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Supernovae as Standard Candles

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V Praze dne Irena Kotíková

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Abstract: Type Ia supernovae are objects whose luminosity is considered to be the same for every star, so called standard candles. This thesis presents a basic outline of methods of distance measurement in astronomy and focuses on SN Ia. The reason why it is important to know the distances in the universe is the fact, that it is necessary for the measurement of cosmic expansion and its rate of change. The discovery of cosmic expansion had a crucial impact on Einstein field equations and the term Λ, the cosmological constant, that was left out from the equations for almost a century. I summarise the history of measurements of the rate of change of cosmic expansion and the role of SN Ia in it. I also present an overview of current results that imply that the cosmic expansion is accelerating and its consequences for Einstein field equations and cosmological models of the universe. In particular, the reintroduction of the cosmological constant into Einstein's equations.
Keywords: distances in the universe, SN Ia, cosmic expansion, cosmological constant

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Abstrakt: Supernovy typu Ia jsou standardní svíčky — hvězdy, které jsou všechny stejně jasné. V této práci uvádím základní metody pro měření vzdáleností v astronomii, se zaměřením na supernovy typu Ia. Význam měření vzdáleností ve vesmíru spočívá v jejich použití pro měření rozpínání vesmíru a jeho rychlosti. Fakt, že se vesmír rozpíná, výrazně ovlivnil Eisteinovy rovnice pro obecnou relativitu. Konkrétně člen obsahující kosmologickou konstantu Λ, se přestal používat po téměř celé 20. století. Práce obsahuje shrnutí historického vývoje měření zrychlování rozpínaní vesmíru a poukazuje na úlohu, kterou v něm supernovy typu Ia sehrály. Uvádím také přehled nejnovějších výsledků, které ukazují, že se rozpínání vesmíru urychluje. Z toho plynou důsledky pro kosmologické modely vesmíru a Einsteinovy rovnice v podobě znovuzavedení kosmologické konstanty.
Klíčová slova: vzdálenosti ve vesmíru, supernovy Ia, rozpínání vesmíru, kosmologická konstanta

Chapter 1

Distances in the universe

People have always been fascinated by the universe and have always tried to understand it. The way to this understanding has been long and complicated and is not over yet. To be able to discover the properties and history of the universe we need to be able to observe and measure it. The size of the universe and distances between stars are fundamental questions that need to be answered in order to move on to more sophisticated problems.

The universe is very complex and contains many objects with different properties. Some objects have properties that make it possible to find out their distances from the Earth, the Solar System or the Milky Way by a combination of different methods. Such combination is sometimes called the Cosmic distance ladder and makes it possible to measure distances of even the most distant galaxies. This ladder has to begin somewhere and for cosmic distances the first step is the parallax.

1.1 Parallax

The measurement of parallax is a very basic way to determine distances in the nearby universe. It is based on an observation of an object (a star) from at least two different positions. By measuring its change of angular position (against a static background), knowing the distance between the two points of observation and using the right triangle geometry, it is possible to calculate the distance to the object (figure 1.1). Parallax is one of the oldest methods of cosmic distance measurement and it works very well within the Solar System and for nearby stars.

The basic unit of length in the Solar System is the Astronomical Unit which is the mean distance between the Earth and the Sun. The first proper measurement of the astronomical unit was done by Jean Richer and Giovanni Domenico Cassini in 1672 [1]. They used an already existing model of the solar system and calibrated it using the parallax of Mars that they

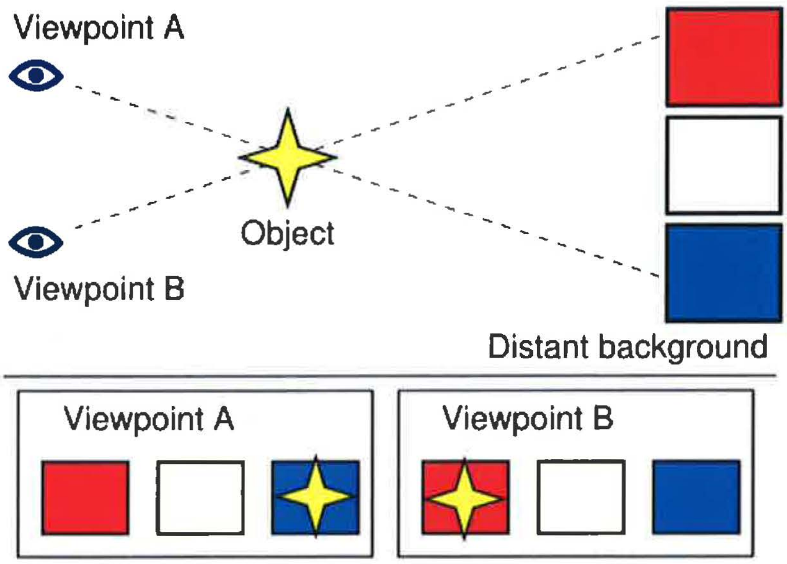

Figure 1.1: Simplified example of parallax measurement, Image from Wikimedia Commons

measured before. The estimated result was 140 000 000 km (the value that we use nowadays is approximately 150 000 000 km).

