's theory 'infringes the law of contradiction, in the sense that a contradiction can be derived from it.'²²⁰ Which contradiction or contradictions? These two are proposed by Russell:

(c1) The existent present King of France exists & the existent present King of France does not exist.

(c2) The round square is round & the round square is not round.

218 *OD*, 482-3.

219 A clearer way of putting it would be by saying that, according to Meinong, the descriptions in question do denote (have denotations).

220 Suter (1967), 513.

Let us begin with (c2). The general idea at work here is that we have such a definite description, D , that D denotes d and, from the *sole* fact that D denotes d , it follows that

$$d \text{ is } P \ \& \ \sim(d \text{ is } P),$$

where P is a property. When $D =$ ‘the round square’, Russell takes Meinong’s theory to imply that ‘ d is round & $\sim(d$ is round)’. In chapter 4, section 4.3, we explored the core part of this argument, making clear that if contradictory objects like the round square are admitted into the realm of what there is, contradictions such as ‘ d is P & $\sim(d$ is P)’ follow. We established that any theory which entails an ontological commitment to contradictory objects is at variance with the Law of Contradiction. As far as Meinong is concerned, the issue comes, of course, with theory-laden phrases such as ‘admitted into the realm of what there is’ and ‘an ontological commitment’.

Suter (1967) reminds us, in his defense of Meinong, that Russell failed to acknowledge Meinong’s separation of subsistence (being) from objecthood. ‘The trick is to distinguish between “being an object,” “subsisting,” and “existing.” These words are not regarded as synonymous by Meinong.’²²¹ A similar objection to Russell’s interpretation of Meinong is presented by Jacquette who says:

Russell foists the confusing being-predication thesis onto Meinong. He misinterprets Meinong as attributing Being even to objects that Meinong expressly says neither exist nor subsist, that are beingless and lack being (Sein) in any sense of the word.²²² (Ibid., 177)

Subsistence or being is the all-encompassing ontological category which, as opposed to the narrower ontological category of existence, includes both entities, those that obtain in time and those that are in some sense, but do not obtain in time. For example, facts subsist, but do not exist. The present queen of England exists, whereas the property of being a queen

221 Suter (1967), 515.

222 Jacquette, J. (2005), 177.

merely subsists. And then there is objecthood. In Meinong's view, objecthood surpasses subsistence. Contradictory objects and possibilities do not subsist. They are not in any sense of the verb *to be*.²²³ In Meinong's parlance, they are beyond being (*Ausserseinde*).

The denotations of 'the present king of France' and 'the round square' are, according to Meinong, being-less objects. Surprisingly perhaps, Russell acknowledges this! When talking in the passage cited above about the present king of France and the round square, he admits that, in Meinong's view, 'such objects do not *subsist*, but nevertheless they are supposed to be objects.' Everything then seems to boil down to how objecthood is to be envisaged.

Meinong's position is based on the idea that it is possible to separate *Sosein* (being thus-and-so) from being. Meinong speaks of *das Aussersein des reinen Gegenstandes* (the extra-being of the pure object). Possibilities and contradictory objects fall into this *phenomenological* category. Russell's objection, it must be admitted, is somewhat dogmatic, as we cannot find in *OD*, as well as in Russell's writings that followed, any direct argument against Meinong's specific views on objecthood. Whenever Russell returns to Meinong, he resorts to remarks about Meinong's lack of the robust sense of reality and the like. This is already present, in a milder form, in the passage cited above when Russell says of Meinong's theory that '[t]his is in itself a difficult view,' most probably meaning by 'difficult' something like 'outright counter-intuitive'. Thinking that Meinong's views are outright counter-intuitive, Russell thought, I believe, that no direct argument against them is necessary. That does not mean, however, that no such argument is possible. The argument which Russell could have proposed is by nature a transcendental one.