As I described above, *two* different points are needed to measure parallax. In the case of Cassini and Richter they used different points on the Earth: Richter was measuring in Cayenne, French Guayana in South America and Cassini in Paris. In this measurement the shortest side of the triangle was half the distance between Paris and Cayenne. However, as the distance from the observed object increases, the parallax angle decreases to such an extent, that it becomes almost undetectable. Consequently, such a parallax cannot be used on interstellar scale.

However, there is a way to improve the measurements of parallax. The basic one is to make the base of the triangle bigger — to find two positions as distant as possible. One way to do it is to use Earth's orbit around the Sun. The first measurement would be done in one position and the other six months later. This creates a base of one astronomical unit (which was already known) and therefore a possibility to measure parallax of even more distant objects. This method is called stellar parallax (figure 1.2). In 1838 Friedrich Bessel was the first to use this method to calculate the distance to another star, 61 Cygni. Bessel's result was a distance of about 3 parsecs which only differs by 15% from the present day value of 3.5.

Parallax has proved to be a good and reliable method when we need to measure distances of roughly hundreds of light years. For many decades scientists were not able to improve these methods and thus improve our

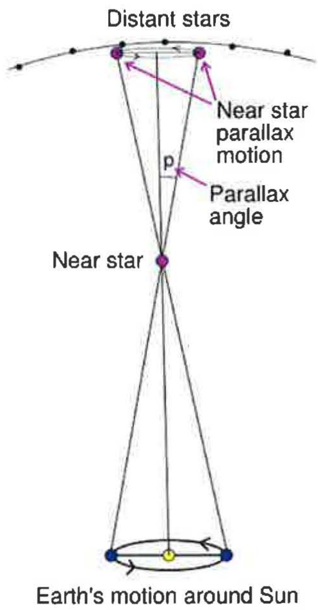

Figure 1.2: Stellar parallax demonstration, Image from Wikimedia Commons

estimation of the size of the universe.

1.2 Cepheid variables

Cepheid variables are variable stars that have a significant correlation between their period and luminosity - the brighter the star, the longer the period. This relation between luminosity and period was discovered by Henrietta Swan Leavitt.

In 1908 she examined hundreds of photographic plates obtained at Harvard College's observatory in Peru. Some stars appeared repeatedly on different plates and she noticed that their luminosity changed with time. She calculated their periods and noticed that brighter variables have longer periods [2]. By plotting the data, Leavitt concluded that there is a clear relation between the brightness of variables and their periods.

Ejnar Hertzsprung realised the importance of Cepheids and tried to calibrate the relation. He knew that Cepheids could be used for distances much bigger than those measured by parallax. His first approximation was:

$$M_v = a + b \cdot \log P, \tag{1.1}$$

where M_v is an absolute magnitude and P is the period in days [3].

This was a good estimate but without concrete values of a and b it would be useless. The problem with the calibration was the fact that there were no Cepheids close enough to be measured by a stellar parallax. Hertzsprung used another improved parallax method — the statistical parallax. In this method the base line of the triangle is the change in the position of the Sun. Its velocity is about 13 km/s resulting in a difference of 2.8 astronomical units per year. Unfortunately, it is not so easy to use this method in reality. Indeed, the Sun does move but so do the objects of observation (Cepheids). That is why the method is called *statistical* because it is necessary to eliminate the peculiar movement of stars and assume their *average* velocity is zero [3].

More importantly, Hertzsprung did not have the best sample of stars to study. In 1940's Walter Baade discovered that there are at least two different types of variable stars. δ Cepheid variables (named after the δ Cepheid star) are population I stars, and W Virginis variables are population II stars. Population I and II differ in the amount of iron and other heavy elements which results in a major difference in their period-luminosity relationship. Hertzsprung tried to calculate the absolute luminosity of W Virginis variables using the formula for δ Cepheid stars. The fact that there were both types of variables in Hertzsprung's data caused a lot of inaccuracy in his computation. In addition, Hertzsprung only had a couple of well observed Cepheids with measured periods. The combination of all these problems resulted in a very poor result [3]

$$M_v = -0.6 - 2.1 \cdot \log P \tag{1.2}$$

which is very different from the present day value of [3]

$$M_v = -1.43 - 2.81 \cdot \log P, \tag{1.3}$$

especially in the value of a (a zero point) which is crucial for correct interpretation.

Once we know the absolute magnitude, it is enough to observe the relative luminosity and use the inverse square law to calculate how far the star is. The reason why Cepheids are suitable for larger distances is their luminosity: the brightest Cepheids (with a period of 30 days) are 10^4 brighter than the Sun which makes them observable on the scales of millions of light years.

Hertzsprung's process of calibration was later (1920's) revised by Shapley and Hubble. They managed to improve some parts of the process but some problems still remained (e.g. two not distinguished populations of Cepheids). Nevertheless, the accuracy and precision of this calibration were enough for crucial extragalactic measurements.

Chapter 2

Supernovae — origin and classification

Astronomers have always been fascinated by supernovae — suddenly appearing bright stars that no-one was able to explain. Firstly it was the supernovae in the Milky Way that were observed, for instance by Johannes Kepler (1604), Tycho Brahe (1572) or ancient Chinese astronomers. In the beginning of the 20^{th} century it was possible (due to better telescopes) to observe even more distant supernovae in other galaxies. It was soon suggested (based on independent distance measurement) that they are much brighter than ordinary stars, brighter than any known objects in the universe. During the 1930's they were finally given a name: supernovae. Further observations have shown that there are differences in the type of explosion that occurs, their light curves and spectra. Nowadays we distinguish four major types of supernovae.

2.1 Type Ia Supernovae

SN Ia are believed to occur in every type of galaxy, in a binary system of a white dwarf and an ordinary star (usually a red giant), where the white dwarf accumulates mass from the companion star until it reaches the Chandrasekhar limit of 1.44 solar masses. This triggers a thermonuclear explosion that completely destroys the original white dwarf.

This way of origin makes them perfect standard candles (objects that have about the same luminosity) but they are also useful because of their absolute luminosity (roughly 4.10^9 solar luminosities at the peak), which makes them observable on very large scales. Nevertheless, there are still minor differences in their composition and therefore luminosity.

Nowadays there are many methods that eliminate these differences and provide a solid picture of type Ia supernovae. One of these is the light-curve shape method. It is based on observational data from the supernova — how luminosity changes with time. It takes about 20 days for the supernova to reach its maximum luminosity, it then decreases to 50% in the next two weeks and slowly fades away for 1.5 years by 1% per day.

Even tough the peak luminosity of the light curves is the same, there are some differences in the rate of fading in each supernova. Using the multi-colour light-curve shape method focusing on differences in supernova colour and brightness it was observed that "intrinsically dim SN Ia's are redder and have faster light curves than the bright ones which are slow and blue" [4]. This observation helped to understand the SN Ia and their classification, as well as the measurement of their distances. It was crucial for the standardisation of SN Ia and their consequent use as standard candles.

The spectrum of SN Ia contains little or no hydrogen (unlike SN II).

2.2 Type II Supernovae

SN II are only found in the arms of spiral galaxies, that contain relatively young stars, and there is a different process behind their explosion. Massive stars (more than 9 times solar mass) have the temperature and pressure to create (fuse) heavy elements up to iron. This creates many layers of elements from the surface (with hydrogen) to the core, where the heavier elements are fused. When the process in the core reaches iron, the fusion stops and as soon as the iron core reaches the Chandrasekhar limit, it is no longer able to support the star and collapses, creating a shock wave that throws away the outer layers of the star.

Depending on the mass of the original star the core either collapses to a neutron star (if it is less than 20 solar masses) or to a black hole [5].

SN II have a different spectrum than SN Ia — it contains a lot of hydrogen and some more heavier elements which are rare in SN Ia. The difference in the spectra of SN II and SN Ia is significant and is an efficient tool to distinguish between these two types.

There exists another classification within SN II based on further differences in their spectra. However, as SN II are not the main subject of interest in this thesis I will not elaborate on this.

2.3 Type Ib and Ic Supernovae

SN Ib and SN Ic are despite their name essentially similar to SN II — they are created by gravitational core collapse.

However, the spectrum is very different from SN II and more similar to SN Ia .

For a long time SN Ib and Ic were mistakenly thought to be SN Ia. It was not until 1980's when it became clear that they are fundamentally different objects.

Chapter 3

Expanding universe

3.1 History of discovery

At the beginning of the 20^{th} century it was believed that the Milky Way was all that there was in the universe. It was not possible to observe objects outside of our galaxy, so it was assumed that there were not any. When astronomers measured velocities of the observed stars using the method of red shift in their spectra, they found out that stars were moving neither closer nor away from us — that they were more or less static. These results were later used as a proof of the model of static universe.

Astronomers eventually noticed strange objects that looked like solar systems in formation: the spiral nebulae. In 1913 Vesto Slipher obtained spectra of these spiral nebulae and calculated their velocities from the Doppler shift in their spectral lines. He concluded that almost all spiral nebulae that he observed were moving away from us [6]. Moreover, these velocities were much higher that any of the observed stars in the Milky Way. There was no explanation for these objects to be moving away so quickly. In addition, their distance still remained unknown.

A couple of years later Edwin Hubble used the newly build 100-inch telescope at Mount Wilson to observe the spiral nebulae more thoroughly and possibly find the answer to their origin. He was looking for Cepheids in the spiral nebulae and he was successful. The Cepheids that he located in what we nowadays call M31 were about 100 times fainter than the Cepheid stars that Henrietta Leavitt observed in the Magellanic clouds [1].

Hubble estimated that M31 had to be 10 times further (from the inverse square law) than the Magellanic clouds, therefore not a solar system in formation but a galaxy, just like the Milky Way itself [7].

Moreover, given that Slipher measured velocities and Hubble measured

[1]The Magellanic cloud is a dwarf galaxy very close the the Milky Way which was at that time thought to be a nebula in the Milky Way.