If Meinong's *Gegenstandstheorie* purports to explain the semantic nature of singular terms, it aims at providing us with a true description of the relations between singular terms and their meanings, the relation between the meaning of a singular term and the meaning of a whole sentence in which the singular term occurs and so on. Now, every such description presupposes that we can quantify over the domain of the constituents of the

223 More precisely, they are not in any ontic sense of the verb *to be*, provided we leave open whether any non-ontic sense of the verb is possible. We will see that Russell's standpoint against Meinong consisted in treating the verb as incapable of having a non-ontic sense.

semantic facts described. So, we can quantify over the domain of singular terms as well as over the domain of their meanings. As long as the round square is the meaning of ‘the round square’, it is something that has being in the sense of being the value of a bound variable! This is an indisputable fact unless we want to claim that Meinong’s *Gegenstandstheorie* does not purport to explain the semantical nature of singular terms. If we want to claim this, Russell is clearly the winner of the debate. So, let us suppose that Meinong’s theory does purport to explain the semantic nature of singular terms. Now, being ontologically committed to an entity is nothing else than acknowledging the being of that entity in the minimal, Quinean-Russellian sense of the being the value of a bound variable. Meinong’s theory, therefore, cannot escape an ontological commitment to objects like the round square. And such a commitment, as was shown in Chapter 4, is inseparable from an infringement of the Law of Contradiction.

Furthermore, Meinong would certainly agree that the meanings of singular terms can be counted. For example, he would agree that the meanings of ‘the round square’ and ‘the present king of France’ make a couple. Unfortunately, this is at odds with an ontological commitment to contradictory entities, as such a commitment makes it impossible to construe arithmetics. The validity of arithmetics is presupposed by Meinong’s theory as long as it treats the meanings of singular terms as countable.

We conclude, on Russell’s behalf, that Meinong’s *Gegenstandstheorie* must either (a) resign from construing a semantics, (b) dispense with its ontological commitment to contradictory entities, or (c) admit that its discourse is beyond the validity of the Law of Contradiction and arithmetics; the last option is equal to admitting that the discourse of Meinong’s theory is unintelligible.

7.72 Argument II

So far we have discussed ‘the round circle’ and (c2). Let us now proceed to ‘the existent present King of France’. Russell says that, if we apply Meinong’s theory to ‘the

existent present King of France’, ‘the existent present King of France exists, and also does not exist’.²²⁴ That is, (c1) is derivable:

(c1) The existent present King of France exists & the existent present King of France does not exist.

This can be misleading. Unlike ‘the round square’, ‘the existent present King of France’ does not describe a contradictory object! The derivation of (c1), then, must proceed in a way different from what we have seen in the case of ‘the round square’. Let us, again, call the description and its denotation ‘*D*’ and ‘*d*’ respectively. Then, the first conjunct of (c1) comes from the fact that *D* denotes *d*, but the second is a contingent fact which is logically independent of the fact that *D* denotes *d* and which is the case because France is a republic nowadays.

We saw that (c2) came *in its entirety* from the fact that ‘the round square’ denotes a contradictory object which is both round and square. This is the main point of difference between Russell’s two examples.

How do we derive the first conjunct of (c1) from the fact that *D* denotes *d*? First, we need to acknowledge that, for Meinong (as well as for the Russell of *Principles* as we saw in Chapter 5), ‘existent’ means ‘obtaining in time’ or ‘actual’. The predicate ‘present’ requires that the denotation is located in a temporal region which includes the moments that make up presence. We start with an instance of the principle associated by Russell with Meinong’s naive semantics. We have:

(1) *D* is meaningful if and only if *D* denotes *d*.

Since *D* is meaningful, we infer that

(2) *D* denotes *d*.

²²⁴ *OD*, 483.

From the fact that D denotes d , we can infer the following triad of characterizations of d :

(3a) d obtains in time (is actual).

(3b) d is located in a temporal region which includes the moments that make up presence.

(3c) d reigns France.

Satisfying the condition of actuality involved in (3a), d is not a possible. It is an actual thing. We have, therefore, this:

(4) d exists (obtains in time, is actual).

Since France is a republic nowadays, Russell's contradiction, (c1), is derivable. Let us call this argument M4.

M4 differs significantly from the argument concerning 'the round circle'. Russell does not indicate this difference, but he does not preclude it either. By being too terse he is inarticulate about this point. Suter, as we saw, appeals in his defense of Meinong, to the difference between objecthood, subsistence (or being) and existence. Before we assess the relevance of this distinction for any successful reply to M4, it should be noted that Suter's reading of the passage concerning 'the existent present King of France' differs from ours. He takes Russell to propose the following argument:

"The present King of France" is a grammatically correct denoting phrase; hence there must be an object which is the present King of France. But we all know that no present King of France exists in 1965; even De Gaulle is not literally King of France. Consequently, we must conclude that, on Meinong's theory, the present King of France both exists and also does not exist.²²⁵

225 Suter (1967), 514.

Russell is taken by Suter to argue that Meinong's theory leads to the consequence that 'all negative existential statements must be false because self-contradictory.'²²⁶ So, Suter identifies the argument concerning 'the existent present King of France' with an instance of M1 from Section 5.5 of this essay. This is a misinterpretation.

In our discussion of M1, we saw that Russell appeals in *Principles* to the distinction between subsistence (or being) and existence. If we make a true judgment that the present king of France does not exist, then, according to the early Russell, we truly deny the existence in the sense of the actuality of something which, nevertheless, is admitted to be (to subsist). In Meinong's case, we truly deny the existence of something which neither has existence, nor has it being, but which, nevertheless, is admitted to be an object. To an extent, this is the same maneuver. Russell's statement, in the passage cited, that the present king of France and the round square 'do not *subsist*, but nevertheless [...] are supposed to be objects', makes it clear that he knew Meinong's solution to the puzzle of negative existentials. How could he, then, possibly accuse Meinong of having no solution to it?

Russell does not in fact accuse Meinong of lacking a solution to the puzzle of negative existentials, as the interpreted passage is not concerned with this puzzle at all. Suter misinterprets Russell. This is clear once we realize Suter's unexplained ignorance, in his interpretation of the passage, of the epithet 'existent' as included in the definite description used by Russell. If we want to read Russell correctly, we need to take seriously that Russell used 'the *existent* present King of France', not 'the present King of France'. In M4, this fact is taken seriously. It remains to ask whether Meinong's trick of distinguishing between 'being an object,' 'subsisting,' and 'existing' is of any help in evading M4.

The second conjunct of (c1) is a contingent fact which can hardly be disputed, so, if Meinong's trick is to be of any help, its role must consist in breaking the inference leading towards the first conjunct of (c1), i.e., line (4) of M4:

(4) *d* exists (obtains in time, is actual).

226 Ibid.

To remind ourselves, d = the existent present king of France. In defense of Meinong, we could, perhaps, say that d is a mere possibile which is beyond being—it lacks existence and subsistence, but it is an object. If this is the case, (4) is not derivable. But, in the same breath, it would follow that $d \neq$ the existent present king of France, because d as a mere possibile does not satisfy the condition of actuality expressed by ‘existent’ involved in Russell’s description. So, insofar as we are concerned with the denotation of ‘the existent present king of France’, we must endorse that d is an entity satisfying the condition of actuality. It follows, d cannot be beyond being; it cannot be a mere possibile. Meinong’s trick is, therefore, of no help in evading M4.²²⁷

227 In the literature on Meinong, it is suggested that Ernst Mally’s distinction between nuclear and extra-nuclear properties could have helped Meinong to cope with such arguments as M4. The solution would consist in taking existence as an extra-nuclear property of things. The general idea would be to establish, in one way or another, that ‘existent’ in ‘the existent d ’ is deflationary. For more details, see, e.g., Jacquette (2015), Chapter 5.