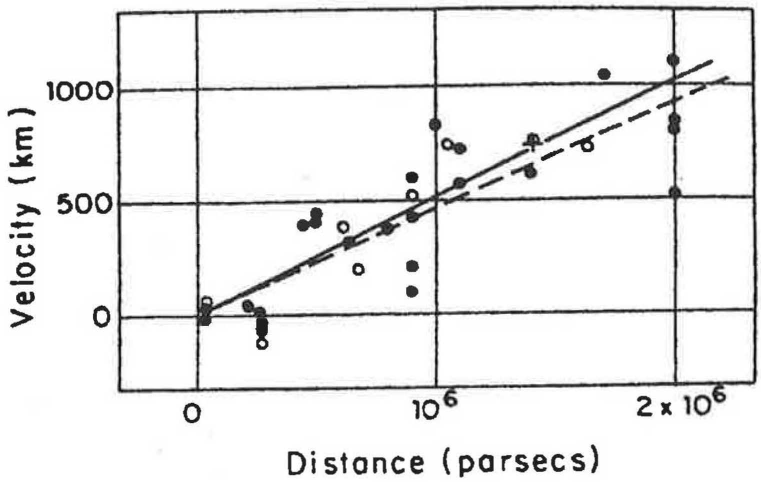

Figure 3.1: Hubble's original diagram published in [7]

distances it was easy for Hubble to plot a graph. The Hubble diagram (figure 3.1) showed that galaxies that were further, moved away faster, than the nearby ones, therefore the universe is expanding [7]. This was a substantial change in the perception of the universe. It was believed to be static for many centuries but now the data showed that velocity is proportional to the distance. If we express this relation in the form of an equation we get the Hubble's law:

$$V = H \cdot D \tag{3.1}$$

where V stands for velocity, D for distance and H represents the relation between them; it is called the Hubble parameter. The Hubble parameter is independent of the spacial coordinates but is a function of time in all viable cosmological models of the universe. It is often called the Hubble constant (H_0), referring to its spacial independence.

Even though Hubble correctly estimated that the universe was expanding his measurements were neither precise nor accurate. He calculated the value of the Hubble constant (which represents the slope of the line in Hubble's diagram) as 528 kilometers per second per megaparsec [7]. Just like Hertzsprung's calibration of Cepheids his result was very different from the present-day value of 71 kilometers per second per megaparsec.

The Hubble constant is also important, because we can use it to estimate the age of the universe. If we assume that H_0 is not time dependent, then we can say that two objects whose distance is now D were separated at the time of

$$t_0 = \frac{D}{v}, \tag{3.2}$$

therefore from the Hubble law (3.1)

$$t_0 = \frac{1}{H_0}. \tag{3.3}$$

If we assume, that the expansion is always decelerating (see Chapter 4.1 for specific models), than the value of t_0 represents the upper bound for the age of the universe.

Hubble's value of H_0 implied that this upper bound was only 2 billion years. At that time it was already known (from radioactive decay) that the age of the Earth was 1.6 – 3 billion years [8]. Hubble's observations were therefore in contradiction with cosmological models assuming constant deceleration of cosmic expansion (see Chapter 4.1). As we will see later, this is the case when the so called cosmological constant is equal to zero.

As I explained in Chapter 1.2 the calibration of Cepheid variables was not very well done at that time but the expansion was such a significant phenomenon that Hubble was able to measure its existence anyway. However, when it came down to calculating H_0, many factors combined to cause his flawed result.

In addition to the mistake in calibration, Hubble made several systematic mistakes. Just like Hertzsprung 10 years earlier, Hubble was not aware of the differences among the variable stars. His data contained some of the population II variables, which besides having a different period-luminosity relation are on average 1.5 times less bright than population I [8]. Moreover, some objects in Hubble's observation were not stars at all — he observed some giant clouds of gas that were glowing because they were ionised [8]. The final problem was not associated with distances but with velocities. To have good data on cosmic expansion one must make sure that the velocities measured are those of the expansion and not peculiar motions of galaxies. This can only be done on very large scales. Hubble was only measuring Cepheids in nearby galaxies where the peculiar motion can have a significant influence on the result.

Although Hubble's data were flawed and full of systematic mistakes, he nevertheless provided sufficient evidence to accept the universe as expanding.

3.2 Consequences of expansion for theoretical physics

When Albert Einstein was working on the General Theory of Relativity his original equations had major consequences for the universe: it was either expanding or contracting. However, the idea of universe at that time was a static one. The reason for this were the observations of stars in the Milky Way that seemed neither to drift away, nor to move closer to us and because the only observable universe at that time was the Milky Way it was only logical to assume that the whole universe was static.

Einstein decided to respect the experimental evidence, changed his initial equations and added a new term to the equations, the cosmological constant (Λ):

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{3.4}$$

where $R_{\mu\nu}$ is the Ricci tensor, R the scalar curvature, $g_{\mu\nu}$ is the metric tensor and $T_{\mu\nu}$ is the stress-energy tensor. The constant $\kappa = 8\pi G/c^4$ (in SI units), where G is the universal gravitational constant and c the speed of light.

He emphasised that he only did so for the sake of the static universe:

The [cosmological] term is necessary only for the purpose of making possible a quasi-static distribution of matter, as required by the fact of the small velocities of the stars. [10]

When Edwin Hubble published his ideas on the expanding universe in 1929, the cosmological constant was no longer necessary. Einstein understood that his initial equations were correct. He abandoned the cosmological term, so did everybody else and the cosmological constant disappeared from Einstein's equations for many decades.

Historically the term containing the "cosmological constant" λ was introduced into the field equations in order to enable us to account theoretically for the existence of a static universe with a finite mean density. It now appears that in the dynamical case this end can be reached without the introduction of λ. [11]

Chapter 4

The rate of change of cosmic expansion — discovery and consequences

Since Hubble's observations in 1929, the expansion of the universe has become a commonly accepted idea. The question was not about the existence of the expansion anymore. It was about its rate of change. Is the expansion constant, increasing or decreasing? Throughout the following decades astronomers and theoretical physicists tried to find the answer to this question.

One way to approach this problem was a theoretical one. Let me outline the method of solving Einstein's equations and interpreting the result in order to find the geometry of the universe.

4.1 Theoretical approach

Firstly, we assume that the universe is homogeneous and isotropic on very large scales (about hundreds of megaparsecs) [1].

These properties of the universe are important because they imply maximum possible spacial symmetry which gives us the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2) \right] \tag{4.1}$$

where k is a curvature constant and $a(t)$ is a scaling factor, which is related to the size of the universe. The value of a(t) is generally time dependent but is normalised to unity for present day.

[1]The universe has actually been confirmed (by the microwave background radiation) to be isotropic up to the order of 10^{-5} which is sufficient for our calculations.

It is important to realise that the equation (4.1) is invariant under the following transformations:

$$k \longrightarrow \frac{k}{|k|}$$
$$r \longrightarrow \sqrt{|k|}r$$
$$a \longrightarrow \frac{a}{\sqrt{|k|}}.$$

Therefore we only have one relevant parameter $k/|k|$ with three possible values, that represent different curvature of space: $k = +1$ corresponds to a positive curvature, which is called closed, $k = -1$ is a negative curvature, which is open and $k = 0$ has no curvature and is therefore called flat.

Having this metric we can find a solution of Einstein's field equations (3.4). Let me follow the argument of [23]. On large scales, matter can be approximated as a perfect fluid which has the energy-momentum tensor

$$T_{\mu\nu} = (p + \varepsilon)u_\mu u_\nu + pg_{\mu\nu} \tag{4.2}$$

where p represents pressure, ε energy density and u_μ is the 4-velocity of the fluid. Because ideal fluids are isotropic in their rest frame, they are at rest in the comoving coordinates. The 4-velocity is therefore

$$u_\mu = (1, 0, 0, 0) \tag{4.3}$$

and consequently, with one index raised, it takes a more convenient form

$$T^\mu_{\ \nu} = diag(-\varepsilon, p, p, p). \tag{4.4}$$

It is useful to find the zero component of the conservation of energy equation:

$$0 = \nabla_\mu T^\mu_{\ 0} = \partial_\mu T^\mu_{\ 0} + \Gamma^\mu_{\mu 0} T^0_{\ 0} - \Gamma^\lambda_{\mu 0} T^\mu_{\ \lambda}, \tag{4.5}$$

where Γ are Christoffel symbols that can be calculated from the metric. As it is quite long and complicated, I will not perform the calculation here. It can be found for instance in [23]. The final result is:

$$\dot{\varepsilon} = -3\frac{\dot{a}}{a}(\varepsilon + p). \tag{4.6}$$

It is also useful to realise that the equation (4.6) can be rewritten as

$$0 = (\dot{\varepsilon a^3}) + p(\dot{a^3}) = \frac{dU}{dt} + p\frac{dV}{dt}, \tag{4.7}$$

where U is the energy in the volume V. This equation represents the first law of thermodynamics for adiabatic process. Cells in the comoving coordinates in a homogeneous universe can therefore be considered adiabatically isolated. It is interesting to realise that Einstein equations implicitly contain laws of conservation of energy and momentum.

The next step depends on the relation between ε and p — an equation of state. For perfect fluids the equation is generally

$$p = \omega\varepsilon \tag{4.8}$$

where ω is constant in time. If we substitute this into the conservation of energy equation (4.6) we find

$$\frac{\dot{\varepsilon}}{\varepsilon} = -3(1+\omega)\frac{\dot{a}}{a} \tag{4.9}$$

which can be integrated to get

$$\varepsilon \propto a^{-3(1+\omega)}. \tag{4.10}$$

This equation depends on the properties of matter that are specified in the equation of state by ω.

One part of the matter in universe is dust. Dust is collisionless, non-relativistic matter whose $\omega = 0$. From (4.10) it is clear that

$$\varepsilon \propto a^{-3} \tag{4.11}$$

which shows how the energy density changes as the universe expands ($a(t)$ increases). It also means that for dust $p \ll \varepsilon$ which will be important later on.

Another part of the universe is radiation which has the equation of state

$$p = \frac{1}{3}\varepsilon. \tag{4.12}$$

If we substitute $\omega = \frac{1}{3}$ into (4.10), then

$$\varepsilon \propto a^{-4}. \tag{4.13}$$

For this reason radiation is only dominant in the early stage of the universe (up to 200 000 years) and its influence is therefore not considered when we describe present day universe.

One other form of matter is considered to be the energy of vacuum represented by the cosmological constant. For Λ the equation of state is

$$\varepsilon = -p. \tag{4.14}$$

This means that $\omega = -1$ and ε is, unlike for dust and radiation, independent of $a(t)$.