Conclusion

It has been conclusively established that W. V. O. Quine's interpretation of Russell's ontological development is correct in describing the effect that the famous theory presented for the first time on the pages of 'On Denoting' (1905), Theory of Descriptions, had upon Russell's views on what there is. It was a revolutionary change of views. As for Russell's earlier position in *Principles*, we can safely state that there is some textual evidence showing that Russell wanted to dispense with possibilia such as the present king of France, that being so before he discovered Theory of Descriptions. The crucial question, however, is this: Could Russell succeed, before he discovered Theory of Descriptions, in providing solid grounds for his desired ontological austerity? Our investigation into the role of aboutness (intentionality) in Russell's thought confirmed that the resources of the semantics endorsed in *Principles* did not provide Russell with a real possibility of dispensing with possibilia. Moreover, we saw that the same reason that led Russell to introduce a distinction between existence and being (subsistence), as he needed to find room for accommodating possibilia among what there is, precluded him from avoiding a disastrous view that contradictory objects have being. We devoted one whole chapter to demonstrating in detail that admitting contradictory objects was not an option for Russell. From this, it follows that Russell's semantical thought was, after the publication of *Principles* in 1903, in need of a substantial revision.

The Theory of Denoting Concepts, the predecessor of Theory of Descriptions, does not lead to Meinongian ontological commitments in the straightforward way a naive semantics of singular terms (i.e., the semantical theory of the sort Russell attributes to Meinong) does, as it assigns descriptions with denoting concepts rather than with their denotations. This is a result we can take to be correctly established by the previously discussed opponents of Quine, Graham Stevens and David Bostock, whose interpretation of the early Russell we criticized in Chapter 5. The opponents of Quine over-interpret this result, going too far in their conclusions when they propose that the Theory of Denoting Concepts was as good in providing grounds for a possibilia-free ontology as Theory of Descriptions was. Considered in the wider context of Russell's semantical thought, the

Theory of Denoting Concepts can be seen as *partially* responsible for the early Russell's temptation to allow for Meinongian ontological commitments.

The threat of Meinongianism, especially, then, of that part of it which concerns contradictory objects, was not the only good reason for a substantial revision of Russell's semantical thought developed in *Principles*. There were several of such reasons, some of them being omitted in this essay. Russell's paradox (in its various incarnations) and the related struggle to avoid it by means of the so-called substitutional theory of propositions or the theory of types (and orders) is a good example of the omitted ones. We did, however, discuss some problems of Russell's early semantical views that lie, to a considerable extent, beyond the debate over the semantics of descriptions, which was our central topic.

In Chapter 1, we explored the elements of the early Russell's realist position, both its semantical-ontological and methodological aspects. We made a considerable effort to elaborate on the notion of a Russellian proposition, as we needed to distinguish Russellian propositions clearly from the special kind of propositions that were supposed to serve as meanings of sentences containing descriptions. This distinction was necessary in order place Russell's semantical theorizing about descriptions properly within the context of his overall philosophical thought. And we discovered that the perhaps seemingly innocuous concept of a Russellian proposition that formed the basis of the 'strictly pluralist' ontology of *Principles* was far from being innocuous. Facing Bradley's regress on one side, a version of Frege's 'concept horse' paradox on the other, Russell was unable to provide a coherent account of the propositional unity (the unity of a Russellian proposition). Russell was stuck between Scylla and Charybdis, but admittedly he was not aware of the whole difficulty due to a repeated failure to grasp and, therefore, fully appreciate Bradley's regressive argument against external relations. Moreover, as we demonstrated, the early Russell's theory of propositions leads to an unacceptable view that every Russellian proposition is true. In the last section of Chapter 6, this problem was revealed as relevant to our main topic, that is, to the debate concerning the early Russell's tendencies towards Meinongianism. The faulty theory of Russellian propositions can be counted as a unique source of Meinongian ontological commitments.

The Theory of Denoting Concepts was in fact a very tentative proposition on Russell's part, one which can be seen to lie in the intersection between Frege's semantics

of *Sinn* and *Bedeutung* and a naive semantics of the type Russell associated with Meinong's *Gegenstandstheorie*. We saw in Chapter 3 that, except for the analysis of definite descriptions, the theory was built upon a questionable idea that denoting concepts denote entities of a peculiar sort called 'combinations'. Russell apparently struggled to separate properly the intensional aspect of a description from its extensional aspect, conflating these in his notion of a combination. The problems plaguing the Theory of Denoting Concepts we identified in Chapter 3 make the theory inferior to the theory of 'On Denoting' regardless of whether it is was (co-)responsible for the early Russell's tendencies towards Meinongianism or not.