If we now take the Einstein field equations for $\Lambda = 0$ (as I explained in Chapter 3.2 this was the general consensus about the cosmological constant during most of the 20^{th} century)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{4.15}$$

and substitute the metric (4.1) and energy-momentum tensor (4.2) we get two equations. The equation for $\mu\nu = 00$ is called the first Friedmann equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\varepsilon + 3p). \tag{4.16}$$

The result for $\mu\nu = ij$ will be:

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G(\varepsilon - p). \tag{4.17}$$

but we can simplify it using the first Friedmann equation (4.16) to get:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\varepsilon - \frac{k}{a^2} \tag{4.18}$$

which is called the second Friedmann equation. Friedmann equations define some key parameters. The rate of expansion is represented by the Hubble parameter:

$$H(t) \equiv \frac{\dot{a}}{a}. \tag{4.19}$$

I already described the Hubble parameter in Chapter 3.1. The Hubble parameter for a particular value of t_0 is called the Hubble constant

$$H_0 = H(t_0). \tag{4.20}$$

The second Friedmann equation (4.18) can also be rewritten using H as

$$\frac{8\pi G}{3H^2}\varepsilon - 1 = \frac{k}{H^2a^2}, \tag{4.21}$$

where ε, H and a are all time dependent.

Because H^2 and a^2 are always positive, the sign of k is determined by the left hand side of the equation (4.21) at any given time. Present day values of ε and H are in principle measurable quantities. If we denote the present day value of ε as ε_0 and H as H_0 than the value of ε_0 for which the left hand side of equation (4.21) is equal to zero, is called the critical density

$$\varepsilon_{crit} = \frac{3{H_0}^2}{8\pi G}. \tag{4.22}$$

It is useful to define the parameter Ω as:

$$\Omega = \frac{8\pi G}{3{H_0}^2}\varepsilon = \frac{\varepsilon_0}{\varepsilon_{crit}}, \tag{4.23}$$

so that from (4.21) follows:

$$\Omega - 1 = \frac{k}{{H_0}^2{a_0}^2}. \tag{4.24}$$

The critical density is called critical because its value in the equations (4.21) and (4.24) determines the sign of k, which as I explained earlier describes the geometry of the universe.

Here are the three possible models of the universe based on Ω [2]:

$\Omega > 1 \Leftrightarrow \varepsilon > \varepsilon_{crit} \Leftrightarrow k = +1$

This model describes an expansion that is significantly slowed down by gravity to such an extent, that it will eventually stop and change into a slow contraction, that will be accelerated by gravity and finally end up with a "Big Crunch".

$0 < \Omega < 1 \Leftrightarrow \varepsilon < \varepsilon_{crit} \Leftrightarrow k = -1$

In this scenario the universe will be expanding at a decreasing rate because the effect of gravity will be monotonously decreasing with the expansion.

$\Omega = 1 \Leftrightarrow \varepsilon = \varepsilon_{crit} \Leftrightarrow k = 0$

This is the case of a decelerating expansion which will asymptotically approach zero and never stop. This model also represents a borderline between the previous models.

As you can see, all of these models are *decreasingly* expanding. The theoretical calculations therefore imply a decelerating universe. Consequently, when the astronomers were trying to measure the rate of change of the expansion they naturally assumed that it will be decreasing based on these cosmological models.

[2]Please note, that we are still working with Λ=0

4.2 Experimental approach

Following Hubble's steps, astronomers tried to measure the rate of change of cosmic expansion. Unfortunately, cosmological observations are complicated and require a necessary combination of quality instruments and theoretical understanding of the problem. Therefore no definite conclusions could be made from the observations until the end of the 1990's. Let me go briefly through the key events that led to the measurement of the rate of change of cosmic expansion using the SN Ia as standard candles.

As I have explained in Chapter 2, supernovae were firstly classified in the 1930's by Fritz Zwicky, but their origin and properties were not known at that time. Moreover, astronomers were not aware of the fact that there were more types of supernovae and therefore were not able to see the standard pattern of SN Ia. An important discovery was made by Minkowski in 1940 when he observed the spectra of supernovae a noticed a clear difference in the spectra of two groups of supernovae [12]. Based on this analysis he split supernovae into two groups: type I and type II (for further explanation see Chapter 2). This distinction definitely brought better understanding of the problem but what was more important was the nature of supernovae. The mechanism behind their explosion still remained unknown.

In the 1960's Willy Fowler and Fred Hoyle were the first to come up with the idea of supernovae being "the same". They studied the nuclear burning of stars with different masses. Knowing that some stars (those with masses of 8 solar masses) end up as white dwarfs they suggested that SN I are nuclear explosions of white dwarfs that have been possibly triggered by added mass from another star in a binary system [14]. Chandrasekhar later calculated this critical mass to be 1.44 solar masses. They also observed heavier stars that seemed to have a different type of death. Stars that are heavier than 8 times the solar mass end their life in a collapse to a neutron star and create SN II [13] (see Chapter 2).

By the late 1970's there already was a theory that suggested the origin of SN I and the data showed that they are all more or less the same which meant that they could be used as standard candles. The problem was with the "more or less" because even though most of the supernovae fitted the pattern, some of them were many times brighter than they were expected to be. This was a major flaw in the theory that made SN I basically useless.

The fact that supernovae are rare (only a couple of them explode in one galaxy during one century) did not help either. What was needed was to find a statistically sufficient sample of supernovae, observe them, get their spectra and analyse them. This was extremely difficult and the astronomers were only able to find a few supernovae in one year.

Major improvement in the knowledge of supernovae came firstly in 1980's when another important distinction was made. Based on even more subtle differences in their spectra SN I were divided into SN Ia and SN Ib (see Chapter 2) and a couple of years later another group of supernovae was distinguished: SN Ic. This majorly improved the situation, for SN Ib and SN Ic were no longer mistakenly included into the calculations.

There are many other problems related to the analysis of spectra from supernovae, for instance their individual differences in luminosity and composition or reddening by cosmic dust. As these problems are complicated and not the primary aim of this thesis, I will not describe their details here. The important thing is that in the beginning the 1990's astronomers were able to minimise their effect and therefore use the spectra of SN Ia with much better precision, as well as accuracy.

During the 1990's it finally all came together. There was a sufficient cosmological understanding of the supernovae explosions and better methods and instruments like the Hubble Space Telescope or the 400-inch Keck telescope. This combination made it possible to find more supernovae in shorter amount of time, get better spectra with more information and finally, to observe more distant supernovae that were far enough to tell us about the expansion of the universe.

Two teams of physicists and astronomers have worked independently during the 1990's to find the answer to cosmic expansion: The Supernova Cosmology Project at Lawrence Berkley Lab and an international High-Z Supernova Search Team. They both combined the most recent developments in the technical field, new methods of analysis and information about the SN Ia. It was The High-Z Supernova Search Team that published their results in February 1998 (SCP reported the same results later in 1999) concluding, that the expansion of universe has been accelerating at least for the last 5 billion years [16] [17].

This was a very surprising result. Based on cosmological models of the universe (see Chapter 4.1), everybody expected the expansion to be slowing down. Many scientists who worked on this problem even called their research "measurement of decelerating expansion" or simply assumed that the cosmic expansion was decelerating [18] [19]. That is why it was such a surprise when the universe turned out to be accelerating.

4.3 Rebirth of Λ

I already mentioned in Chapter 3.2 that Hubble's observations played a key role in Einstein's rejection of the cosmological constant. I also pointed out the contradiction between Hubble's observations and the estimated age of the Earth. The cosmological constant, was sometimes being used to explain this discrepancy by assuming a different model of the universe where

t_0 is not the upper bound (see Chapter 3.1). However, during the 20^{th} century Hublle's observations were improved and they were finally consistent with the age of Earth. Once again, there was no need for the cosmological constant and around the 1990's Λ seemed to be dead.

This changed with the observations of the High-Z Supernova Search Team.

The fact that the expansion is accelerating means that there must be something missing in the cosmological models. There must be some other force that is responsible for the acceleration. This missing element is called the dark energy. Although its nature is unknown, we know its effects. It acts against the force of gravity to accelerate the cosmic expansion. The first idea how to interpret the dark energy was the cosmological constant. Introduced by Einstein 80 years ago as a way to balance the universe to be static, Λ was a natural candidate for a force that works against gravity and accelerates the cosmic expansion.

We have to go back to Einstein's equations (4.15) where we made the assumption that $\Lambda = 0$. Based on the observational evidence we now consider $\Lambda > 0$:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{4.25}$$

which gives us a different set of Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\varepsilon + 3p) - \frac{\Lambda}{3} \tag{4.26}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\varepsilon - \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{4.27}$$

If we again rewrite these equations using Ω we find out that it is not just the value of Ω that determines the geometry of space:

$$(\Omega + \Omega_\Lambda) - 1 = \frac{k}{{a_0}^2{H_0}^2} \tag{4.28}$$

We see that the curvature of space (and therefore a particular cosmological model) does not depend only on the energy density of matter but also on the vacuum energy represented by $\Omega_\Lambda = \frac{\Lambda}{3{H_0}^2}$. From now on I will use the symbol Ω_m to refer to the ratio of energy densities that is determined by the amount of matter in the universe and to Ω as

$$\Omega = \Omega_m + \Omega_\Lambda. \tag{4.29}$$

We now have a parameter that depends on the density of the universe and on its dark energy. It is the sum of those two that determines the geometry of the universe. We can again distinguish 3 possible geometries for 3 different values of Ω just like in Chapter 4.1. However, the rest of the properties (the rate of expansion and ultimate fate) no longer depend on the value of Ω (or at least not explicitly).

Let me go back to the first Friedmann equation (4.26) for $\Lambda > 0$:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\varepsilon + 3p + \frac{\Lambda}{4\pi G}). \tag{4.30}$$

By substituting for p and ε from the equations (4.12) and (4.14), using the property of dust that $\rho \ll \varepsilon$ and assuming that the effect of radiation is negligible we get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\frac{\varepsilon_0}{a^3} + \frac{\Lambda}{4\pi G} - \frac{3\Lambda}{4\pi G}\right). \tag{4.31}$$

where ε_0 and Λ are constant. If we then factorise $1/a$ from the right hand side we get an equation for $\ddot{a}$

$$\ddot{a} = -\frac{4\pi G}{3}\left(\frac{\varepsilon_0}{a^2} - \frac{2\Lambda}{4\pi G}a\right) \tag{4.32}$$

which describes the acceleration of the expansion of universe. We see that there are two forces that influence it - the energy density of dust, which is effective in the early stages of the universe and the dark energy, which acts against it and is on the other hand negligible in the early universe and gets bigger as the universe expands.