Is Meinongianism that bad? By all means it is clear that in this essay I promote, rather indirectly though, a school of philosophy which operates under the imperative associated with Russell's philosophy after the discovery of Theory of Descriptions, that is, the imperative urging us to reduce ontological commitments as much as possible. The reduction of ontological commitments specific to the Russellian approach consists in replacing categorematic terms with syncategorematic ones construed by means of the machinery of Theory of Descriptions. In this, I follow Prof. Gregory Landini who sees this methodological orientation of Russell's philosophical programme (logical atomism) as leading to an interesting sort of structural realism, and also to placing philosophy in the context of sciences that has proven fruitful. As for Meinongianism, our investigation into it was tied up with the goal of reconstructing Russell's semantical-ontological views before 'On Denoting'. Within this constraint, we can conclusively affirm that Meinongianism is indeed a very bad option. Alexius Meinong's theory of objects must be distinguished from the early Russell's realism. Meinong held to a view that it is possible to separate *Sosein* (being thus-and-so) from being, speaking of the 'extra-being of the pure object'. This is incompatible with the fundamental assumption of Russell's philosophy, later adopted by Quine, that to be is to be the value of a bound variable. The Russell-Meinong antithesis boils down to the status of this core assumption and the related notion of objecthood and existence.

In his early book titled 'Methods of Logic' (1950), Quine rather briefly examines what an error theory of Meinongianism could be. 'The mistaken view that the word 'Cerberus' must name something in order to name anything,' says Quine, 'turns [...] on

confusion on naming and meaning.’²²⁸ In other words, the ‘mistaken view’ turns on conflating the intensional aspect of a singular term with its extensional aspect. Another error theory suggested a bit earlier in the book is that Meinongianism may arise due to conflating what is named with the idea that we entertain when we know what the object designated by the name is. We read:

‘Parthenon’ names the Parthenon and only the Parthenon, whereas ‘the Parthenon-idea’ names the Parthenon-idea. Similarly, not ‘Cerberus’, but ‘the Cerberus-idea’, names the Cerberus-idea; whereas ‘Cerberus’, as it happens, names nothing.²²⁹

So, according to the second of the error theories, Meinongianism may arise due to conflating the intentional aspect of our use of a singular term and the extensional aspect of the singular term used. As long as we link in a substantial manner the semantical notion of intensionality with the psychological notion of intentionality, the two error theories might actually form one.

I have suggested the hypothesis that the first of the error theories can explain what I take to be the main failure of Russell’s Theory of Denoting Concepts. I am talking about the dubious notion of a combination. I believe that, in Chapter 3, this hypothesis was shown to be correct, although, admittedly, a more thorough investigation is necessary in order to arrive at a sufficiently rigorous analysis of the matter. The exegetical part of such an investigation should include a comprehensive study of Russell’s manuscripts containing his experimentation in the period of 1903-4 with the Theory of Denoting Concepts that eventually led to discovering Theory of Descriptions.

A similar concern should apply to Meinong’s unorthodox notion of objecthood based on an idea that being-thus-and-so is independent of being, as this core assumption of *Gegenstandstheorie* seems *prima facie* to result, in the context of semantics, in an outrageous conflation of extensionality with intensionality. The issue does not lie, it must be said, in assuming that the intensional aspect of a singular term (being-thus-and-so) is existentially independent from being (the extension). We are familiar with this in Frege’s

228 Quine (1950/1966), 200.

229 Ibid., 199.

mature semantical theory which can hardly be taken to embrace Meinong's notion of objecthood. The issue with Meinong seems to lie in treating the intensional aspect of a singular term as a posit that can function as if it were its extensional aspect whenever Meinong needs it in order to maintain the universality of his intentionality thesis, i.e., in the contexts in which we truly deny the being of something. This looks like a case of magical thinking, one of believing in the myth that it is possible to turn water into wine. Yet, there are scholars who believe that Meinong's theory goes in the right direction, since it builds upon the undeniable fact that we can think and talk about entities of which it is true that they do not have being. This brings us back to Quine.

Quine's two error theories which we discussed above do not exhaust his critical opinion of Meinongianism. As evidenced by the following passage, Quine was clearly aware of the role that the notion of aboutness plays in Meinongian strategies of dealing with the puzzle of negative existentials. We read:

[T]he view [= Meinongianism, MS] is encouraged also by another factor, viz., our habit of thinking in terms of the misleading word 'about'. If there is no such thing as Cerberus, then, it is asked, what are you talking *about* when you use the word 'Cerberus' (even to say that there is no such thing)? [...] The remedy here is simply to give up the unwarranted notion that talking sense always necessitates there being things talked about.²³⁰

Quine suggests that we can avoid Meinongianism, or more precisely: one prominent reason to adopt it, by refuting the principle we have dubbed in Chapter 5 the 'Intentionality Rule'. Interestingly enough, Russell did not have to give up this principle when he finally dodged Meinongian ontological commitments. The unique feature of Theory of Descriptions is that the ontological austerity achieved by means of it does not necessitate giving up the principle that earned such a notorious reputation due to its use in Meinongian strategies to cope with the puzzle of negative existentials. It is an important outcome of the research presented in this essay that the Intentionality Rule alone should not be treated as a source of Meinongian ontological commitments. Only when coupled with a particular style of

230 Ibid., 200-1.

semantics of singular terms, exemplified by Russell's Theory of Denoting Concepts, does this rule end up giving grounds for the unwanted ontological commitments.²³¹

When Russell adopted Theory of Descriptions, he did preserve the rule that 'talking sense always necessitates there being things talked about,' but he did not do that in a way that affirms the commonplace usage of 'about'. To remind ourselves, 'I met a unicorn' is not, according to Russell in 'On Denoting' and later, about a magical animal, but, technically, it is about a concept; and so is, e.g., 'I met Cerberus'. The commonplace usage of 'about' is, accordingly, deemed irrelevant with regard to the precondition of sentential meaning whenever Theory of Descriptions is applied. This is a view on the matter that lies in between Quine's more sweeping standpoint and the standpoint of those who favor Meinong(ianism).

Theory of Descriptions certainly does not depend on preserving the Intentionality Rule and, presumably, would perfectly survive if we are to refute it. Quine might be right we should refute it altogether. In the continuation of the passage above, we read that the Intentionality Rule springs from essentially the same confusion as the one between meaning and naming: '[N]ow it is confusion,' Quine adds, 'between meanings and things talked about.'²³²

Quine's critical take on Meinongianism is very useful in rising fundamental questions—those that need to be answered in order to bring more clarity into the ongoing Russell-Meinong debate, and, in general, the questions that need to be answered if we want to arrive at a solid, tenable conception of semantics. The distinction between intension and extension, the notions of intentionality or aboutness, naming, meaning—all these and many others, like the notions truth and falsehood, quantification, representation and on, must be put under scrutiny and be properly placed within the complex network of sciences that deal with the working of language and, by extension, within the network of the sciences that deal with the working of mind.

231 That being said, it should also be pointed out that with regard to common names, as opposed to descriptions, Russell's solution to the Meinongian conundrum by means of Theory of Descriptions relies on the associated theory according to which that sort of names of which we admit that they can fail to name something - the names of fictitious figures such as 'Cerberus' being fine examples of these - has to be treated, in a semantical analysis of sentences containing them, as truncated definite descriptions.

232 Ibid., 201.

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Abbreviations:

The Principles of Mathematics = *Principles*

On Denoting = *OD*

The Philosophy of Logical Atomism = *LA*

The Collected Papers of Bertrand Russell, Volume 4: Foundations of Logic = *CP4*

Theory of Knowledge: The Manuscript = *TK*

Introduction to Mathematical Philosophy = *IMP*

Principia Mathematica, Vol. 1 = *PM / Principia*

The Existential Import of Propositions = *EIP*

The Theory of Denoting Concepts = *TDC*

Theory of Descriptions = *TD*