The cosmological models of the universe are therefore based on the particular values of ε_0 and Λ. If there is very little dark energy, the universe will collapse before Λ starts to influence it. However, if there is enough dark energy to balance the force of gravity it will eventually prevail and cause an increasing acceleration of the cosmic expansion — just like the observations suggest.

This means that if we were able to look even further back into the history of universe we should be able to observe the decelerating expansion slowed down by the force of gravity.

This theory was confirmed by observation at the very end of the 20^{th} century. From December 1997 to January 1998 the Hubble Space Telescope took many images of the Hubble Deep Field — North. Accidentally, in almost all of these pictures there was a small dot — supernova 1997ff. It was purely by chance that this supernova was in the field of view of the Hubble Space Telescope. It was important because this 1997ff was SN Ia with a redshift of 1.7 which was exactly what was needed to observe the possible cosmic deceleration more than 5 billion years ago. Adam Riess and his team then used these data to calculate the distance and concluded that 1997ff confirms the theory of decelerating expansion in the early universe [20]. However, this was just one case and it takes more evidence to prove

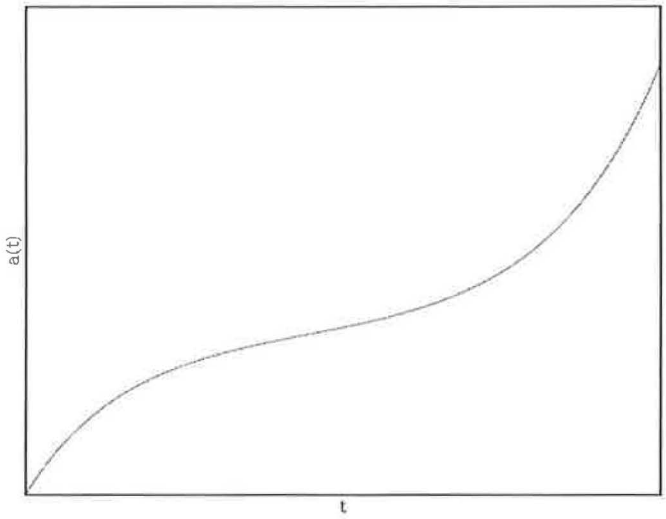

Figure 4.1: An example of a solution of the Friedmann equation (4.32) in arbitrary units

that the theory is right. The High-Z Supernova Search Team continued its work using the HST to look for more supernovae like 1997ff. They were successful and confirmed the early deceleration of cosmic expansion [21]. We now believe that the universe is of the type shown in Figure 4.1.

Let me go back to the equations (4.28) and (4.29) that determine the geometry of the universe. As I explained, the values of Ω_m and Ω_Λ are independent of the cosmic expansion. Therefore, we cannot use the observational evidence from expansion to determine the geometry. In addition there is a problem with the value of Ω_m. It is supposed to include all the matter in the universe which is not as easy as it may seem.

In 1933 Fritz Zwicky was trying to measure masses of galaxies in clusters. Zwicky combined different methods based on the orbital velocities of galaxies or the virial theorem and came to a disturbing result: the masses of galaxies are much larger that their expected masses based on observation of the emitted light [15]. It was obvious that galaxies contain some matter that cannot be seen but has significant gravitational effects. It was given the name "dark matter". Nowadays, with much more sophisticated instruments there has been enough observational evidence to confirm the existence of dark matter and its properties. What is important is that there is much more dark matter than there is the visible one. This is essential for the calculation of Ω_m. When trying to determine the current model of universe dark matter must be taken into consideration.

Overall, it is quite complicated to calculate the values of Ω_m and Ω_Λ and therefore the geometry of the universe. Nevertheless, there are sophisticated methods how to find the geometry of the universe as well as its age. One of them is a measurement of small differences in the cosmic microwave radiation which has been done by many projects like Boomerang, COBE or most recently WMAP. Their results suggest that we live in a universe that is 13.7 billion years old and flat with the values of $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$.

Even though there are still many unanswered questions about the existence of dark energy and consequently cosmological constant, the provided experimental evidence was sufficient to confirm the model of cosmic expansion which was slow at the beginning and later on reached an accelerating stage.

Chapter 5

Conclusions

Type Ia supernovae have proved to be a useful tool for measuring the universe. Their luminosity which is extremely high and almost the same, makes them perfect standard candles. SN Ia played a key role in the measurement of the rate of change of cosmic expansion, which completely changed our perception of the universe and led to the discovery of the dark energy. Nowadays we consider the universe to be accelerating and there has been experimental evidence for deceleration in the early age of universe, which is consistent with theoretical models.

Although, the experimental evidence of the dark energy seems to be indisputable, its interpretation is still not clear. The cosmological constant is just one way to approach dark energy and there are many questions about its nature and origin. Nevertheless, a lot of research is still being done and there is a possibility of even more precise results or perhaps some surprising discoveries in near future.

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